Single User Massive MIMO

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Introduction

Multi user MMIMO

- Base station (BS) has large number of antennas, mobile handset (MH) has a single antenna.
- Beamforming in the downlink.
- Spatial multiplexing may not be possible due to single antenna at MH.

Single user MMIMO

- Both BS and MH have large number of antennas.
- Beamforming in both uplink & downlink.
- Spatial multiplexing is possible in both uplink and downlink.

Introduction

Beamforming

- Low spectral efficiency (bits/transmission). Same signal, with delay, is transmitted from a large number of antennas.
- Uses highly directive pencil beam.

Spatial Multiplexing

- High spectral efficiency. Independent signals are transmitted from a large number of antennas.
- No directivity. A rich scattering channel is required for effective operation.

> “Therefore SU-MIMO can have both beamforming and spatial multiplexing in the uplink and downlink.”

A very useful feature, as opposed to MU-MMIMO which can have beamforming only in the downlink and no spatial multiplexing.

Of course, beamforming and spatial multiplexing cannot be done simultaneously.

Signal Model

Without Precoding

- The received signal is
$$\mathbf{R_k} = \mathbf{H_k S} + \mathbf{W_k} \qquad (1)$$
- $\mathbf{H_k}$ is $N \times N$ channel matrix.
- $\mathbf{S}$ is $N \times 1$ turbo coded QPSK symbol vector.
- $\mathbf{R_k}$ is the $N \times 1$ received signal vector.
- $\mathbf{k}$ denotes re-transmission $(1 \leq \mathbf{k} \leq N_{rt})$.

With Precoding

- The received signal is
$$\mathbf{R_k} = \mathbf{H_k H_k^H S} + \mathbf{W_k} \qquad (2)$$
- $\mathbf{H_k}$ is $N_r \times N_t$ channel matrix.
- $\mathbf{S}$ is $N_r \times 1$ turbo coded QPSK symbol vector.
- $\mathbf{R_k}$ is the $N_r \times 1$ received signal vector.
- N_r receive and N_t transmit antennas.

Problem Statement & Remarks

- Find $\mathbf{S}$ given $\mathbf{R}_\mathbf{k}$.
- Elements of $\mathbf{H}_\mathbf{k}, \mathbf{W}_\mathbf{k}$ are independent, zero-mean and complex Gaussian.
- Easier to transmit QPSK using large number of antennas, rather than transmitting a large constellation using a single antenna, since QPSK has the lowest peak-to-average power ratio (PAPR).
- Easier to introduce channel coding, e.g. turbo code, for QPSK compared to a large constellation.
- Each data bit generates two coded QPSK symbols. Hence each QPSK symbol carries 0.5 bits of information.
- It is possible to have a large number of antennas in mmwave frequencies (>100 GHz).

Solution

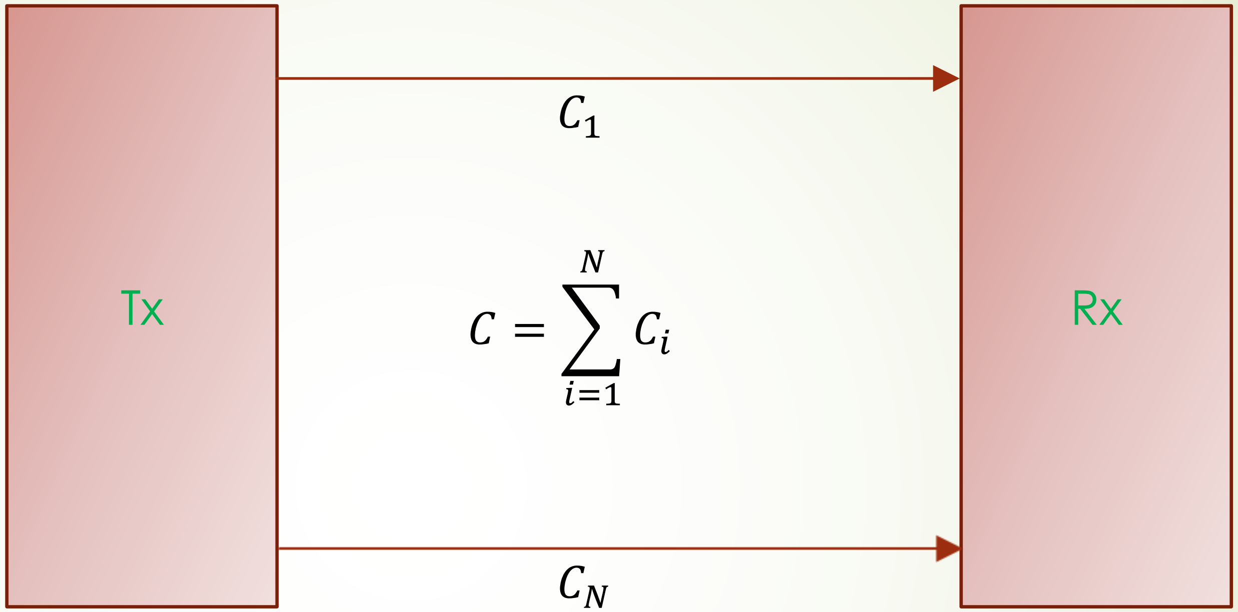
- We use the property

$$E\left[\mathbf{H_k}\mathbf{H_k^H}\right] = C_1\mathbf{I}$$
$$E\left[\mathbf{H_k^H}\mathbf{H_k}\right] = C_2\mathbf{I}$$

- C_1 & C_2 are constants and $\mathbf{I}$ is identity matrix of appropriate size.
- Replace $E[\cdot]$ by time averaging, using re-transmissions.

Channel Capacity

- **Proposition 1**: *The channel capacity is additive over the number of complex dimensions. In other words, the channel capacity over many complex dimensions, is equal to the sum of the capacities over each complex dimension, provided the information is independent across the complex dimensions. Independence of information also implies that, the bits transmitted over one complex dimension is not the interleaved version of the bits transmitted over any other complex dimension.*
- **Proposition 2**: *Conversely, if C bits per transmission are sent over N complex dimensions, it seems reasonable to assume that each complex dimension receives C/N bits per transmission, on the average.*
- *A "complex dimension" denotes a communication link between a transmitter and receiver.*



$$C = \sum_{i=1}^{N} C_i$$

Figure 1

Illustration of Proposition 1. This is achieved by time and frequency division multiplexing.

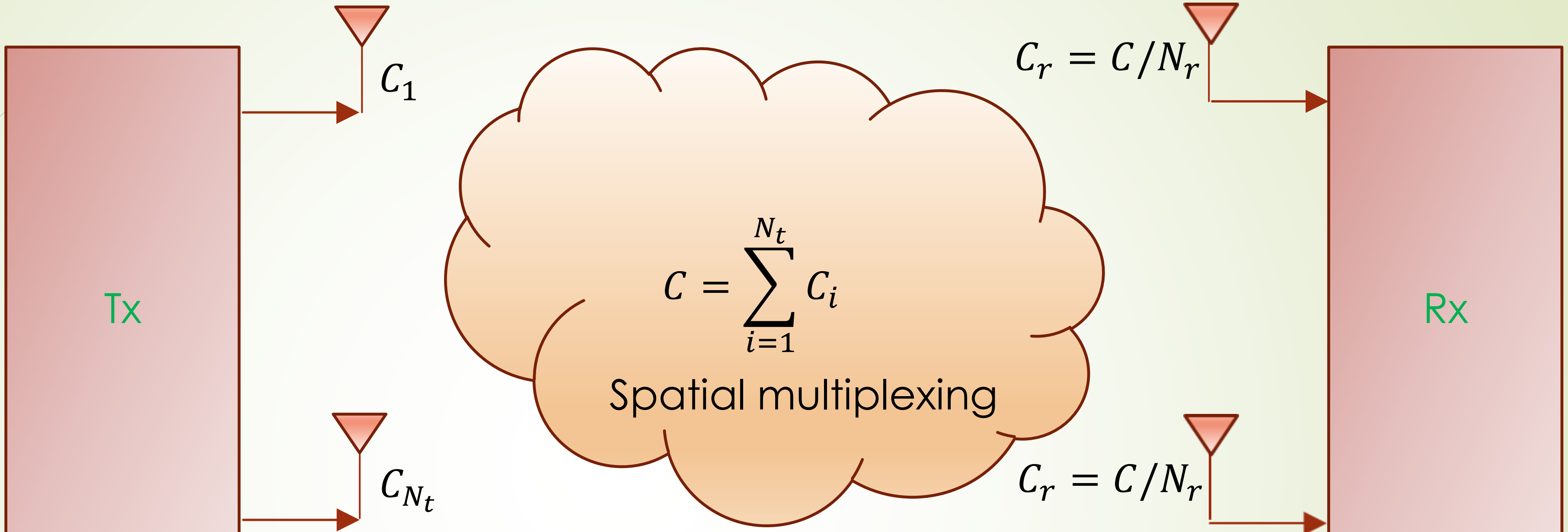


Figure 2

Illustration of Proposition 2 and spatial multiplexing.

Channel Capacity

- Recall that for each complex dimension

$$C_r = \log_2(1 + \mathrm{SNR}) \quad \text{bits/transmission (4)}$$

- The $\mathbf{i}^{\mathbf{th}}$ element of $\mathbf{R}_{\mathbf{k}}$ in (1) receives on average $\mathrm{C}_{\mathrm{r}} = 1/(2N_{rt})$ bits per transmission **(Proposition 2)** .
- We also have

$$\mathrm{SNR} = \frac{\text{Power of } \mathbf{i}^{\mathbf{th}} \text{ element of } \mathbf{H}_{\mathbf{k}}\mathbf{S} \text{ in } (1)}{\text{Power of } \mathbf{i}^{\mathbf{th}} \text{ element of } \mathbf{W}_{\mathbf{k}} \text{ in } (1)} \qquad (4a)$$

- The average SNR per bit is

$$\mathrm{SNR}_{\mathrm{av},b} = \frac{\mathrm{SNR}}{\mathrm{C}_r} \qquad (5)$$

- Therefore

$$C_r = \log_2\left(1 + C_r\, \mathrm{SNR}_{\mathrm{av},b}\right) \qquad (6)$$

Channel Capacity

- We have

$$\mathrm{SNR}_{\mathrm{av},b} = \frac{2^{C_r} - 1}{C_r} \qquad (7)$$

- When $C_r \to 0$, $\mathrm{SNR}_{\mathrm{av},b} \to \ln 2 \equiv -1.6$ dB.

Channel Capacity

The minimum average SNR per bit for error-free transmission over fading channels is identical to that of the AWGN channel, and is equal to -1.6 dB.

- *It is necessary to specify the operating average SNR per bit of all telecommunication systems.*
- *For example, currently in a mobile phone, only the received signal strength is specified e.g. -100 dBm. However, this is not the average SNR per bit.*

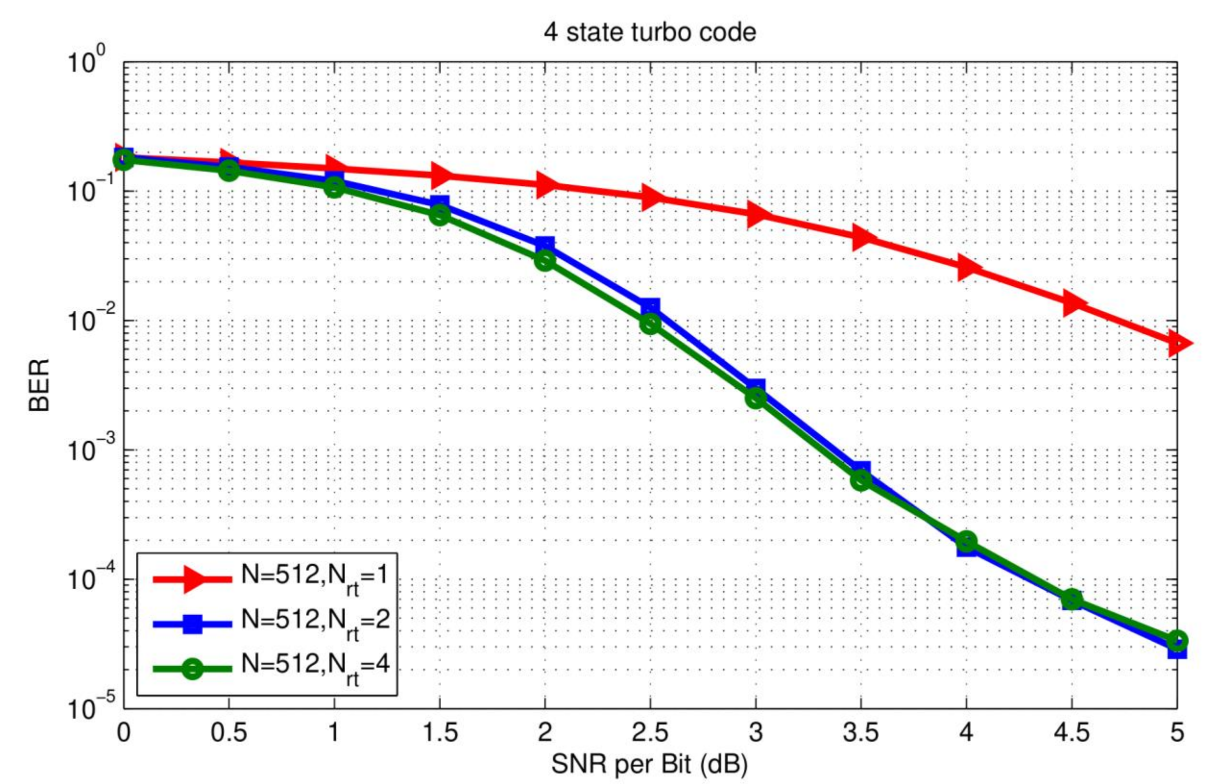


Figure 3

Performance of 512×512 SU-MMIMO without precoding. The spectral efficiency is $N/(2N_{rt})$.

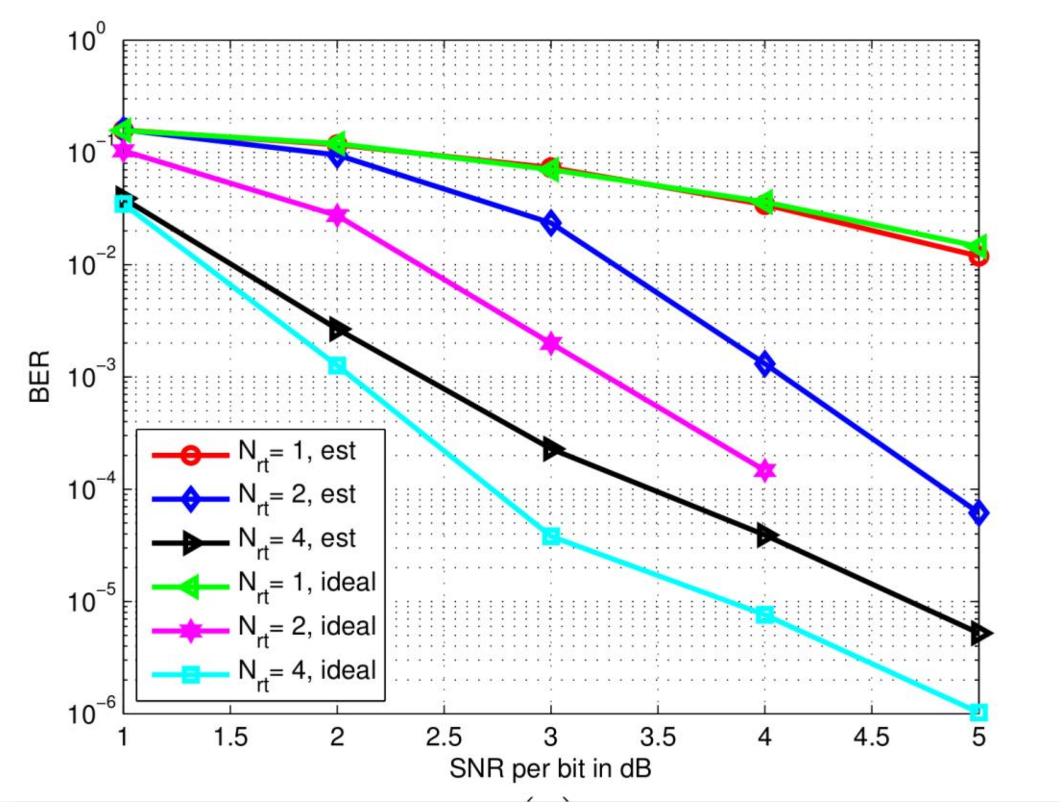


Figure 4

Performance of 8×8 SU-MIMO with estimated carrier, timing and channel and no precoding.

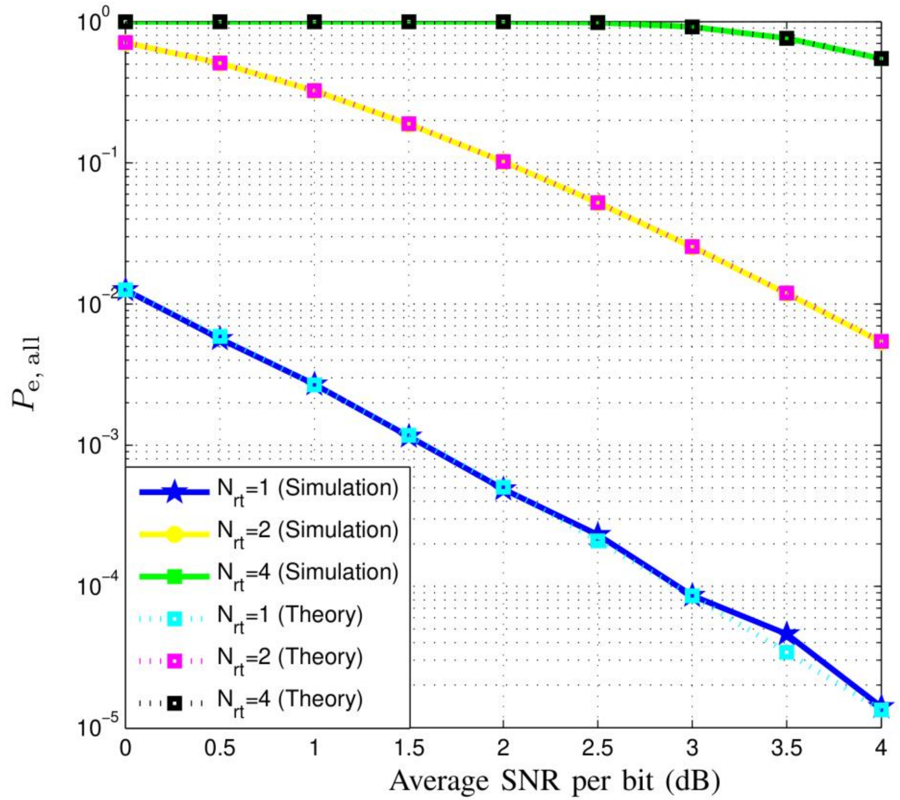


Figure 5

Probability of not detecting a frame (packet) for 8×8 SU-MIMO, without precoding.

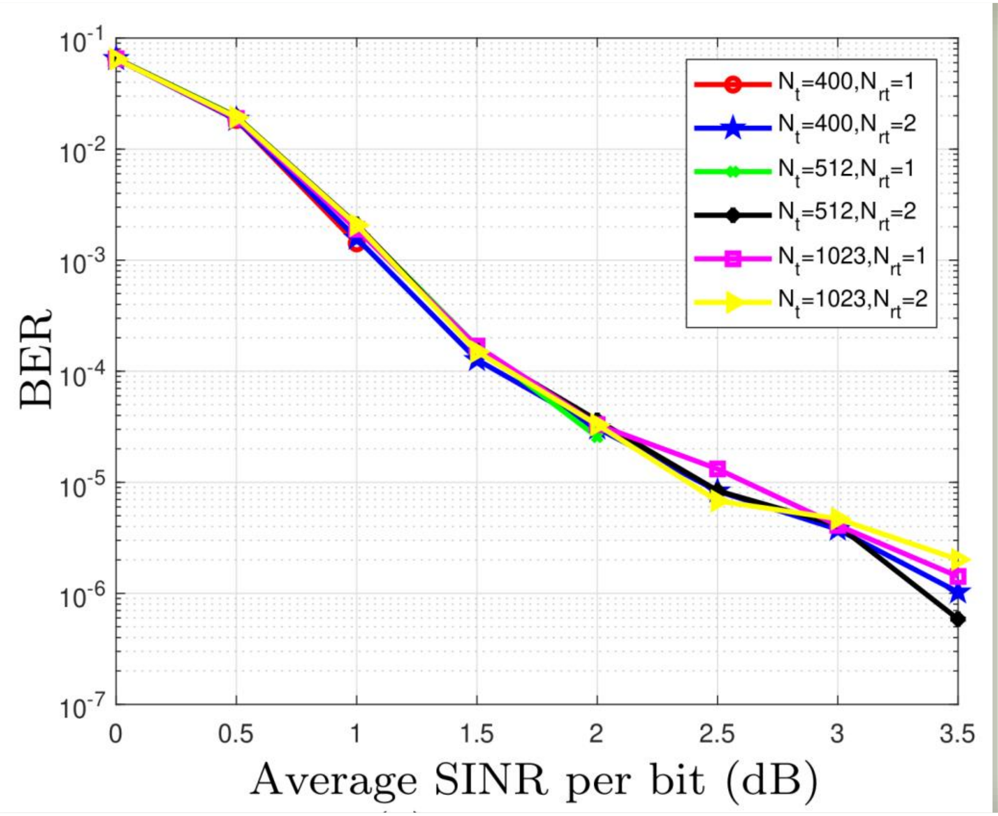


Figure 6

Performance of precoded SU-MMIMO with different transmit and receive antennas. Here $N_t + N_r = 1024$. Performance is independent of re-transmissions when SINR is used instead of SNR.

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