Department of Physics, IIT-Kanpur

Instructors: TKG & DpC PHY103A (2014-15, II-SEM) Problem Set 5

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1. A vacuum diode consists of a heated cathode kept at zero potential and an anode at positive potential V_0. Thermal electrons are emitted by the cathode and accelerate to the anode. The potential difference V_0 controls the net current. In a very short time, the system reaches steady state condition (charge density and current become independent of time). The cloud of the moving electrons within the gap is called space charge.

   Suppose the plates have area A and separation d and are perpendicular to the x axis at $x = 0$ and $x = d$. Assuming $A \gg d^2$, so that the edge effects can be neglected.

   Write down the Poisson's equation for the region between the plates. Assume that electrons start from rest at the cathode, what is their speed at point x, where the potential is $V(x)$? In the steady state, I is independent of x. What is the relation between $\rho(x)$ and $v(x)$? Use these results to obtain a differential equation for $V(x)$, by eliminating ρ and v. Solve this equation for $V(x)$ as a function of x, V_0 and d. Plot $V(x)$ and compare it with the potential without space charge. Also, find ρ and v as a function of x.

2. An infinite straight wire carrying line charge density λ, which is kept at a distance d above a grounded conducting plane. Find the potential in the region above the plane. Find the charge density induced on the conducting plane.

3. A sphere of radius R, centered at the origin, carries charge density

$$\rho = k\frac{R}{r^2}(R - 2r)\sin\theta,$$

   where k is a constant and r, θ are the spherical coordinates. Find the appropriate potential for points on the z axis far from the sphere.

Exercises

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1. A point dipole $\mathbf{p} = p_0\hat{k}$ is situated at the origin. (a) What is the force on a point charge q at $(a, 0, 0)$ (Cartesian coordinates)? (b) What is the force on q at $(0, 0, a)$? (c) How much work does it take to move q from $(a, 0, 0) to (0, 0, a)$?

2. The locations of point charge particles q, $3q$, $-2q$ and $-2q$ are $(0, 0, -d)$, $(0, 0, d)$, $(0, d, 0)$ and $(0, -d, 0)$, respectively. Find a simple approximate formula for the potential valid at points far from the origin. Express your answer in spherical coordinates.

3. Consider a spherical shell of radius R, which carries a surface charge density $\sigma = \sigma_0 \cos\theta$. Calculate the dipole moment of this charge configuration. Find the approximate potential far away from the shell.

4. Consider an electric dipole $\mathbf{p} = p_0\hat{j}$ located at the point $(0, 0, z_0)$ above the grounded conducting plane at $z = 0$. What is the electric field on the z axis for $z \gg z_0$?