Single-User Massive MIMO

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Introduction

- Massive MIMO (MMIMO) can be classified as
  - Multi user (MU-MMIMO): Base station (BS) has large number of antennas, mobile handset (MH) has a single antenna. Beamforming in the downlink required for effective operation. Beamforming improves directivity at the cost of spectral efficiency.
  - Single user (SU-MMIMO): Both BS and MH have a large number of antennas. Rich scattering channel required for effective operation. High spectral efficiency. Beamforming can also be used.

Introduction

- Single user massive MIMO is used to increase the spectral efficiency
  - Spatial multiplexing
- Large number of antennas in tx and rx in mm-wave freq
  - Antenna size is small
- Operating SNR per bit is not known
  - Nobody can see what is inside a mobile phone anyway

System Model

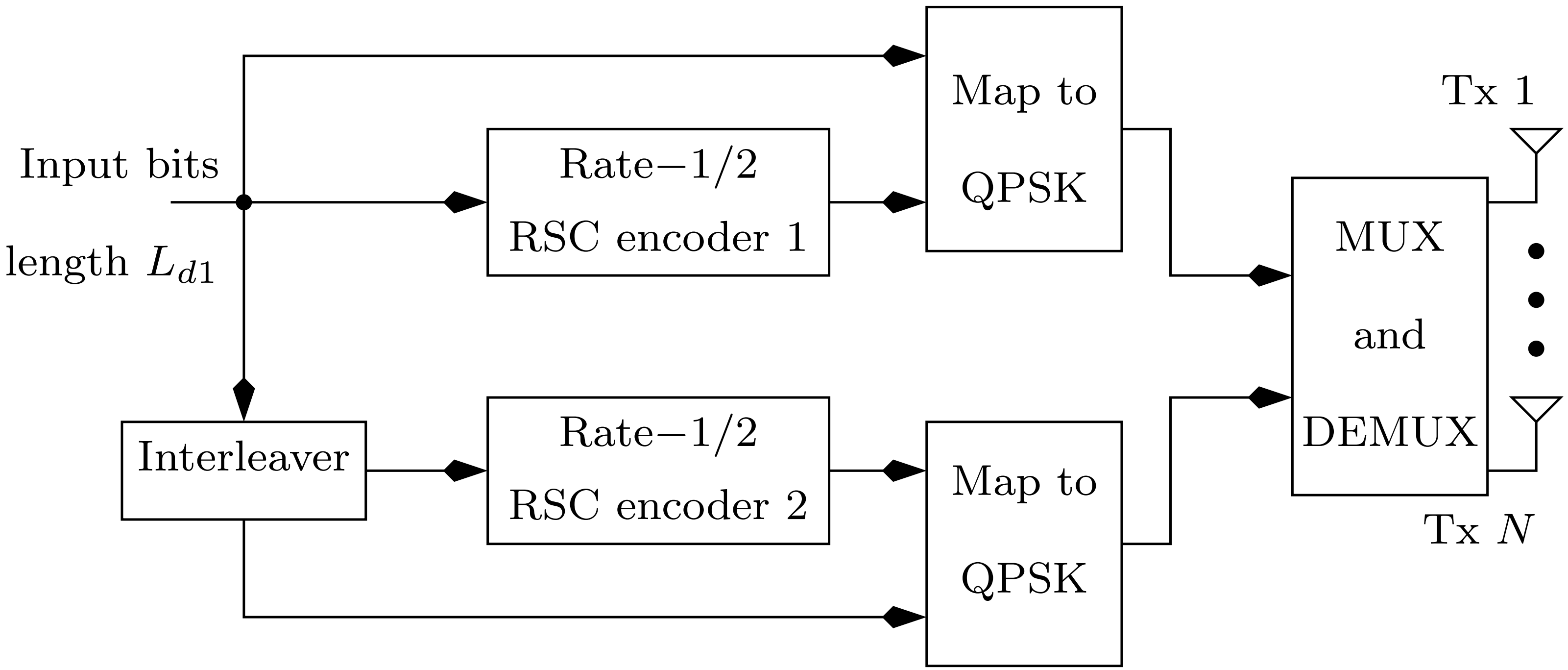

Figure 1: Transmitter.

System Model

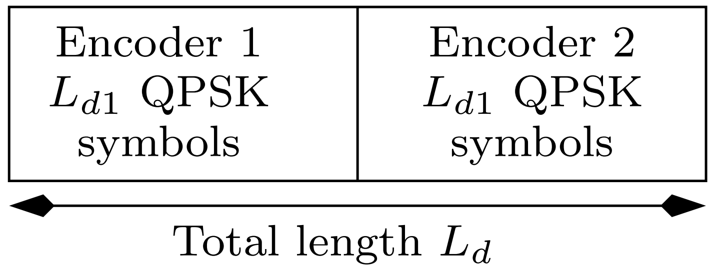

Figure 2: The frame structure.

System Model

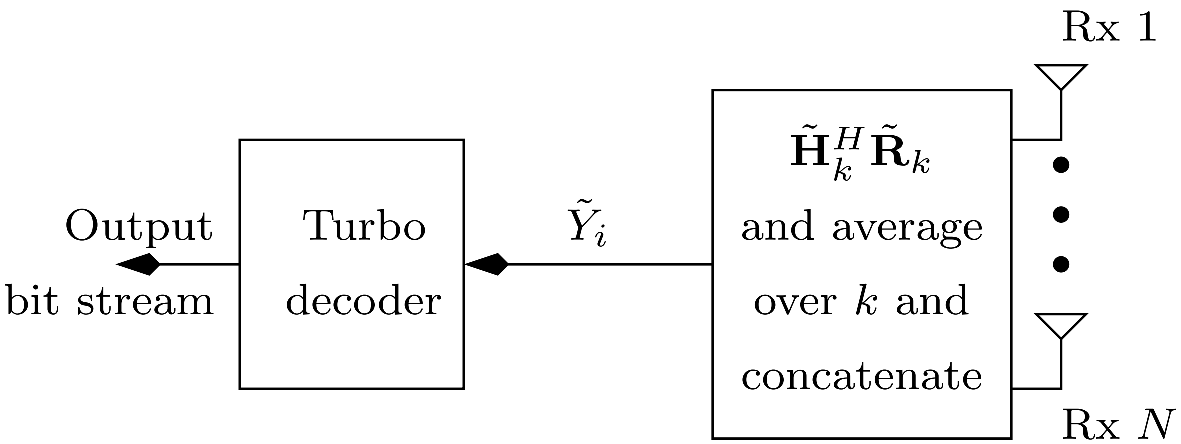

Figure 3: Receiver.

Signal Model

- The received signal in the k^{th} ($0 \leq k \leq N_{rt} - 1$, k is an integer), re-transmission is given by

$$\tilde{\mathbf{R}}_k = \tilde{\mathbf{H}}_k \mathbf{S} + \tilde{\mathbf{W}}_k \tag{1}$$

Signal Model

- $\tilde{\mathbf{R}}_k \in \mathbb{C}^{N \times 1}$ – received vector
- $\tilde{\mathbf{H}}_k \in \mathbb{C}^{N \times N}$ – channel matrix
- $\tilde{\mathbf{W}}_k \in \mathbb{C}^{N \times 1}$ – AWGN vector
- The transmitted symbol vector is $\mathbf{S} \in \mathbb{C}^{N \times 1}$, M-ary constellation
- Boldface letters denote vectors or matrices
- Complex quantities are denoted by a tilde
- Tilde is not used for $\mathbf{S}$

Signal Model

- $$\frac{1}{2}E\left[\left|\tilde{H}_{k,\,i,\,j}\right|^2\right] = \sigma_H^2 \tag{2}$$

- $$\frac{1}{2}E\left[\left|\tilde{W}_{k,\,i}\right|^2\right] = \sigma_W^2 \tag{3}$$

- $$\begin{aligned} \frac{1}{2}E\left[\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_l^H\right] &= N\sigma_H^2\delta_K(k-l)\mathbf{I}_N \\ \frac{1}{2}E\left[\tilde{\mathbf{W}}_k\tilde{\mathbf{W}}_l^H\right] &= \sigma_W^2\delta_K(k-l)\mathbf{I}_N \end{aligned} \tag{4}$$

Problem Statement

- Find $\mathbf{S}$ given $\tilde{\mathbf{R}}_k$
  - ML solution – complexity $\mathscr{O}(M^N)$
  - Inversion of $\tilde{\mathbf{H}}_k$ (zero-forcing solution) requires $\mathscr{O}(N^3)$ operations – noise enhancement
  - Sphere decoding

The Proposed Solution

- Consider

$$\begin{aligned}\tilde{\mathbf{Y}}_k &= \tilde{\mathbf{H}}_k^H \tilde{\mathbf{R}}_k \\ &= \tilde{\mathbf{F}}_k \mathbf{S} + \tilde{\mathbf{V}}_k\end{aligned} \tag{5}$$

where

$$\begin{aligned}\tilde{\mathbf{F}}_k &= \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k \\ \tilde{\mathbf{V}}_k &= \tilde{\mathbf{H}}_k^H \tilde{\mathbf{W}}_k.\end{aligned} \tag{6}$$

The Proposed Solution

- Observe that

$$\frac{1}{2}E\left[\tilde{\mathbf{H}}_k^H\tilde{\mathbf{H}}_l\right] = N\sigma_H^2\delta_K(k-l)\mathbf{I}_N. \tag{7}$$

- However

$$\frac{1}{2}\tilde{\mathbf{H}}_k^H\tilde{\mathbf{H}}_l \neq N\sigma_H^2\delta_K(k-l)\mathbf{I}_N. \tag{8}$$

- Replace $E[\cdot]$ by time-averaging – using re-transmissions

Channel Capacity

- Consider the signal

$$\tilde{r}_i = \tilde{x}_i + \tilde{w}_i \tag{9}$$

- The channel capacity is

$$C = \log_2 (1 + \text{SNR}) \qquad \text{bits per transmission} \tag{10}$$

  over a complex dimension.

- The average SNR is

$$\text{SNR} = \frac{E\left[\left|\tilde{x}_i\right|^2\right]}{E\left[\left|\tilde{w}_i\right|^2\right]} = \frac{P'_{\text{av}}}{2\sigma_w^2} \tag{11}$$

  over a complex dimension.

Channel Capacity

Proposition 0.1 *The channel capacity is additive over the number of complex dimensions. In other words, the channel capacity over N complex dimensions, is equal to the sum of the capacities over each complex dimension, provided the information is independent across the complex dimensions. Independence of information also implies that, the bits transmitted over one complex dimension is not the interleaved version of the bits transmitted over any other complex dimension.*

Channel Capacity

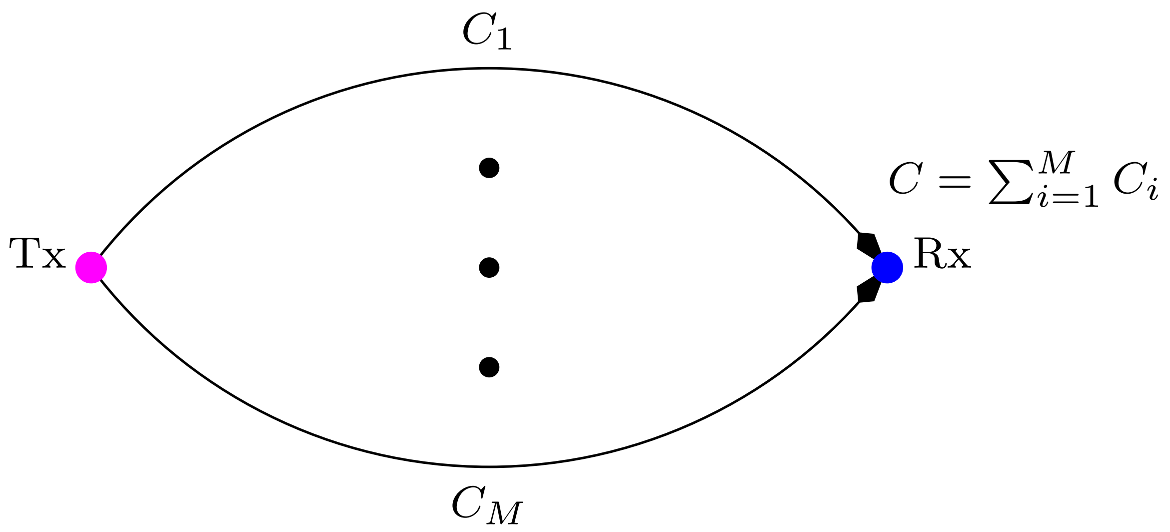

Figure 4: Proposition 1.

Channel Capacity

Proposition 0.2 *Conversely, if C bits per transmission are sent over N complex dimensions, it seems reasonable to assume that each complex dimension receives C/N bits per transmission*

Channel Capacity

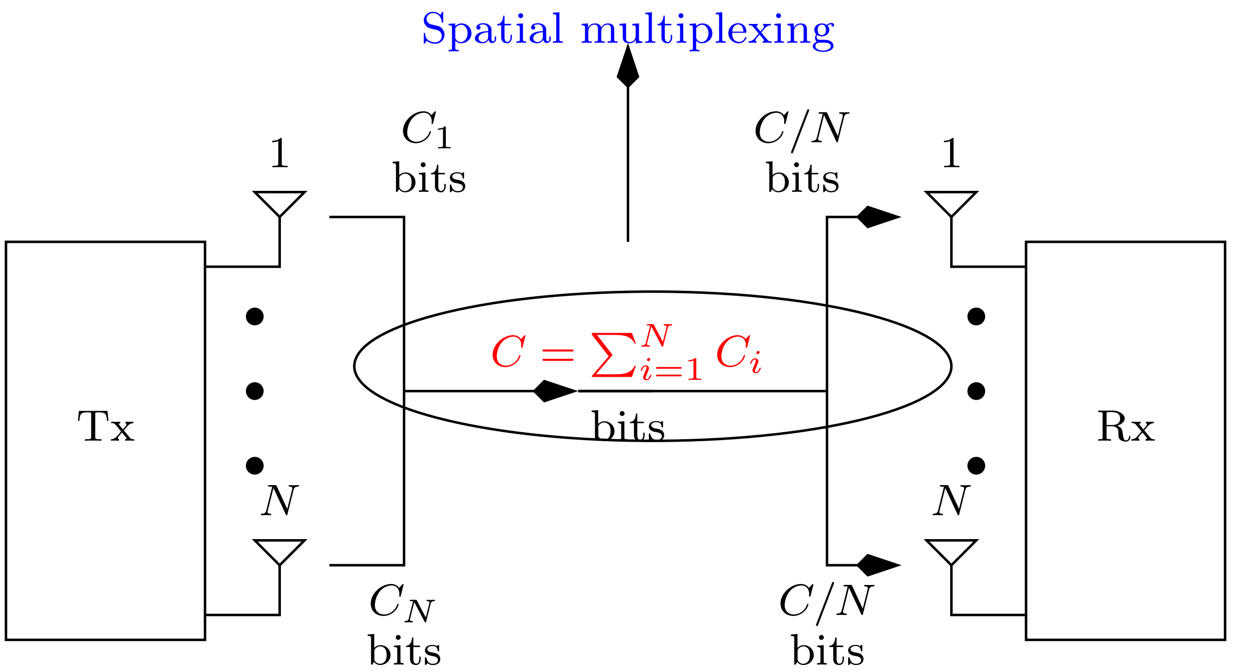

Figure 5: Proposition 2.

Channel Capacity

- The i^{th} element of $\tilde{\mathbf{R}}_k$ in (1) is

$$\tilde{R}_{k,\,i} = \sum_{j=1}^{N} \tilde{H}_{k,\,i,\,j} S_j + \tilde{W}_{k,\,i}. \tag{12}$$

- Now substitute

$$\begin{aligned} \tilde{x}_i &= \sum_{j=1}^{N} \tilde{H}_{k,\,i,\,j} S_j \\ \tilde{w}_i &= \tilde{W}_{k,\,i} \end{aligned} \tag{13}$$

Channel Capacity

- The average SNR at each rx antenna is

$$\text{SNR} = \frac{2NP_{\text{av}}\sigma_H^2}{2\sigma_W^2}. \tag{14}$$

- $C = 1/(2N_{rt})$ bits/transmission at each rx antenna
- The average SNR per bit is

$$\text{SNR}_{\text{av},\, b} = \frac{2NP_{\text{av}}\sigma_H^2 \cdot 2N_{rt}}{2\sigma_W^2} = \frac{\text{SNR}}{C} \tag{15}$$

Channel Capacity

- Capacity relation at each rx antenna is

$$C = \log_2 \left(1 + C\, \mathrm{SNR}_{\mathrm{av},\, b}\right) \qquad \text{bits per transmission} \tag{16}$$

- Re-arranging terms in (16) we get

$$\mathrm{SNR}_{\mathrm{av},\, b} = \frac{2^C - 1}{C}. \tag{17}$$

- As $C \to 0$, $\mathrm{SNR}_{\mathrm{av},\, b} \to \ln(2) \equiv -1.6$ dB.

Table 1: Simulation parameters.

Parameter	Value
L_{d1}	512
L_d	1024
N	1, 16, 512
N_{rt}	1, 2, 4
No of frames simulated	10^5, 10^6
No of turbo decoder iterations	8

Simulation Parameters

- 4-state turbo code with generating matrix given by

$$\mathbf{G}(D) = \left[\begin{array}{cc} 1 & \frac{1+D^2}{1+D+D^2} \end{array} \right]. \tag{18}$$

- Spectral efficiency $N/(2N_{rt})$ bits/transmission

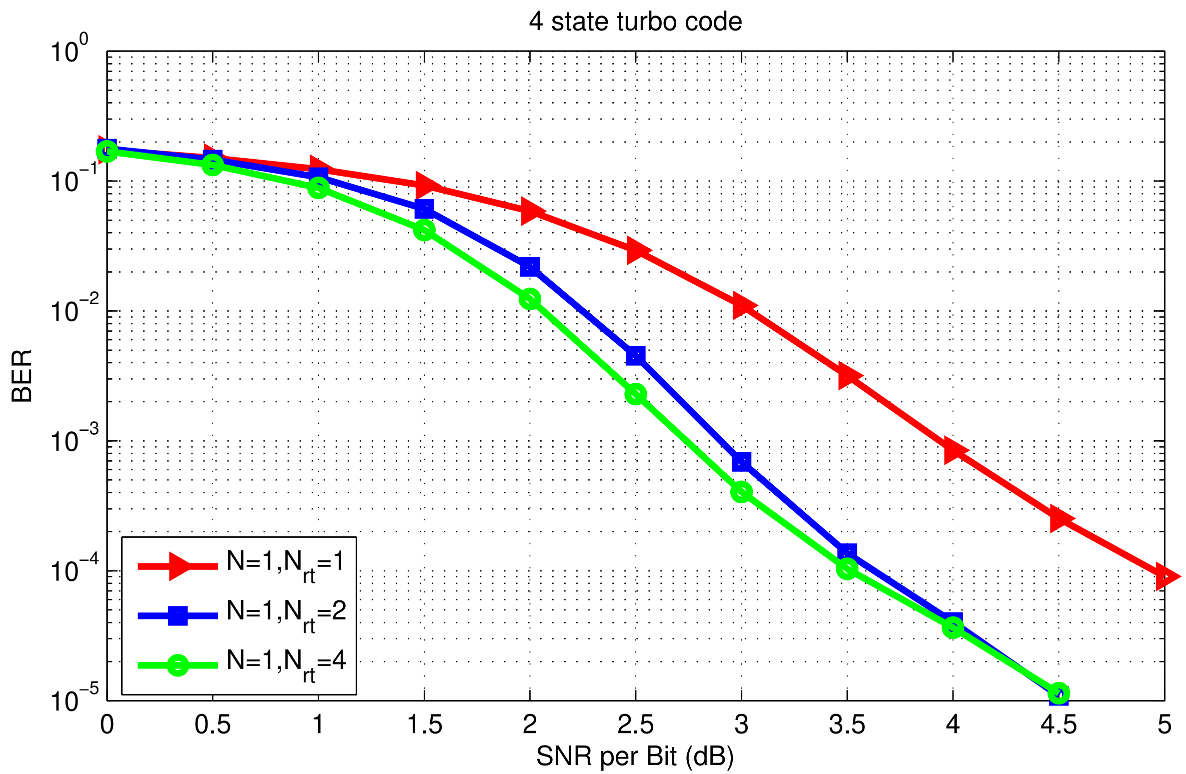

Figure 6: Results for 4-state turbo code, $N = 1$.

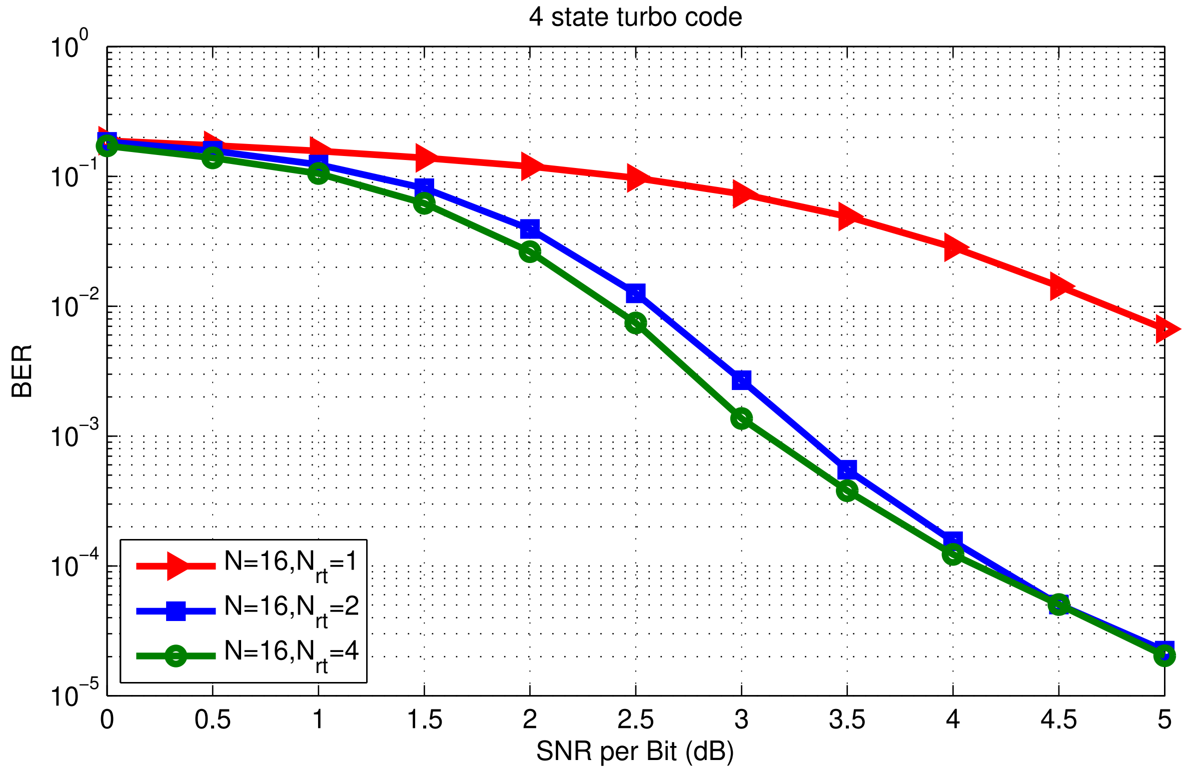

Figure 7: Results for 4-state turbo code, $N = 16$.

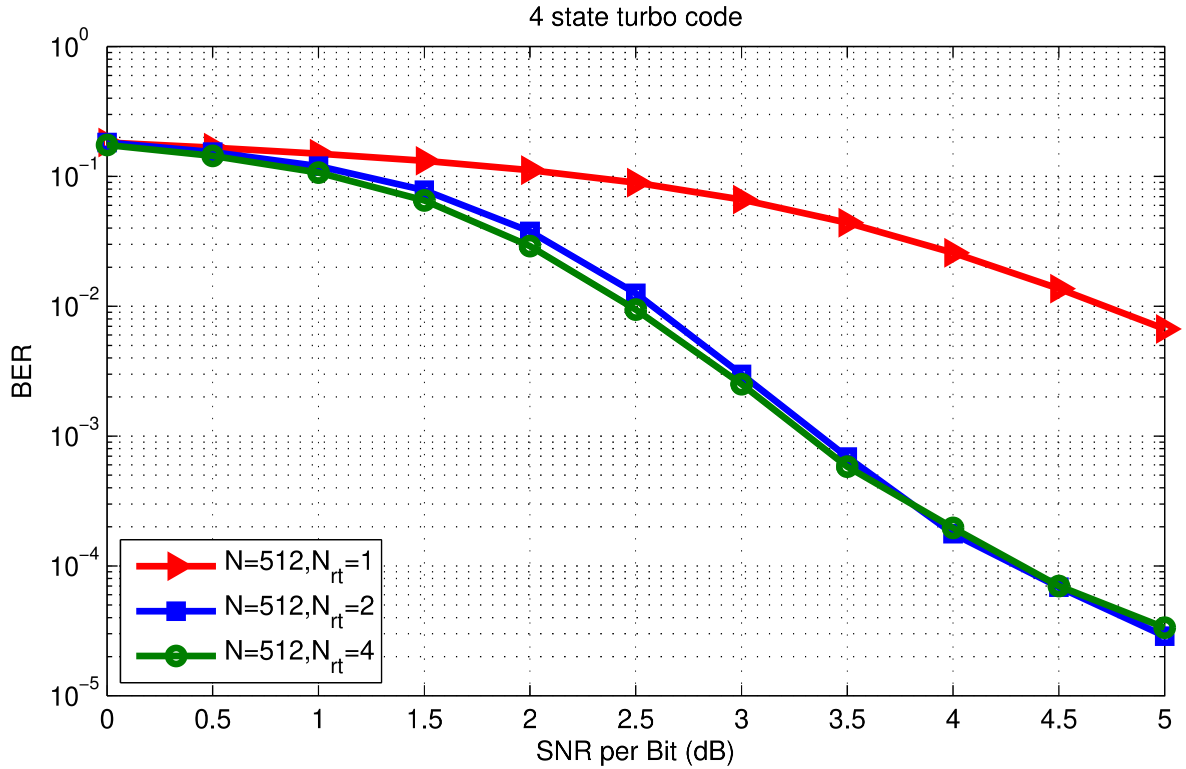

Figure 8: Results for 4-state turbo code, $N = 512$.

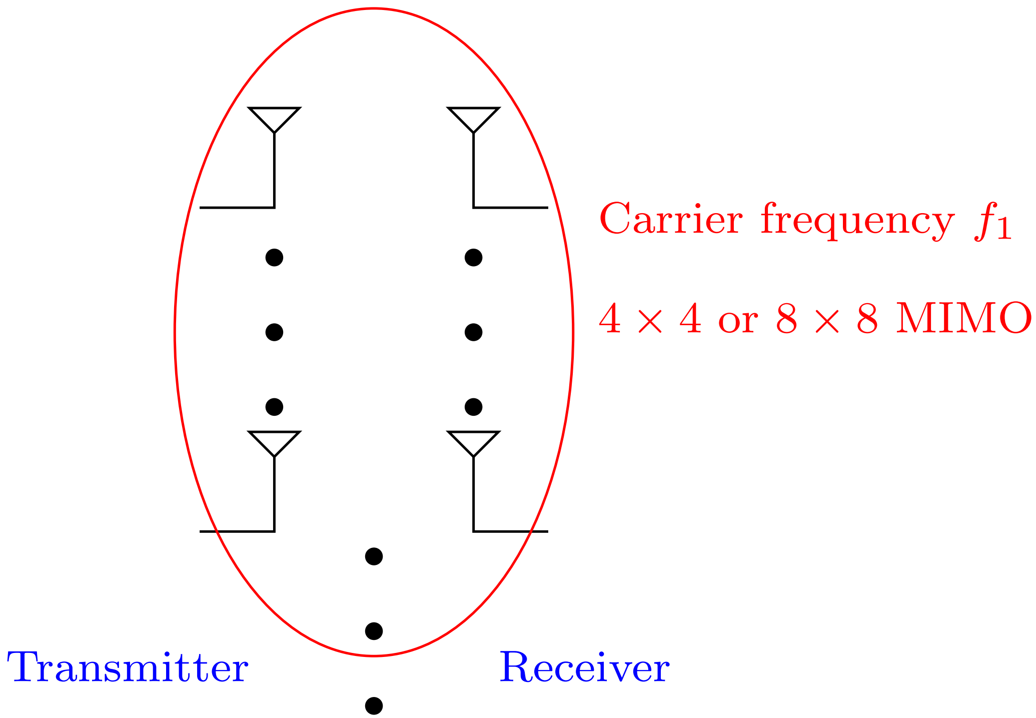

Figure 9: How to implement massive MIMO.

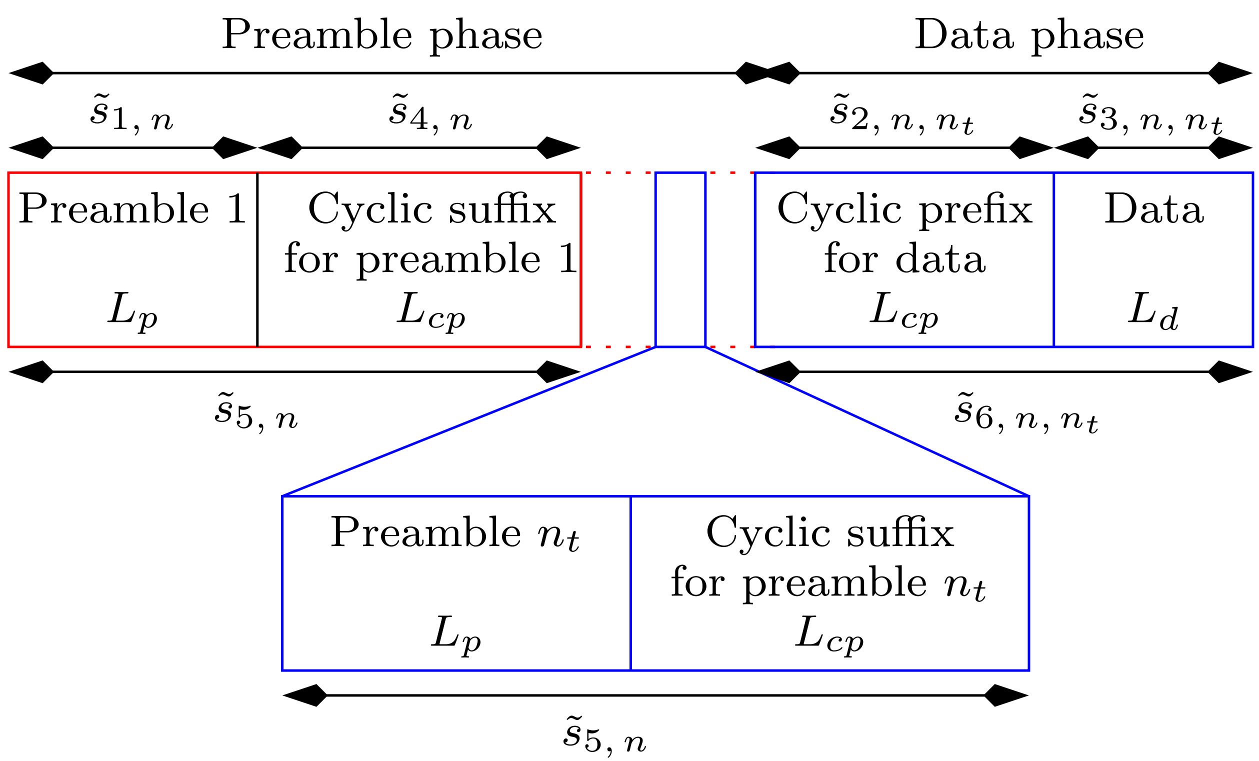

Figure 10: Frame structure for coherent massive MIMO.

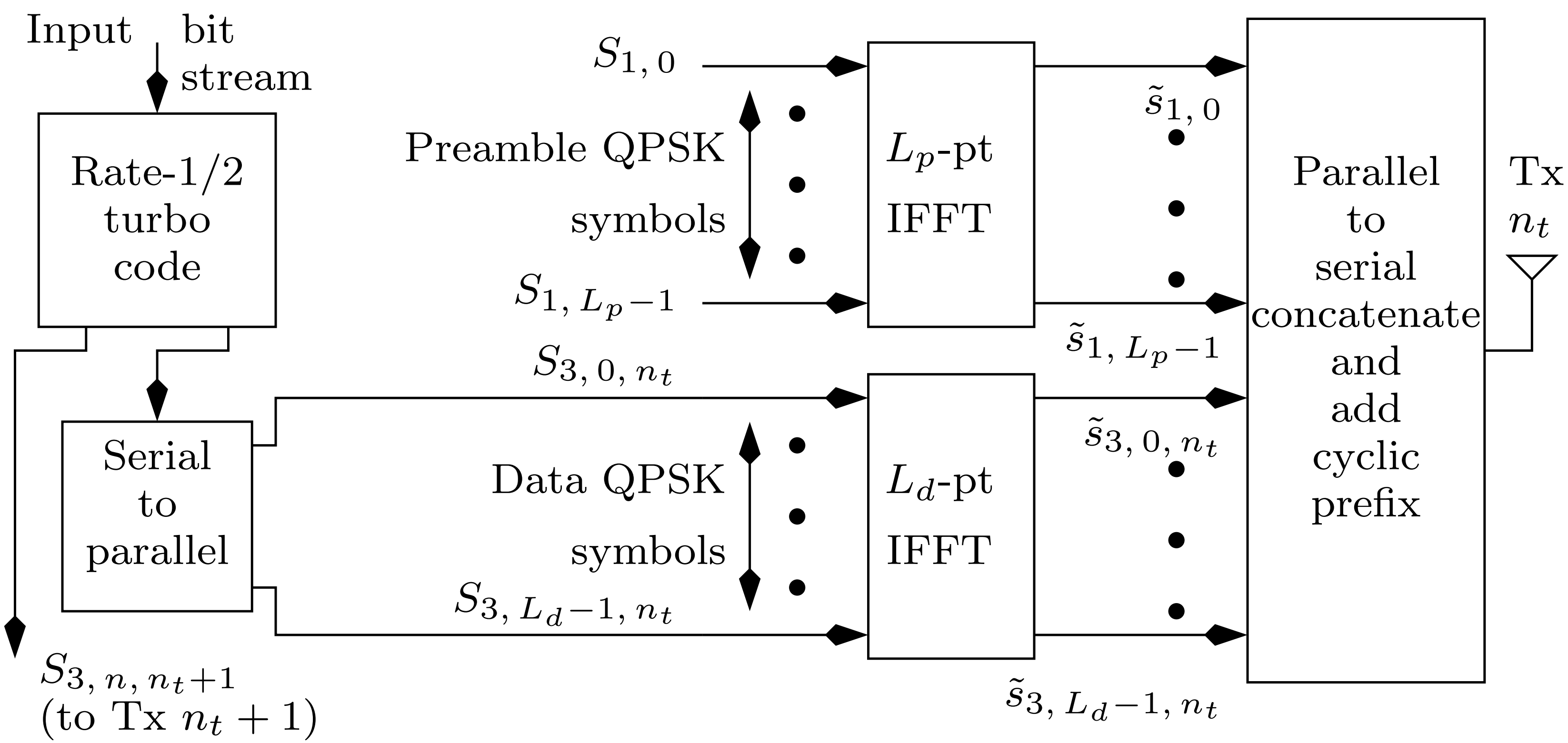

Figure 11: Tx n_t for coherent massive MIMO.

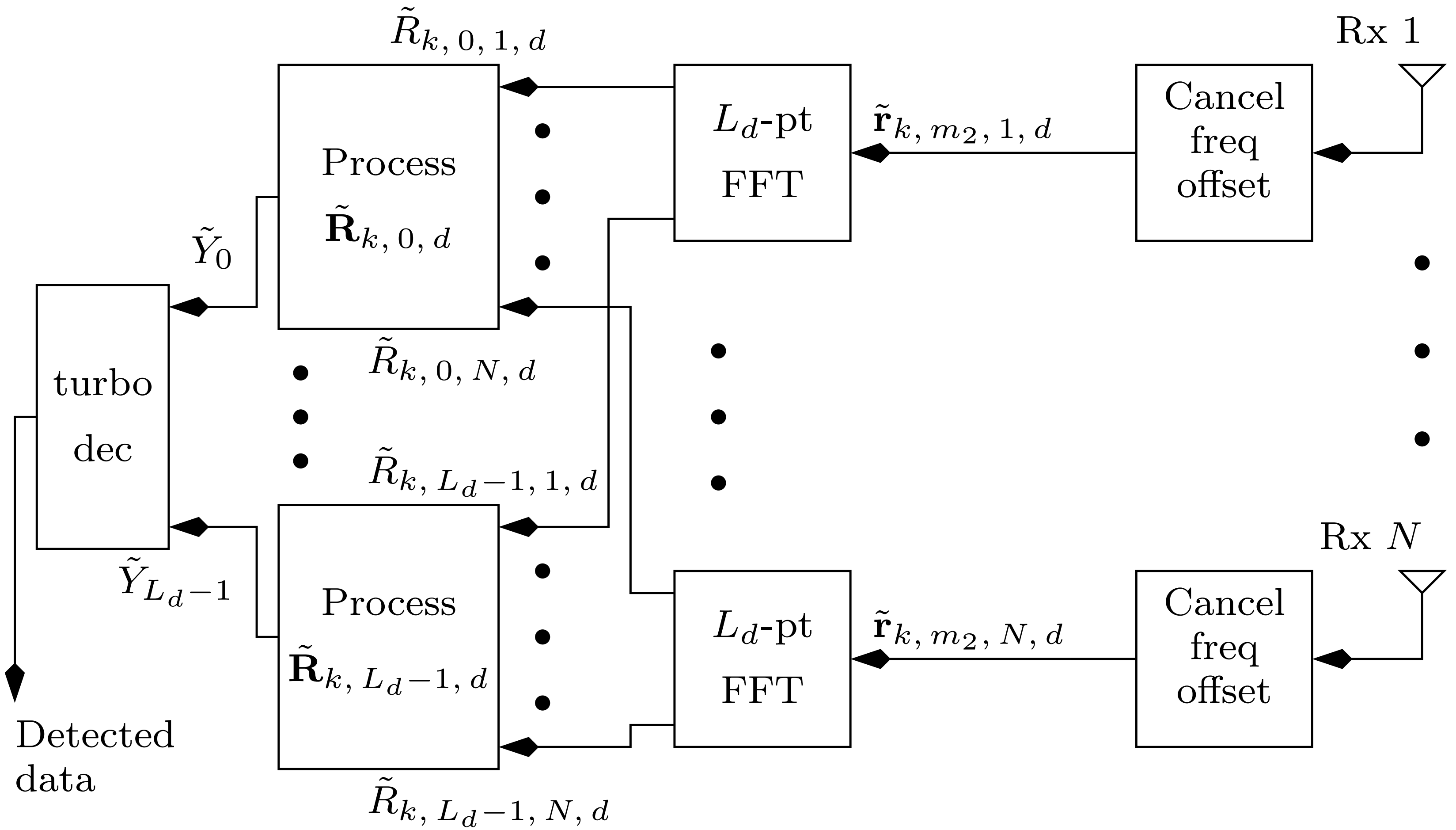

Figure 12: Rx for coherent massive MIMO.

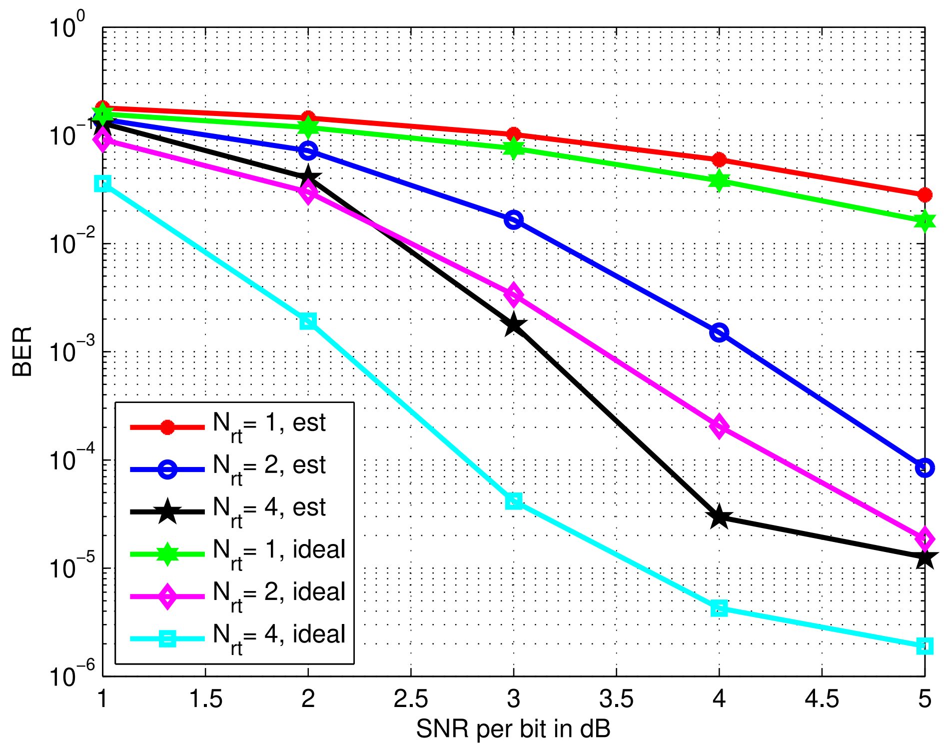

Figure 13: Results for 4×4 coherent MIMO.

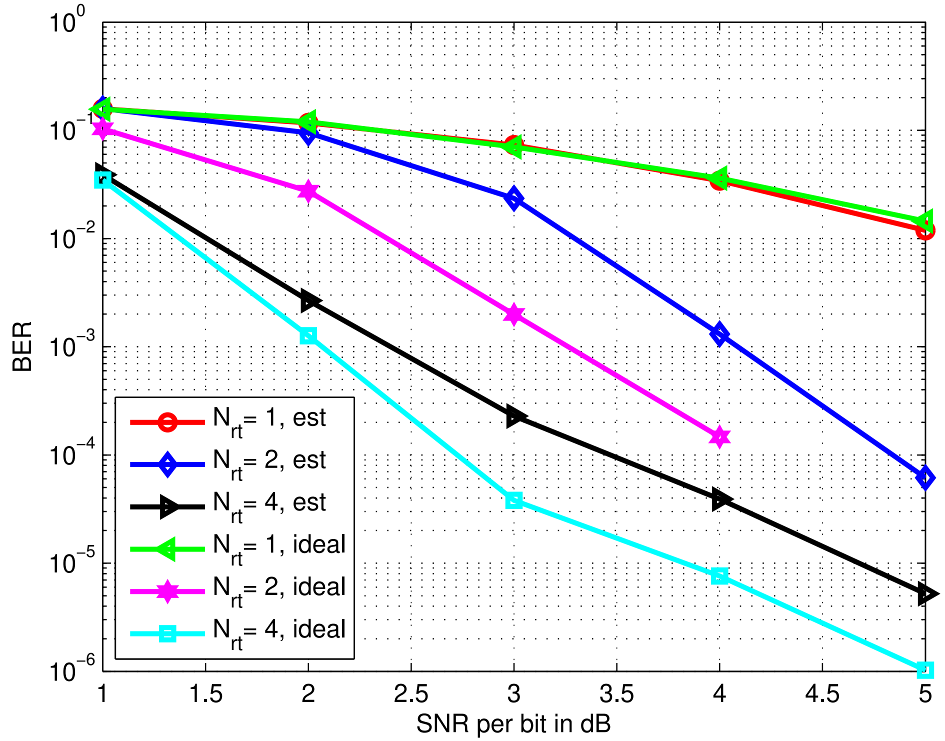

Figure 14: Results for 8×8 coherent MIMO.

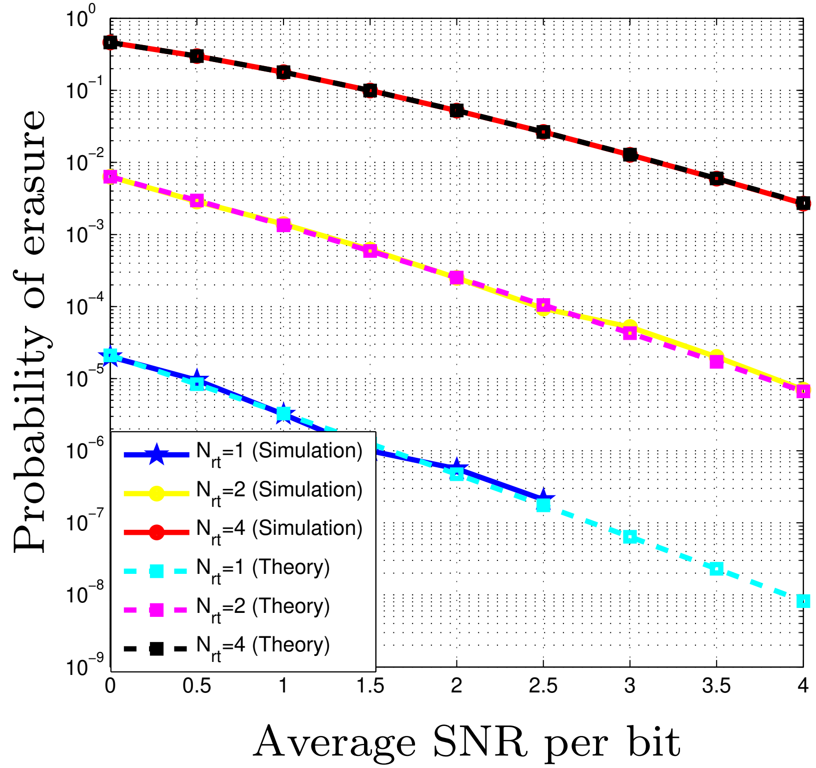

Figure 15: Probability of erasure for 4×4 MIMO.

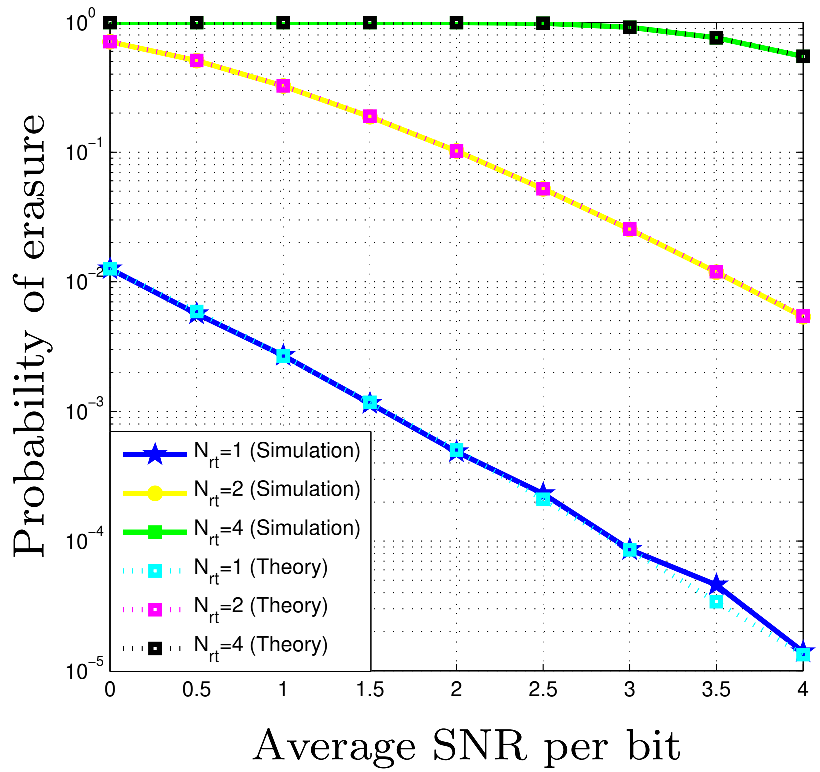

Figure 16: Probability of erasure for 8×8 MIMO.

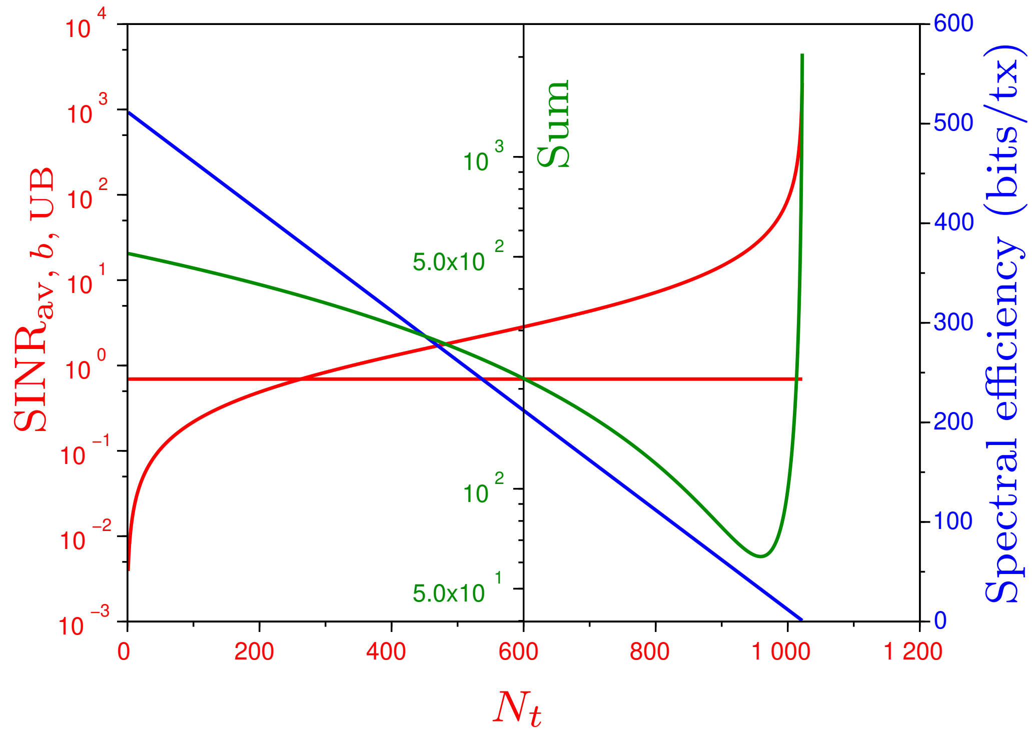

Figure 17: Upper bound on SINR, spectral efficiency and sum vs N_t.

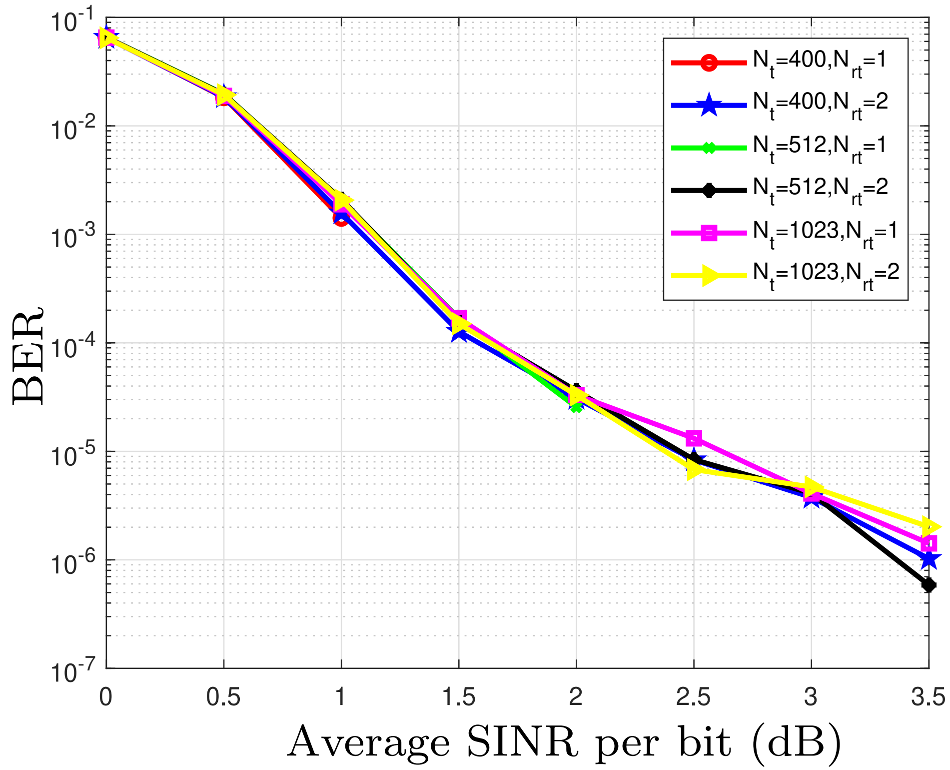

Figure 18: Simulation results with precoding.

Conclusions

- Re-transmissions may not be required.

References

- K. Vasudevan, K. Madhu, Shivani Singh, ““Data Detection in Single User Massive MIMO Using Re-Transmissions”, The Open Signal Processing Journal, vol. 6, pp. 15–26, March 2019. Also available at: https://arxiv.org/abs/1811.11369
- K. Vasudevan, Shivani Singh and A. Phani Kumar Reddy, “Coherent Receiver for Turbo Coded Single-User Massive MIMO-OFDM with Retransmissions”, IntechOpen, April 2019. Available at: https://www.intechopen.com/books/multiplexing/coherent-receiver-for-turbo-coded-single-user-massive-mimo-ofdm-with-retransmissions

References

- K. Vasudevan, A. Phani Kumar Reddy, Gyanesh Kumar Pathak, Shivani Singh, "On the Probability of Erasure for MIMO-OFDM", Semiconductor Science and Information Devices, Bilingual Publishing Co., Singapore, vol. 2, issue 1, April 2020.
- K. Vasudevan, Gyanesh Kumar Pathak and A. Phani Kumar Reddy, "Turbo Coded Single User Massive MIMO with Precoding", submitted. Preprint: https://arxiv.org/abs/2007.15959