DpC

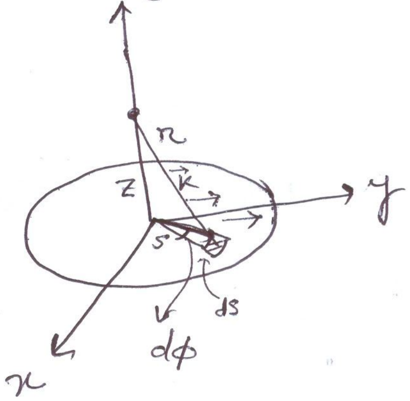
Indian Institute of Technology Kanpur

PHY103 **Quiz-2** **Date: March 24, 2015**

Name Model Solutions Roll No.

Section Max Marks: 25 Time: 35 min

1. Consider a surface current $\vec{K} = K_0\hat{y}$ is flowing on the surface of a disc of radius R, lying in the x-y plane with its centre at the origin. Find the magnetic vector potential $\vec{A}$ on any point on the z-axis above and below the x-y plane. Sketch $|\vec{A}|$ as a function of z. **[6+2]**



$$\vec{A}(z) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}')\,da'}{r}$$

$$= \frac{\mu_0}{4\pi} K_0 \hat{y} \int \frac{da'}{r}$$

In cylindrical co-ordinates

$$da' = s\,ds\,d\phi \quad \text{(see figure)}$$

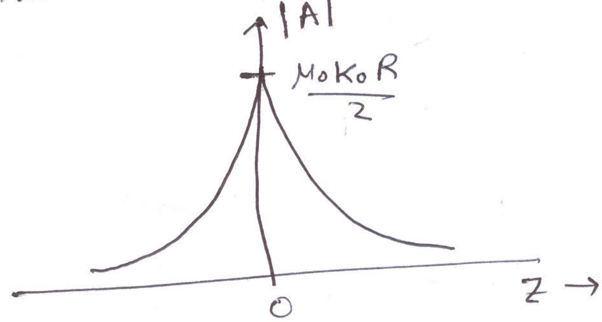
$$\vec{A}(z) = \frac{\mu_0 K_0}{4\pi}\hat{y} \int_{s=0}^{R}\int_{\phi=0}^{2\pi} \frac{s\,ds\,d\phi}{\sqrt{s^2+z^2}}$$

$$= \frac{\mu_0 K_0}{2}\left[\sqrt{R^2+z^2} - \sqrt{z^2}\right]\hat{y}$$

$\Rightarrow$ The expression is same for $\pm z$ $\Rightarrow$ The above result is true for both above & below the disc.

As $z \to \infty$ $|\vec{A}| \to 0$

at $z = 0$, $|\vec{A}| = \frac{\mu_0 K_0 R}{2}$

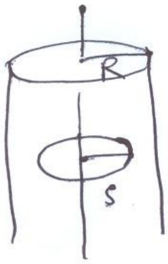


2. A long cylindrical wire of radius R with magnetic susceptibility χ_m is placed along z-axis. It carries a free current density $\vec{J} = J_0 \hat{z}$. Calculate $\vec{H}$ and from $\vec{H}$ evaluate $\vec{B}$, inside and outside the cylinder. Calculate the magnetization $\vec{M}$ and the bound currents. Calculate $\vec{B}$ from the currents. **[10]**

In cylindrical coordinates: $\nabla \times \vec{v} = [\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}]\hat{s} + [\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}]\hat{\phi} + \frac{1}{s}[\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}]\hat{z}$

By symmetry $\vec{B}$ & $\vec{H}$ are in $\hat{\phi}$ direction.

For $s \leq R$



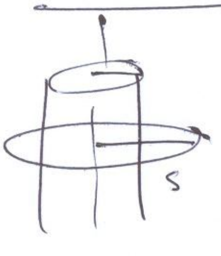
Consider an Amperian loop of radius s as shown in the figure

$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$.

$\Rightarrow \quad 2\pi s H = \pi s^2 J_0 \quad \Rightarrow \quad H = \frac{J_0 s}{2}$

$\Rightarrow \vec{H} = \frac{J_0 s}{2} \hat{\varphi} \qquad (s \leq R)$

for $s \geq R$ $\Rightarrow$ Amperian loop with $s > R$



$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc} \Rightarrow \quad 2\pi s H = J_0 \pi R^2$

$\vec{H} = \frac{J_0 R^2}{2s} \hat{\varphi} \qquad (s \geq R)$

$\vec{B} = \mu \vec{H}$ where $\mu = \mu_0(1+\chi_m)$ for $s \leq R$, $\mu = \mu_0$ for $s \geq R$.

$$\left.\begin{aligned} B(s \leq R) &= \mu_0 (1+\chi_m) \frac{J_0 s}{2} \hat{\varphi} \\ B(s \geq R) &= \mu_0 \frac{J_0 R^2}{2s} \hat{\varphi} \end{aligned}\right\}$$

$$\begin{cases} \vec{M} = \chi_m \vec{H} = \chi_m \frac{J_0 s}{2} \hat{\phi} \text{ for } s \le R \\ \vec{M} = 0 \text{ for } s > R \end{cases}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = \chi_m J_0 \hat{z}$$

$$\vec{K}_b = \vec{M} \times \hat{n}\Big|_{s=R} = -\frac{\chi_m J_0 R}{2} \hat{z}$$

<u>Calc. of $\vec{B}$ from currents</u>

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl.}$

<u>for $s \le R$</u> [consider the same Amperian loop as before]

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \int (\vec{J}_f + \vec{J}_b) \cdot \vec{da}$$

$$= \mu_0 \left[J_0 \pi s^2 + \chi_m J_0 \pi s^2 \right]$$

$$\Rightarrow B = \frac{\mu_0 J_0 s (1 + \chi_m)}{2} = \frac{\mu J_0 s}{2}$$

$$\Rightarrow \vec{B} = \frac{\mu J_0 s}{2} \hat{\phi} \quad (\text{for } s \le R)$$

<u>for $s \ge R$</u>

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left[\int (\vec{J}_f + \vec{J}_b) \cdot \vec{da} + \int \vec{K}_b \cdot \vec{dl}' \right]$$

$$\Rightarrow B \cdot 2\pi s = \mu_0 \left[J_0 \pi R^2 + \chi_m J_0 \pi R^2 - \chi_m \frac{J_0 R}{2} \cdot 2\pi R \right]$$

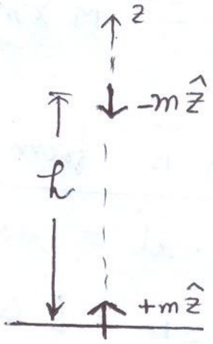
$$= \mu_0 J_0 \pi R^2$$

$$\Rightarrow B = \frac{\mu_0 J_0 R^2}{2s}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 J_0 R^2}{2s} \hat{\phi} \quad (s \ge R)$$

($\vec{B}$ is same as derived from $\vec{H}$)

3. Two magnetic dipoles(each of strength m and mass μ) can slide on a frictionless vertical rod. When the dipole moments are in opposite direction to each other, the upper one floats at a height L. Find the value of L. [7]



Magnetic field due to a dipole

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$\Rightarrow$ Field at a height z, due to the lower dipole (at ground) $[\theta = 0]$

$$\vec{B} = \frac{2\mu_0 m}{4\pi z^3}\hat{z}$$

Force on the upper magnet

$$\vec{F} = \vec{\nabla}(\vec{m}\cdot\vec{B}) = -\frac{\partial}{\partial z}\left(\frac{2\mu_0 m}{4\pi z^3}\right)\hat{z} = \frac{3}{2}\frac{\mu_0 m^2}{\pi z^4}\hat{z}$$

For equilibrium at a height $z = L$

$$\frac{3}{2}\frac{\mu_0 m^2}{\pi L^4} = Mg$$

$$\Rightarrow \quad L = \left[\frac{3\mu_0 m^2}{2\pi M g}\right]^{1/4}$$