Department of Physics, IIT-Kanpur

Instructors: TKG & DpC PHY103A (2014-15, II-SEM) Problem Set 7

---

1. A very long cylinder of radius a, carries a uniform polarization $\mathbf{P}$ perpendicular to its axis. Find the electric field inside the cylinder. Show that the field outside the cylinder can be expressed in the form
$$\mathbf{E}(\mathbf{r}) = \frac{a^2}{2\epsilon_0 s^2}\Big[2(\mathbf{P}\cdot\hat{s})\hat{s} - \mathbf{P}\Big].$$

2. Suppose the field inside a large dielectric is $\mathbf{E}_0$, so that the electric displacement is $\mathbf{D}_0 = \epsilon_0\mathbf{E}_0 + \mathbf{P}$. Now a small spherical cavity is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$.

3. A point charge q is imbedded at the center of a sphere of linear dielectric material (with susceptibility χ_e and radius R). Find the electric field, the polarization, and the bound charge densities. What is the total bound charge on the surface? Where is the compensating negative bound charge located?

4. A circular disk of radius R carries a uniform surface charge density σ. If it rotates at an angular velocity ω, what is the surface current density $\mathbf{K}$ at a distance s from the center?

   A uniformly charged solid sphere of radius R and total charge Q is centered at the origin and spinning at a constant angular velocity ω about the z axis. Find the current density $\mathbf{J}$ at any point within the sphere.

Exercises

---

1. A dielectric slab of side a, centered at the origin, carries a "frozen-in" polarization $\mathbf{P} = k\mathbf{r}$, where k is a constant. Calculate all the bound charges.

2. A parallel plate capacitor with plate area 40 cm^{-2} and plate separation 5 mm is given a charge $q = 2\times10^{-12}$ C. The battery is disconnected and a parallel faced dielectric slab ($k = 5$) of area 40 cm^{-2} and thickness 3 mm is placed centrally between the two plates. Find σ, σ_b. Calculate $\mathbf{D}/\epsilon_0, \mathbf{E}$ and $\mathbf{p}/\epsilon_0$ in the various regions between the plates. What is the capacitance for the three different situations: i) no dielectric materials, ii) the dielectric material is placed centrally as described in this problem and iii) the dielectric material fills the entire space between the plates.

3. A current I flows down a wire of radius a. If it is uniformly distributed over the surface, what is the surface current density? If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis, what is J?