OFDM-OQAM using Hilbert Transform

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Outline I

Introduction

- Orthogonal frequency division multiplexing (OFDM), see Figure 1.1, is a popular modulation technique for mitigating intersymbol interference (ISI) introduced by frequency selective channels.
  - OFDM is spectrally equivalent to quadrature carrier multiplexing (QCM) or double sideband suppressed carrier (DSB-SC) modulation.
  - Is it possible to transmit only one sideband and at the same time mitigate ISI?
  - This would increase symbol density in time-frequency space.
- Use single sideband (SSB) modulation.
  - Requires real-valued message for simple (matched filter) receiver implementation – reduces symbol densty.
  - *Controlled ISI* can be introduced to improve symbol density.

Introduction

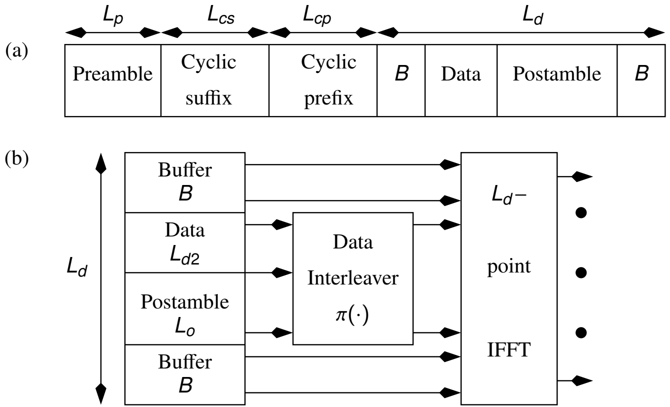

Figure 1.1: Frame structure of OFDM in the time domain.

Introduction

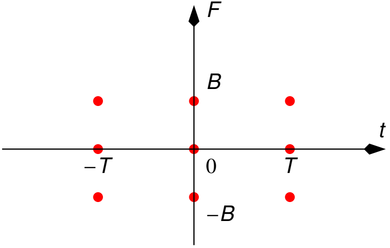

Figure 1.2: Symbol density of OFDM in time-frequency space.

- In Figure 1.1, T is one 2D/1D symbol duration (*not* an OFDM symbol or "frame").
- $B = 2(1 + \alpha)/(2T)$, where $0 < \alpha \leq 1$ is the roll-off factor of the root-raised cosine (RRC) spectrum. Clearly, $1/(BT) < 1$.

System Model – Digital SSB Modulation

- With SSB we have $B = (1 + \alpha)/(2T)$, which implies $1/(BT) > 1$, since only one sideband (upper or lower) is transmitted.
- This is illustrated in Figure 2.1.

System Model – Digital SSB Modulation

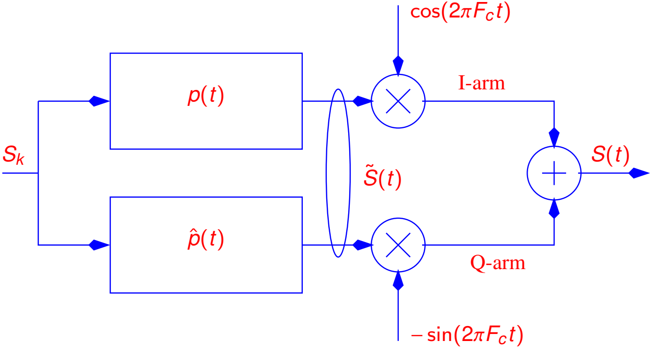

Figure 2.1: Digital SSB modulation – transmitter.

System Model – Digital SSB Modulation

- We have
$$S(t) = \Re\left\{\tilde{S}(t)\mathrm{e}^{\mathrm{j}\,2\pi F_c t}\right\} \tag{2.1}$$
where
$$\tilde{S}(t) = \sum_{k=-\infty}^{\infty} S_k \tilde{p}(t - kT) \tag{2.2}$$
and
$$\tilde{p}(t) = p(t) + \mathrm{j}\,\hat{p}(t) \tag{2.3}$$
is the complex-valued transmit filter.
- Here $\hat{p}(t)$ is the Hilbert transform of $p(t)$.
- $p(t)$ is a square root Nyquist pulse e.g. having RRC spectrum.
- S_k denotes real-valued symbols from a pulse amplitude modulation (PAM) (1D) constellation.

Analysis

- The power spectral density (psd) of $\tilde{S}(t)$ in (2.2) is (for uncorrelated symbols)

$$S_{\tilde{S}}(F) = \frac{P_{\text{av}}}{2T} \left|\tilde{P}(F)\right|^2 \tag{3.1}$$

where P_{av} is the average power of the PAM constellation and

$$\tilde{p}(t) \rightleftharpoons \tilde{P}(F) \tag{3.2}$$

constitute a Fourier transform pair.

Analysis

- Let

$$p(t) \rightleftharpoons P(F). \tag{3.3}$$

- Then

$$\begin{aligned} \tilde{P}(F) &= P(F) + \mathrm{j}\left(-\mathrm{j}\,\mathrm{sgn}(F)P(F)\right) \\ &= \begin{cases} 2P(F) & \text{for } F > 0 \\ P(0) & \text{for } F = 0 \\ 0 & \text{for } F < 0 \end{cases} \end{aligned} \tag{3.4}$$

- Thus, the psd of $\tilde{S}(t)$ in (3.1) is one-sided – hence the psd of $S(t)$ in (2.1) is also one-sided about the carrier frequency F_c.

Analysis

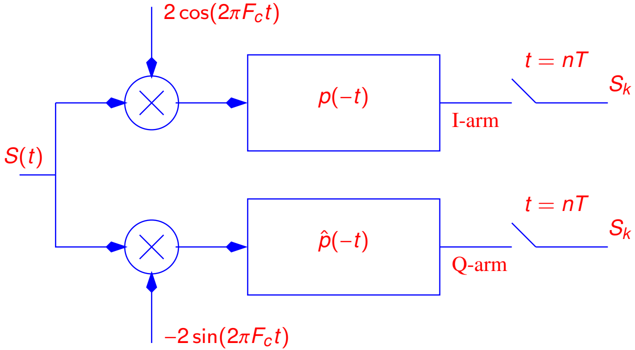

Figure 3.1: Digital SSB modulation – receiver.

- Scope for error correction since S_k is available in both receiver arms.

Analysis

- Note that

$$p(t) \star p(-t)\Big|_{t=nT} = R_{pp}(0)\delta_K(nT)$$
$$\hat{p}(t) \star \hat{p}(-t)\Big|_{t=nT} = R_{\hat{p}\hat{p}}(0)\delta_K(nT) \tag{3.5}$$

  where "$\star$" denotes convolution, $\delta_K(\cdot)$ is the Kronecker delta function, $R_{pp}(\cdot)$ is the autocorrelation of $p(t)$ and $R_{\hat{p}\hat{p}}(\cdot)$ is the autocorrelation of $\hat{p}(t)$.

- Note that

$$R_{pp}(0) = R_{\hat{p}\hat{p}}(0). \tag{3.6}$$

- No ISI at the sampler output – channel is ideal and introduces only additive white Gaussian noise (AWGN).

Analysis

Issues

- PAM constellations are power inefficient compared to QAM.
- In real-life both $p(t)$ and $\hat{p}(t)$ are time-limited. How would this effect ISI?

Analysis

Table 3.1: Power of QAM vs PAM.

Bits/symbol	P_{av}	
	QAM	PAM
1	1	1
2	2	5
3	$3+\sqrt{3}$	21
4	10	85

- In Table 3.1 minimum Euclidean distance is 2.
- QAM cannot be used – only offset QAM (OQAM) can be used, which introduces ISI – receiver is more complex.
- PAM does not introduce ISI – simple matched filter receiver can be used.

Analysis

- For an RRC spectrum

$$p(t) = \frac{1}{\pi\sqrt{2B}(1 - 64B^2\rho^2 t^2)}\left[8B\rho\cos(\theta_1 + \theta_2) + \frac{\sin(\theta_1 - \theta_2)}{t}\right] \tag{3.7}$$

where

$$\begin{aligned}
\rho &= \alpha \\
B &= 1/(2T) \\
\theta_1 &= 2\pi Bt \\
\theta_2 &= 2\pi B\rho t.
\end{aligned} \tag{3.8}$$

Analysis

- The HT of $p(t)$ is

$$\hat{p}(t) = I_1 + I_2 \tag{3.9}$$

where for $F_1 = B(1-\rho)$

$$\begin{aligned}
I_1 &= \frac{2}{\sqrt{2B}} \int_{F=0}^{F_1} \sin(2\pi F t)\, dF \\
&= \frac{1}{\pi t \sqrt{2B}} \left(1 - \cos(2\pi F_1 t)\right) \\
I_2 &= \frac{2}{\sqrt{2B}} \int_{F_1}^{2B-F_1} \cos\left(\frac{\pi(F-F_1)}{4B-4F_1}\right) \sin(2\pi F t)\, dF \\
&= \frac{8B\rho}{\pi\sqrt{2B}(1-64B^2t^2\rho^2)} \left[\sin(2\pi Bt(1+\rho)) \right. \\
&\quad \left. - 8Bt\rho \cos(2\pi Bt(1-\rho))\right].
\end{aligned} \tag{3.10}$$

Analysis

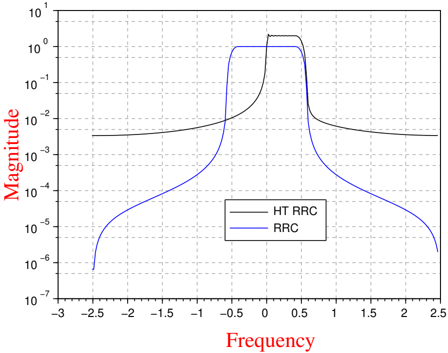

Figure 3.2: Magnitude response of $p(t)$ and $\tilde{p}(t)$. $P(0) = \tilde{P}(0) = 1$.

- In Figure 3.2 $T = 1$ sec, $\rho = 0.161$, $F_s = 5$ Hz.

Analysis

Table 3.2: SIR of $R_{\hat{p}\hat{p}}(mT)$ and $R_{pp}(mT)$.

N	SIR	
	$R_{\hat{p}\hat{p}}(mT)$	$R_{pp}(mT)$
100	16.931	56.921
80	15.934	49.426
40	12.975	40.602
20	9.363	25.212

- In Table 3.2 $T = 1$ sec, $\rho = 0.161$, $F_s = 5 = 1/T_s$ Hz.
- Samples $p(nT_s)$, $\hat{p}(nT_s)$ considered for $-N \leq n \leq N$.

Analysis

- The ratio of signal-to-interference (SIR) power is

$$\text{SIR} = \frac{R_{gg}^2(0)}{\sum_{\substack{m \\ m\neq 0}} R_{gg}^2(mT)} \tag{3.11}$$

where $g(\cdot)$ is $p(\cdot)$ or $\hat{p}(\cdot)$.

Analysis

- The reason for the low SIR for $\hat{p}(\cdot)$ is due to the discontinuity of the Fourier transform of the Hilbert transformer at $F = 0$, as given by

$$H(F) = \begin{cases} +\mathrm{j} & \text{for } F < 0 \\ 0 & \text{for } F = 0 \\ -\mathrm{j} & \text{for } F > 0. \end{cases} \quad (3.12)$$

where $g(\cdot)$ is $p(\cdot)$ or $\hat{p}(\cdot)$.

Analysis

- A possible solution lies in redefining the Fourier transform of the Hilbert transformer as follows (see Figure 3.3)

$$H(F) = \begin{cases} +\mathrm{j} & \text{for } F \leq -aF_1 \\ \mathrm{e}^{\mathrm{j}(\pi(F+aF_1)/(2aF_1)+\pi/2)} & \text{for } -aF_1 \leq F \leq aF_1 \\ -\mathrm{j} & \text{for } F \geq aF_1. \end{cases} \tag{3.13}$$

where $F_1 = B(1-\rho)$, as given in slide 16 and

$$0 < a < 1. \tag{3.14}$$

Analysis

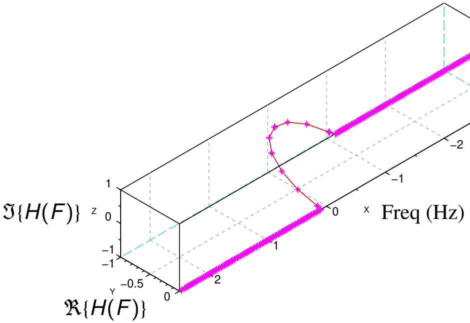

Figure 3.3: Proposed frequency response of the Hilbert transformer.

Analysis

- Using (3.13) we get

$$\hat{p}(t) = I_2 + I_3 + I_4 \tag{3.15}$$

where I_2 is given by (3.10) and

$$\begin{aligned} I_3 &= \frac{1}{\sqrt{2B}} \int_{F=-aF_1}^{aF_1} \mathrm{e}^{\mathrm{j}\,(\pi(F+aF_1)/(2aF_1)+\pi/2)}\, \mathrm{e}^{\mathrm{j}\,2\pi F t}\, dF \\ &= \frac{-2aF_1}{\sqrt{2B}} \operatorname{sinc}(aF_1 A_1) \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} A_1 &= \frac{1}{2aF_1} + 2t \\ \operatorname{sinc}(x) &= \frac{\sin(\pi x)}{\pi x}. \end{aligned} \tag{3.17}$$

Analysis

- Finally

$$I_4 = \frac{-\mathrm{j}}{\sqrt{2B}} \int_{F=aF_1}^{F_1} \mathrm{e}^{\mathrm{j}\,2\pi F t}\, dF + \frac{\mathrm{j}}{\sqrt{2B}} \int_{F=-F_1}^{-aF_1} \mathrm{e}^{\mathrm{j}\,2\pi F t}\, dF$$
$$= \frac{2(1+a)F_1}{\sqrt{2B}} \mathrm{sinc}(F_1(1+a)t)\sin(F_1(1-a)t). \quad (3.18)$$

Analysis

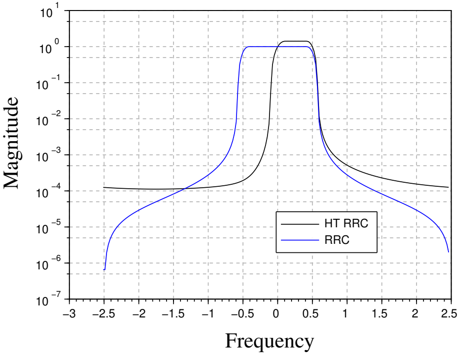

Figure 3.4: Magnitude response of $p(t)$ and $\tilde{p}(t)$. $P(0) = \tilde{P}(0) = 1$.

- In Figure 3.4 $T = 1$ sec, $\rho = 0.161$, $F_s = 5$ Hz, $a = 0.25$.

Analysis

Table 3.3: SIR of $R_{\hat{p}\hat{p}}(mT)$ and $R_{pp}(mT)$.

N	SIR	
	$R_{\hat{p}\hat{p}}(mT)$	$R_{pp}(mT)$
100	52.787	56.921
80	45.168	49.426
40	33.298	40.602
20	17.93	25.212

- In Table 3.3 $T = 1$ sec, $\rho = 0.161$, $F_s = 5 = 1/T_s$ Hz, $a = 0.25$.
- Samples $p(nT_s)$, $\hat{p}(nT_s)$ considered for $-N \leq n \leq N$.

Conclusions

- OFDM-OQAM is not a new modulation technique.
  - Has it been hyped?
- Conceptually similar to SSB modulation that was commonly used in analog communication.
- The proposed model for OFDM-OQAM is much simpler than the ones given in literature.
- Same symbol is available in both I - and Q - arms, resulting in 3 dB improvement in error rate.