Department of Physics, IIT-Kanpur

Instructors: TKG & DpC PHY103A (2014-15, II-SEM) Problem Set 6

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1. Two identical spheres of radius R carrying uniform volume charge densities $-\rho$ and $+\rho$, respectively, are placed so that the **overlap is almost complete** (see problem # 4 of Assignment 3). (a) Show that this system is equivalent to a spherical shell of radius R with the surface charge density $\sigma = \sigma_0 \cos\theta$. Here, $\sigma_0 = \rho\, d$ with d is the infinitesimal separation between the centers of the two spheres and θ is the angle between the position vector of a point on the surface and the z axis. (b) Find the electric field and potential inside the sphere. (c) Find the electrostatic potential and the electric field at a point outside the sphere. (d) Find the energy of the charge configuration using the potential and the electric field separately

2. According to quantum mechanics, the electron cloud for a hydrogen atom in the ground state has a charge density
$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a},$$
where q is the electron charge and a is the Bohr radius. Find the atomic polarizability of such an atom.

3. Consider two point dipoles $\mathbf{p}_1 = p_1\hat{k}$ and $\mathbf{p}_2 = p_2\hat{j}$ located at the origin and on the y axis $(0, a, 0)$, respectively. There are no other charges/fields present. What is the torque on $\mathbf{p}_2$ due to $\mathbf{p}_1$? What is the torque on $\mathbf{p}_1$ due to $\mathbf{p}_2$? First calculate the torque on the dipole about its own center and then calculate the torque about the origin.

Exercises

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1. Four point charges are located at the following places: $+q$ at $(a, a, 0)$, $-q$ at $(a, -a, 0)$, $-q$ at $(-a, a, 0)$ and $+q$ at $(-a, -a, 0)$. Find the potential and electric field at any arbitrary point (x, y, z).

2. A point dipole $\mathbf{p}$ is at a distance r from a point charge q at the origin. The dipole makes an angle θ with the vector $\mathbf{r}$ from q to $\mathbf{p}$. What is the force on $\mathbf{p}$? What is the force on q?

3. A circular plate of radius R in the xy plane has surface charge density given by $\sigma(s, \phi) = \sigma_0 s \cos\phi$. Here, s and ϕ are the cylindrical coordinates. Show that the dipole moment of this charge distribution is $\mathbf{p} = (\sigma_0 \pi R^4/4)\hat{i}$. Show that the every moment of the quadrupole moment tensor is zero.