IITK/PHY103, Problem set-10 Date: March 14, 2015 [DpC/TKG]

1. Consider an infinite slab of conducting material with magnetic susceptibility χ_m, is placed parallel to xy plane, between $z = -a$ and $z = a$. It carries a free volume current density $\vec{J_f}(z) = (J_0 z/a)\hat{x}$, i.e., the integrated current density is zero. Outside the slab is vacuum. Find $\vec{H},\ \vec{B}, \vec{M}$ and the bound current densities $\vec{K_b}$ and $\vec{J_b}$. Plot the field H as a function of z.

2. An iron rod of length L and square cross-section (side a), is given a uniform longitudinal magnetization $\vec{M}$, and then bent around into a circle with a narrow gap (width w). Find the the magnetic field at the centre of the gap, assuming $w << a << L$.

3. Suppose the field inside a large piece of a magnetic material is $\vec{B}_0$, so that $\vec{H}_0 = (1/\mu_0)\vec{B}_0 - \vec{M}$. Find $\vec{B}$ in terms of $\vec{B}_0,\ \vec{M}$, and $\vec{H}$ in terms of $\vec{H}_0,\ \vec{M}$ at the centre of
   (a) a small spherical cavity within the material.
   (b) a long needle-shaped cavity running parallel to $\vec{M}$.
   (c) a thin wafer-shaped cavity perpendicular to $\vec{M}$.

4. A sphere of linear magnetic material of susceptibility χ_m is placed in an otherwise uniform magnetic field $\vec{B}_0$. Find the new field inside the sphere.

Practice problems:

1. A coaxial cable consists of two long cylinders separated by a linear magnetic material of susceptibility χ_m. Current **I** flows through inner cylinder and returns through the outer cylinder with uniform surface current. Find the magnetic field in the region between the tubes. Calculate the magnetization and the bound currents and confirm that they produce the correct field.

2. A short circular cylinder of radius a and length L carries a frozen-in uniform magnetization M parallel to its axis. Find the bound currents. Sketch the magnetic field of the cylinder for (i) $L >> a$, (ii) $L << a$ and (iii) $L \approx a$.

3. A long solenoid of radius R is wound with n turns unit length of wire carrying current I. There is a cylindrical core of iron (susceptibility $\chi_m \approx 10^4$). Find $\vec{B},\ \vec{H}\ \vec{M}$ everywhere. If the radius of the core is $R/5$, find the ratio of magnetic flux through a cross-section of the core to that outside the core.