Robins-Magnus Effect: A Continuing Saga.

Tapan K. Sengupta & Srikanth B. Talla
Dept. of Aerospace Engineering
Indian Institute of Technology Kanpur
Kanpur 208 016, UP

Abstract: The experimental observation by Robins[1], that a projectile spinning about its axis of travel experiences a transverse force (lift), was refuted by Euler[2] purely as a contradiction to expected symmetry of fluid flow. This, undoubtedly had taken away the precedence of finding this effect by Robins and subsequently the same was credited to Magnus[3], which is a testimony of "belief" overtaking physical observation. In the last century Prandtl[4] looked at this problem once again and came up with an upper limit on lift that a spinning cylinder will experience. This is now considered a fundamental tenet in explaining aerodynamic phenomenon of lift generation. However, over the last two decades evidences are accumulating via experimental[5,6] and numerical investigations[7,8] that a new temporal instability affects this flow at high rotation rates, negating the above mentioned upper limit. In this note, we trace the origin of this particular effect to its present day status with respect to flow past a rotating cylinder.

1. The Beginning:

Benjamin Robins' contribution[1] to fluid mechanics and aerodynamics has received little recognition than it deserves due to various reasons. One of the major reasons is that he propounded too many new ideas in a short span of time and he was also busy defending Newton's contribution to calculus. He was largely a self taught person with a desire to take up teaching profession. Upon proving Newton's 'Treatise of quadratures', he received FRS at the early age of twenty. However, he switched his attention to engineering constructing bridges, mills, harbors, making rivers navigable and draining fens. That he had multifaceted talent is evident, when one notes that he is now acknowledged as the father of science of ballistics[9,10] (introducing the concept of rifling the bore of guns, holding the importance of air-resistance in deciding the range of artillery shots and improving the accuracy of projectiles by spinning them), credited for fundamental contributions to aerodynamics[2] (the complex relationship between drag, shape of the body, its angle of attack and air-velocity could not be explained by the then simple theory propounded by Newton and he suggested that ground testing of vehicle is a pre-requisite for a successful design), experimental fluid mechanics[1,2] (developing the whirling arm, the predecessor of present day wind tunnels, the only experimental device at that time and ballistic pendulum for the measurement of velocity of projectiles) and his many contributions to mathematics. He also noted the drag rise at transonic flight regime almost two hundred years ago, before its importance was re-discovered around Second World War[2]. Additionally, he dabbled in contemporary politics and also got involved in controversies related to writing the accounts of Lord Anson's voyage around the world. To this, one must add the fact that he left the center stage of England, when he was appointed the engineer-general of the East India Company to improve the fortifications at

St. David, Madras, where he died of fever at an early age of forty-four. In writing the book on ballistics[9], he made the observation that a spinning projectile experiences a transverse force based on his experiments with ballistic-pendulum and whirling arm. This was not supported by Euler, the leading hydrodynamicist of the time, and this fact was rediscovered by Magnus[3], almost a century later. In this note, we will discuss this particular effect with respect to flow past a rotating cylinder starting with Robins' work to its present day status.

It is incorrectly stated in some references[11] that Robins was responsible for finding the lift force acting on rotating sphere. Undoubtedly, he experimented on spherical shots used for artillery purposes, but he was also first to suggest that a teardrop or egg-like shape of projectile with a center of gravity near the front of it. The observation of Robins' for spinning projectiles was made using the whirling arm-- not a very satisfactory experimental device by present day standard. A whirling arm was used to measure aerodynamic forces at low speed, where the tested body was used to be hung at the end of a long arm that was free to rotate. This arm was rotated by a falling weight via a shaft with cable-pulley arrangement. The rotation of the arm produces the relative motion in air, the same principle that is even used today in wind tunnels to measure forces for steady flight. However, sustained rotation of whirling arm will impart angular momentum to the surrounding air, thereby making the accuracy of such measurements a point of concern. It is with this equipment that he reported his findings[9] in the year 1742. It is noted[2] that, – *Euler was so excited about Robins' book that he personally translated it into German in 1745 adding some commentary….. Euler's interest in Robins' work was both a hindrance and a help. The hindrance concerned Robins' observation of the side force exerted on a spinning projectile moving through the air. Euler considered that to be a spurious finding, due to manufacturing irregularities in the projectile. Recognized as the dominant hydrodynamicist of the eighteenth century, Euler far overshadowed Robins, and thus Robins' finding was not taken seriously for another century, until Gustav Magnus (1802-1870) verified the phenomenon as a real aerodynamic effect.* This book[9] was also translated into French in 1751, the year of Robins' death. It is to be noted that Napoleon read the latter translation from Euler's German translation of the original book[3], while he was a young artillerist at Auxonne, France.

It must be pointed out that both Euler and Robins had mutual admiration for each other's work. For example, Robins published in 1739 "Remarks on M. Euler's Treatise of Motion". So the misinterpretation of Robins' work was based truly upon the personal belief of Euler. We also note that Euler enunciated his famous equations of fluid motion only in 1752, the first mathematical model of inviscid flow. Even today, the Robins-Magnus effect cannot be explained by solution of Euler's equation. Hence, Euler's observations were his intuitive feeling that a spinning symmetric body (with top-down symmetry) cannot experience an asymmetric force in the symmetric direction. In trying to explain the occurrence of transverse force experienced by a spinning body Prandtl[4] advanced an explanation that was based on steady irrotational flow model. In doing so, he also advanced a maximum limit to this transverse force. This is the next development in this subject area that is interesting and discussed next.

2. Maximum Principle Enunciated:

The first qualitative explanation of lift force experienced by a aerodynamic shape was put forward by using Kutta-Jukowski theorem[11]. The lift experienced by the quintessential shape- namely the aerofoil- is obtained by forcing a stagnation point at the sharp trailing edge. Hence, the same is not directly applicable for flow past bodies without sharp trailing edges- as in case of a rotating cylinder. It was Prandtl[4] who explained the flow past a rotating cylinder heuristically by considering the flow to be inviscid and irrotational.

In putting forward his results, Prandtl[4] came up with an upper limit on the value of this transverse force that a rotating cylinder will experience, as its rotation rate is increased. This can be readily explained with the sketches of the flow field in Figure 1. If one defines a non-dimensional rotation rate by $\Omega = \Omega^* D / 2U_\infty$, where the cylinder of diameter D rotates at Ω^* while being placed in a uniform stream of velocity U_∞, then one can define a Reynolds number by $\mathrm{Re} = \dfrac{U_\infty D}{\nu}$ for this flow field. In Figure 1, the top frame (a) depicts the steady inviscid irrotational flow field when the cylinder does not rotate and one can note a perfect top-down and fore-aft symmetry of the flow field. In frame (b), a case is depicted for $\Omega < 2$, where both the front and rear stagnation points (half-saddle points) are deflected downwards, causing the flow to exert an upward force on the cylinder. With increase of Ω to 2, these stagnation points move towards each other and merge at the lowermost point on the cylinder (as shown in Figure 1(c)). For this location of stagnation point, it is easy to show that the corresponding non-dimensional lift value is given by the coefficient $C_{L_{\max}} = 4\pi$. The lift value attained is a maximum was reasoned by Prandtl heuristically, because if the flow field continues to be steady, then with further increase of rotation rate the single half-saddle point of Figure 1(c) would move in the flow field as a full saddle-point located on the closed streamline that demarcates the flow field into two parts (as shown in Figure 1(d)). The region located inside the closed streamline is insulated from the region outside and would be permanent if the flow is steady. This suggests that the circulation cannot increase beyond the rotation rate for the case shown in Figure 1(c). Thus, the Prandtl's observation is based on a steady flow using an inviscid irrotational flow model. In an actual viscous flow, the circulation will be created at the solid wall continually that is convected and diffused according to the governing Navier- Stokes Equation. A steady flow model, as proposed by Prandtl, presupposes that an equilibrium exist between the process of creation of vorticity at the wall and its viscous diffusion. While in the proposed model of Prandtl such an equilibrium is assumed for all rotation rates up to the critical value $(\Omega = 2)$ - it is assumed that the equilibrium is maintained even when the rotation rate is increased further. How realistic is the model proposed by Prandtl? It seemed very real as no counter-examples were encountered for decades where it was violated and it became a very standard argument in all the text books about the infallibility of Prandtl's limit. However, some recent experimental and numerical observations seems to suggest otherwise and they are casting serious doubts about the correctness of Prandtl's logic.

3. Maximum Principle Violated:

The experimental observations in Tokumaru & Dimotakis[6] have cast a doubt about the validity of this upper limit suggested by Prandtl. This observed[6] violation was noted in an indirect manner, as the vertical velocity component ahead of the cylinder was measured and an inviscid point vortex model was used to calculate the lift. It was reported that the maximum lift was violated by 20% for Re = 3800 and $\Omega = 10$ - in a time average sense. The variation of the lift coefficient was provided for different rotation rates for the same Reynolds number of 3800. The authors considered diffusion, unsteady flow processes and three dimensional end effects as likely causes for the supposed violation of maximum lift limit. For a real fluid flow, all these can be a determining factor in breaking the equilibrium and creating circulation that violates the maximum principle. For supercritical rotation rates ($\Omega > 2$), the vorticity will be generated at a larger rate for some period at the solid wall than it is dissipated by viscous action, thus violating the stated equilibrium. If this vorticity remains trapped within the recirculating streamline, then the circulation will increase for the cylinder while a part of it is dissipated by viscous diffusion. The role of diffusion is thus to peg the net circulation at a lower level. However, as we will see in the next section that viscous diffusion also plays a subtle role in supporting enhanced lift when it interferes with physical instability processes. This is clearly seen in computations that use excessive numerical dissipation to *stabilize computations*. Of course, the unsteady processes are most relevant for this supposed violation. In an unsteady scenario, the continuous generation of wall vorticity, if not balanced by viscous dissipation, will cause the lift to increase. The three-dimensionality can also indirectly increase circulation. However, for supercritical rotation rates, the flow being rotational, one can show that three- dimensionality will be suppressed via the application of Taylor- Proudman theorem[12], as the Coriolis force will be significant in this case. Considering all these, it is instructive to compute the actual flow field by solving time dependent Navier-Stokes equation. To begin with, one can perform two- dimensional calculations as have been reported in[13-15] for this flow field. The reported results in[13-14] is particularly noteworthy, as the calculations are based on spectral method using a variable grid that is very fine at early times. Unfortunately, the reported results are only for low rotation rates and short period of integration times that did not show violation of maximum lift limit. The results in[15] were obtained using a higher order accurate method of calculations for a range of Reynolds numbers, but for lower rotation rates. This same method was then used for high Reynolds numbers and rotation rates[16,7,8]. It is noted that the computed lift coefficients in[7,8,16] matches well with the experimental results in[6] i.e. for the first time numerical calculation revealed the violation of the maximum limit, that was in conformity with the experimental observations. The appearance of numerical results that supports experimental observations does more than simply validating the experiments. It also allows one a detailed time accurate account of the physical events, which is otherwise very difficult to track experimentally. In fact, the computational results revealed a new physical instability in the process- that is what we recount next.

4. A New Instability Uncovered

The computations[7,8,16] showed a series of temporal instabilities at early stages of flow evolution. During these periods the computed results displayed overshoot of values of instantaneous lift coefficients in such a manner that the time averaged values match closely with the experimentally reported values. Are these instabilities real or an artifact of the computations?

It is interesting to note that in some experiments by Werle[5], a layer of co-rotating liquid in contact with the cylinder surface suffering aperiodic instabilities were noticed for supercritical rotation rates of the cylinder for $\mathrm{Re} = 3300$ - a value somewhat closer to the reported value of Reynolds number in[6]. The author intentionally called them as aperiodic instabilities that were visually recorded.

In[7,8,16], a third order upwind scheme was used for capturing these temporal instabilities by taking very small time steps ($\Delta t = 10^{-5}$). In accurate numerical computations, apart from achieving high spectral resolution it has to be ensured that the discretization should not alter the physical dissipation that is only a second derivative of the variable. This is the cause for the success of third order upwind schemes that adds fourth order dissipation only. There are other methods that have been also used[17,18] for this problem and they have produced results that are totally different from experimental observations. For example, in[17], the author used a "Streamline- Upward/Petrov-Galerkin" (SUPG) method, along with Pressure- Stabilizing/ Petrov-Galerkin (PSPG) numerical stabilization terms for a case where the cylinder rotated eccentrically. The author stated that – *these computations are also important from the point of view that in a real situation it is almost certain that the rotating cylinder will be associated with a certain degree of wobble.* This was prompted by the author's earlier attempt[18] in computing the flow for $\mathrm{Re} = 3800$ and $\Omega = 5$ that resulted in an error of 400% at $t = 250$. The reason for the failure by SUPG/ PSPG is due to the massive artificial stabilization of the numerical scheme by added second derivative term that interferes with physical dissipation. Additionally, the time steps used are about 10^4 times larger than that used in[7,8,13-16] that would be simply not able to capture the transient events recorded in[7,8,13-16].

A typical set of results are shown in Figure 2, where the time variation of the force and moment coefficients have been computed for $\mathrm{Re} = 3800$ and $\Omega = 5$. Here, more accurate compact different scheme has been used[19] for computing the same flow field. The instability displayed in this case consists of sharp discontinuous jump in the values of force and moment coefficients at discrete times. Such instabilities were also seen in the results reported in[7,8,16]. The physical mechanism behind the instability is already explained in[8] by an equation based on energy principle derived from full Navier-Stokes equation without any approximation. It was noted that the basic equilibrium solution is destabilized by an intense interaction between the velocity and vorticity fields of the primary and disturbance field. In the context of computations, the disturbance field arises via truncation error and accumulated round off error. In simulating physical instabilities, the chosen numerical schemes must be neutrally stable. If there is numerical dissipation present that interferes with the physical dissipation, then the physical instabilities will be subdued. This is the case with the results of[17,18], where excessive second order dissipation suppresses physical instabilities. It is interesting to note that the time at which instabilities appear would depend on the amplitude of accumulated error. As the compact scheme[19] has higher accuracy than the third order upwind scheme[8], the first temporal instability occurs later for the compact scheme. However, the value of C_l at which the first instability appears remains the same.

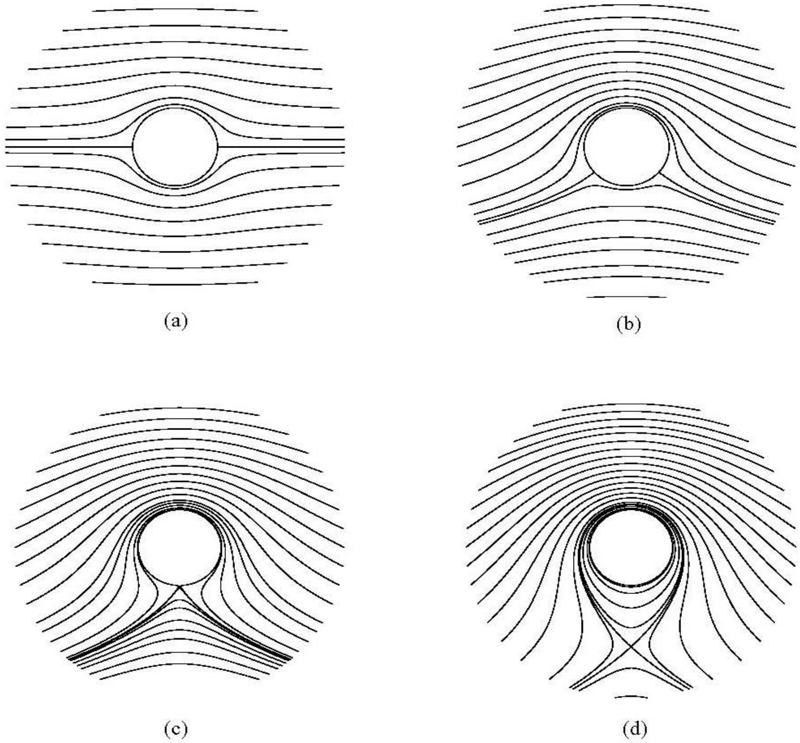
Closing Remarks:

The evolution of this problem has run parallel with the webs and tides of developments in fluid mechanics through last three hundred years. It began with experimental observation of Robins that was negated by heuristic observation of Euler. However, when the analytical fluid mechanics was in its prime, Prandtl, another leading aerodynamicist, not only explained the phenomenon theoretically, but also proposed a new limit on the phenomenon based on heuristic logic and this survived for many decades before evidences started accumulating that this limit may be violated. It is interesting to note that the two experimental observations on this supposed violation is visual in one case and uses analytical model to arrive at the observation in the other case. In contrast, the computational evidences are based on the full governing equations. However, the detection by numerical calculation depends on the existence of ambient noise (numerical error) and the accuracy of the numerical method used. It shows the need to study and develop models for the actual background noise that is present in experiments. A realistic noise model with very high accuracy computational algorithms- that preserves fundamental physical principles- would provide conclusive evidence of this and many other problems of instabilities.

References:

1. Robins, B., *Mathematical Tracts,* **1 & 2**. J. Nourse, London, 1761.
2. Anderson, J.D., *A History of Aerodynamics And Its Impact on Flying Machines,* Cambridge University Press, U.K., 1997.
3. Magnus, G. Uber die abweichung der gesechesse, *Pogendorf's Ann. Der Phys. Chem.,* **88**, 1-14, 1853.
4. Prandtl, L., Application of the "Magnus Effect" to the wind propulsion of ships, *NACA TM 387*, 1926.
5. Werle, H., Hydrodynamic visualization of the flow around a streamlined cylinder with suction. Cousteau-Malavard turbine sail model, *Le Recherche Aerospatiale*, **4,** 29-38, 1984.
6. Tokumaru, P.T. & Dimotakis, P.E., The lift of a cylinder executing rotary motions in a uniform flow, *J. Fluid Mech.*, **255**, 1-10, 1993.
7. Sengupta, T.K., Gupta, K. & Nair, M.T., Lift generation and limiting mechanism via unsteady flow development for Magnus-Robins effect, In *The Proc. of 8th Asian Cong. Fluid Mech.* (Ed. E. Cui), International Acad. Publishing, Beijing, 413-416, 1999.

8. Sengupta, T.K., Kasliwal, A., De, S. & Nair, M.T., Temporal flow instability for Magnus-Robins effect at high rotation rates, *J. Fluids and Structures*, **17** (7), 941-953, 2003.
9. Robins, B., *New Principles of Gunnery Containing The Determination of The Force of Gunpowder and Investigation of The Difference in The Resisting Power of The Air to Swift and Slow Motion, 1742.*
10. Steele, B.D., Muskets and Pendulum: Benjamin Robins, Leonhard Euler and the Ballistic Revolution (1742-1753), *Technology and Culture,* **35,** 345-382, 1994.
11. Tritton, D.J., *Physical Fluid Dynamics,* Van Norstrand Reinhold, Berkshire, England, 1980.
12. Hide, R., Ibbetson, A. & Lighthill, M.J., On slow transverse flow past obstacles in a rapidly rotating fluid, *J. Fluid Mech.,* **32,** 251-272 , 1968
13. Badr, H.M. & Dennis, S.C.R., Time dependent viscous flow past an impulsively started rotating and translating circular cylinder, *J. Fluid Mech.,* **158**, 447-488, 1985.
14. Badr, H.M., Coutanceau, M., Dennis, S.C.R. & Menard, C., Unsteady flow past a rotating circular cylinder at Reynolds numbers 1000 and 10,000, *J. Fluid Mechanics*, **220**, 459-484, 1990.
15. Nair, M.T., Sengupta, T.K. & Chauhan, U.S., Flow past rotating cylinders at high Reynolds numbers using higher order upwind scheme, *Computers & Fluids*, **27**, 47-70, 1998.
16. Nair, M.T. & Sengupta, T.K., Magnus effect at high speed ratios, In *The Proc. of 3rd Asian CFD Conference*, (Eds. Prahlad, T.S., Deshpande, S.M. & Saxena, S.K.), NAL press, Bangalore, 336-341, 1998.
17. Mittal, S., Flow past rotating cylinder: Effect of eccentricity, *J. Applied Mech.,* **68**, 543-552, 2001.
18. Mittal, S., Control of flow past bluff bodies using rotating control cylinders, *J. Fluids and Structures*, **15**, 291-326, 2001.
19. Sengupta, T.K., G. Ganeriwal & De, S., Analysis of central and upwind compact schemes. *J. Computational Physics,* **192 (2),** 677-694, 2003.



Fiigure 1 Inviscid irrotational flow past a rotating circular cylinder for (a) zero (b) subcritical (c) critical and (d) supercritical rotation rates.

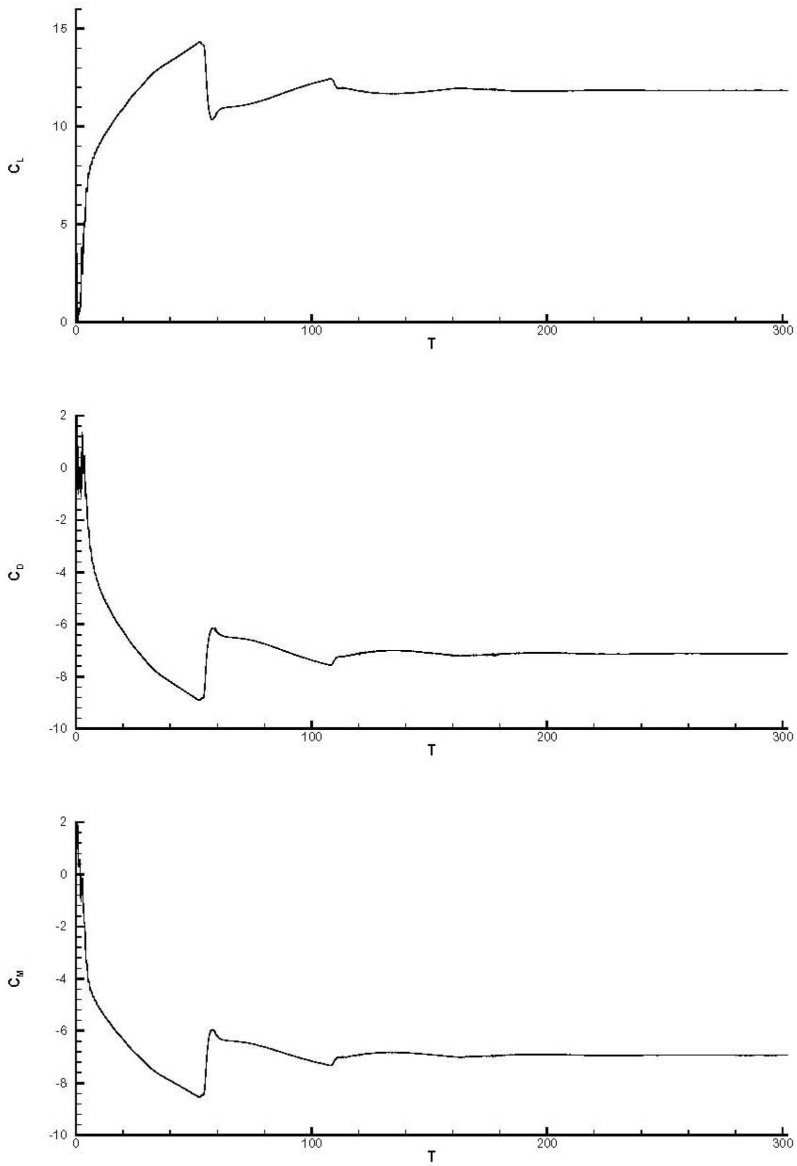


Figure 2 Variation of force and moment coefficients with time for Re = 3800 and $\Omega = 5$: (a) C_L (b) C_D and (c) C_M

The cylinder flows with circulation, Figs. 8.14*b* to *d*, develop an inviscid downward *lift* normal to the free stream, called the *Magnus-Robins force*. This lift is proportional to stream velocity and vortex strength. Its discovery, by experiment, has long been attributed to the German physicist Gustav Magnus, who observed it in 1853. It is now known [40, 45] that the brilliant British engineer Benjamin Robins first reported a lift force on a spinning body in 1761. We see from the streamline patterns that the velocity on top of the cylinder is less, and, thus, from Bernoulli's equation, the pressure is higher. On the bottom, we see tightly packed streamlines, high velocity, and low pressure; viscosity is neglected. Inviscid theory predicts this force.

The surface velocity is given by Eq. (8.39). From Bernoulli's equation (8.3), neglect-

The problem in airfoil analysis, Sec. 8.7, is thus to determine the circulation Γ as a function of airfoil shape and orientation.

Lift and Drag of Rotating Cylinders[2]

The flows in Fig. 8.14 are mathematical: a doublet plus a vortex plus a uniform stream. The physical realization could be a rotating cylinder in a free stream. The no-slip condition would cause the fluid in contact with the cylinder to move tangentially at velocity $v_\theta = a\omega$, setting up a net circulation Γ. Measurement of forces on a spinning cylinder is very difficult, and no reliable drag data are known to the author. However, Tokumaru and Dimotakis [22] used a clever auxiliary scheme to measure lift forces at $Re_D = 3800$.

Figure 8.15 shows lift and drag coefficients, based on frontal area ($2ab$), for a rotating cylinder at $Re_D = 3800$. The drag curve is from CFD calculations [41]. Reported CFD drag results, from several different authors, are quite controversial because they do not agree, even qualitatively. The writer feels that Ref. 41 gives the most reliable results. Note that the experimental C_L increases to a value of 15.3 at $a\omega/U_\infty = 10$. This contradicts an early theory of Prandtl, in 1926, that the maximum possible value of C_L would be $4\pi \approx 12.6$, corresponding to the flow conditions in Fig. 8.14*c*. The inviscid theory for lift would be:

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2(2ba)} = \frac{2\pi\rho U_\infty K b}{\rho U_\infty^2 ba} = \frac{2\pi v_{\theta s}}{U_\infty} \tag{8.44}$$

where $v_{\theta s} = K/a$ is the peripheral speed of the cylinder.

Figure 8.15 shows that the theoretical lift from Eq. (8.44) is much too high, but the measured lift is quite respectable, much larger in fact than a typical airfoil of the same chord length, as in Fig. 7.25. Thus rotating cylinders have practical possibilities. The Flettner rotor ship built in Germany in 1924 employed rotating vertical cylinders that developed a thrust due to any winds blowing past the ship. The Flettner design did not gain any popularity, but such inventions may be more attractive in this era of high energy costs.

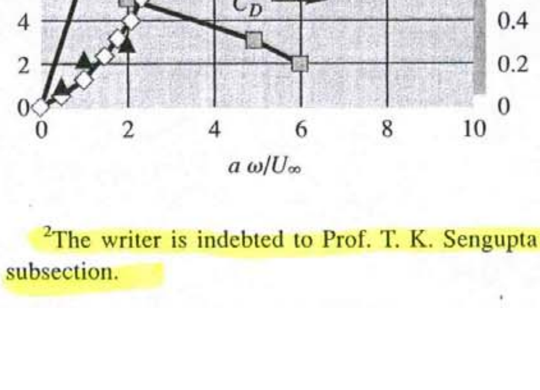


Fig. 8.15 Drag and lift of a rotating cylinder of large aspect ratio at $Re_D = 3800$, after Tokumaru and Dimotakis [22] and Sengupta et al. [41].

[2]The writer is indebted to Prof. T. K. Sengupta of I.I.T. Kanpur for data and discussion for this subsection.