Course: BSO 203 Instructor: S. Raychaudhuri

Session 2002-2003, Semester I

PROBLEM SET VI

Bound States I

1. Consider the potential

$$\begin{aligned} V(x) &= \infty \ \text{ for } \ x < 0 \\ &= 0 \ \text{ for } \ x \geq 0 \ . \end{aligned}$$

   (a) Sketch the potential and discuss what will happen in the classical case.

   (b) Show that it is not possible to have bound states of this potential.

   (c) What physical situation do the scattering states describe?

2. A particle in in the ground state of a box $[-a, +a]$ where $a > 0$. Suddenly the sides of the box are moved to $[-b, +b]$. Assume that the change is practically instantaneous.

   (a) Discuss what will happen in the classical case.

   (b) Write down the energy eigenvalues and eigenfunctions of the old and the new box.

   (c) If the particle was initially in the ground state of the old box, calculate the probability that it will be in one of the eigenstates of the new box.

   (d) What are the differences between the cases $b < a$ and $b > a$?

3. A particle is placed in an asymmetric square well potential of the form

$$\begin{aligned} V(x) &= 0 \ \text{ for } \ x < -a \\ &= -V_1 \ \text{ for } \ -a \leq x < a \\ &= -V_2 \ \text{ for } \ x > a \ , \end{aligned}$$

   where V_1 and V_2 are positive constants and $a > 0$.

   (a) Write the potential in a single line using the θ-function.

   (b) Sketch the potential and discuss what will happen in the classical case.

   (c) Solve the Schrödinger equation using the matrix technique and find the energy eigenvalues and energy eigenfunctions.

   (d) Discuss what happens as E is varied.

4. A particle is placed in a double well potential of the form

$$\begin{aligned} V(x) &= \infty \ \text{ for } \ |x| > a \\ &= 0 \ \text{ for } \ b < |x| \le a \\ &= V_0 \ \text{ for } \ |x| \le b \ , \end{aligned}$$

where V_0 is a positive constant and $b > 0$.

(a) Sketch the potential and discuss what will happen in the classical case.

(b) Solve the Schrödinger equation using the matrix technique and find the energy eigenvalues and energy eigenfunctions.

(c) Discuss what happens in the cases $E < V_0$, $E = V_0$ and $E > V_0$.

5. Use the Frobenius method to solve the following equations:

$$\frac{dy}{dx} - \lambda y = 0 \ . \tag{1}$$

$$\frac{d^2y}{dx^2} + \omega^2 y = 0 \ . \tag{2}$$

$$\frac{d^2y}{dx^2} + 2\beta\frac{dy}{dx} + \omega^2 y = 0 \ . \tag{3}$$