Course: BSO 203 Instructor: S. Raychaudhuri

SESSION 2002-2003, SEMESTER I

QUIZ VI

(5× **1**)

1. Given a normalised wavefunction

$$\psi(x) = \sqrt{2a^3}\ x\ e^{-a|x|}$$

use Schrödinger's equation to obtain the form of the potential $V(x)$.

Answer: The wavefunction is such that at $x = 0$, $\psi(x)$ is smooth, $\psi'(x)$ has a kink, and $\psi''(x)$ has a discontinuity.

For $x > 0$,

$$\begin{aligned} \psi(x) &= \sqrt{2a^3}\ x\ e^{-ax} \\ \Rightarrow \psi'(x) &= \sqrt{2a^3}\left[e^{-ax} - ax\ e^{-ax}\right] = \left(\frac{1}{x} - a\right)\psi(x) \\ \Rightarrow \psi''(x) &= a\left(a - \frac{2}{x}\right)\psi(x) \end{aligned}$$

For $x < 0$,

$$\begin{aligned} \psi(x) &= \sqrt{2a^3}\ x\ e^{ax} \\ \Rightarrow \psi'(x) &= \sqrt{2a^3}\left[e^{ax} + ax\ e^{ax}\right] = \left(\frac{1}{x} + a\right)\psi(x) \\ \Rightarrow \psi''(x) &= a\left(a + \frac{2}{x}\right)\psi(x) \end{aligned}$$

Thus, we have, $\forall x$,

$$\frac{\psi''(x)}{\psi(x)} = a\left(a - \frac{2}{|x|}\right) .$$

Now, Schrödinger's equation leads to the form

$$\begin{aligned} V(x) &= E + \frac{\hbar^2}{2m}\,\frac{\psi''(x)}{\psi(x)} \\ &= E + \frac{\hbar^2}{2m}\,a\left(a - \frac{2}{|x|}\right) \\ &= E + \frac{\hbar^2 a^2}{2m} - \frac{\hbar^2 a}{m}\,\frac{1}{|x|} \end{aligned}$$

Thus, choosing the zero level of potential as $E + \frac{\hbar^2 a^2}{2m}$ and writing $\lambda = \frac{\hbar^2 a}{m}$, the potential assumes the form

$$V(x) = -\frac{\lambda}{|x|}$$

which can immediately be recognised as a one-dimensional Coulomb well.