Coherent Detection of OFDM

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Motivation for Coherent OFDM

- For a given bit-error-rate, coherent OFDM requires least signal power
  - This translates to longer battery life
  - Energy efficient (green) communication technology
- The full system design has not been considered earlier in the literature

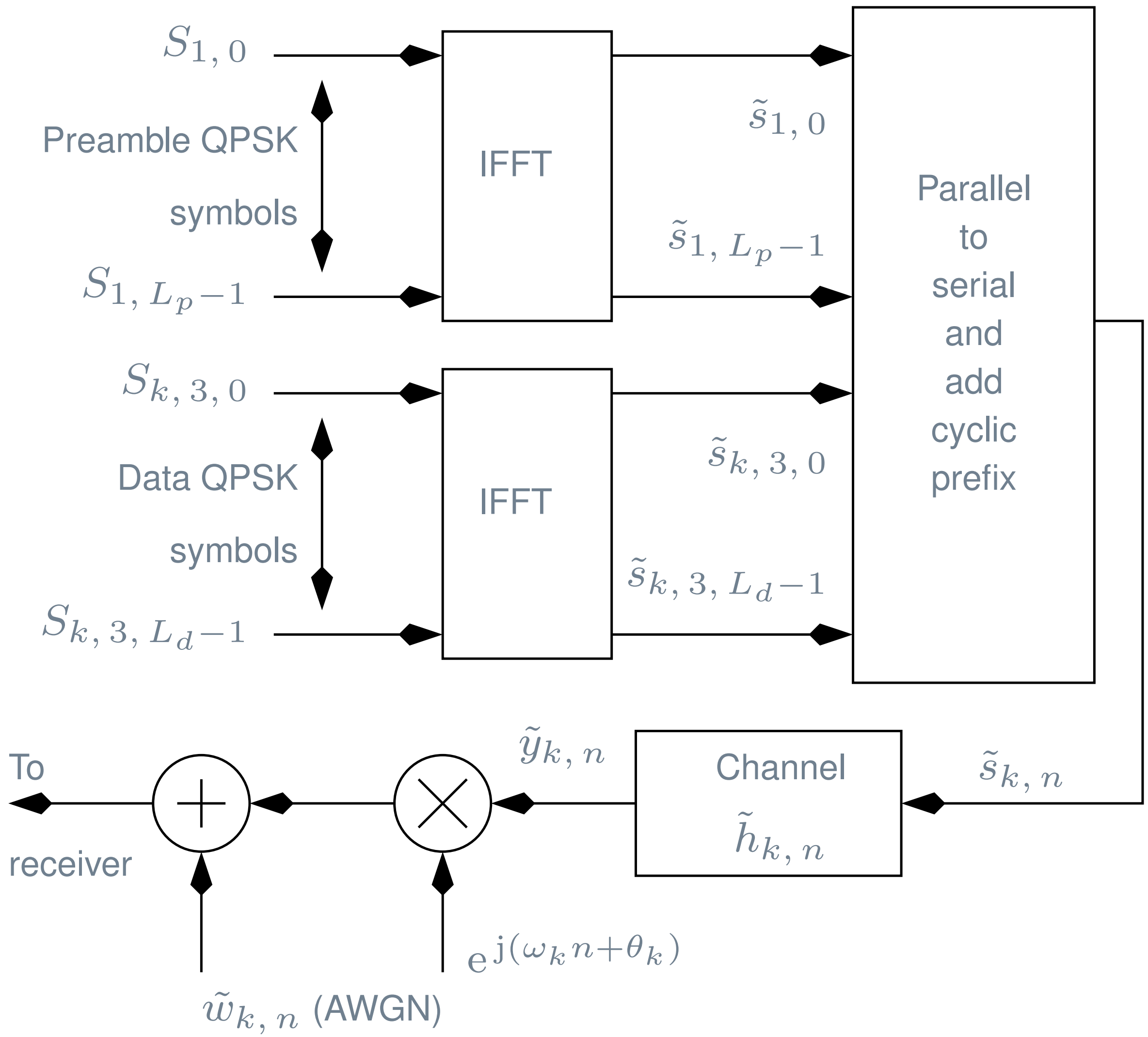
Notation

- Complex quantities denoted by tilde (e.g. $\tilde{s}_n$)
- Subscript n denotes time index
- $\tilde{s}_{1,\,n}$ denotes a known preamble of length L_p
- $\tilde{h}_{k,\,n}$ – channel response for the k^{th} frame
  - $\mathscr{C}\mathscr{N}(0,\ \sigma_f^2)$, independent over n
  - channel length L_h
- $\tilde{w}_{k,\,n}$ – AWGN for k^{th} frame
  - $\mathscr{C}\mathscr{N}(0,\ \sigma_w^2)$, independent over n

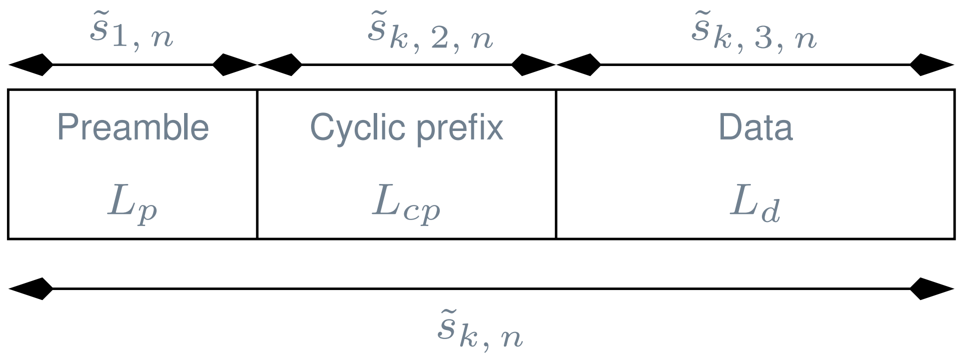
System Model

- Frequency selective Rayleigh fading channel having a uniform power delay profile is assumed
- Data is divided into frames, QPSK modulation
- Channel is static over one frame, span is L_h
- Channel span assumed by the receiver is L_{hr} $(> L_h)$

System Model – contd



System Model – contd



- Observe that
  - Preamble length is L_p
  - Length of cyclic prefix is $L_{cp} = L_{hr} - 1$
  - Length of data is L_d
- Total length of the frame is $L = L_p + L_{cp} + L_d$

System Model – contd

- Note that

$$\tilde{s}_{1,n} = \frac{1}{L_p}\sum_{i=0}^{Lp-1} S_{1,i} e^{\mathrm{j}\,2\pi ni/L_p}$$

(1)
$$\tilde{s}_{k,3,n} = \frac{1}{L_d}\sum_{i=0}^{Ld-1} S_{k,3,i} e^{\mathrm{j}\,2\pi ni/L_d}.$$

System Model – contd

- We assume $S_{k,3,i} \in \pm 1 \pm \mathrm{j}$. Since we require:

$$E\left[\left|\tilde{s}_{1,n}\right|^2\right] = E\left[\left|\tilde{s}_{k,3,n}\right|^2\right] = 2/L_d \triangleq \sigma_s^2 \quad (2)$$

  - We must have $S_{1,i} \in \sqrt{L_p/L_d}\,(\pm 1 \pm \mathrm{j})$.

System Model – contd

- The received signal for the k^{th} frame can be written as (for $0 \leq n \leq L + L_h - 2$):

$$
\begin{aligned}
\tilde{r}_{k,\,n} &= \left(\tilde{s}_{k,\,n} \star \tilde{h}_{k,\,n}\right) \mathrm{e}^{\mathrm{j}(\omega_k n + \theta_k)} + \tilde{w}_{k,\,n} \\
&= \tilde{y}_{k,\,n} \mathrm{e}^{\mathrm{j}(\omega_k n + \theta_k)} + \tilde{w}_{k,\,n} \qquad (3)
\end{aligned}
$$

  where "$\star$" denotes convolution and

$$
\tilde{y}_{k,\,n} = \tilde{s}_{k,\,n} \star \tilde{h}_{k,\,n}. \qquad (4)
$$

- $\omega_k \in [-0.04,\, 0.04]$ radians, $\theta_k \in [0,\, 2\pi)$ radians

System Model – contd

- The set of received samples can be denoted by the vector:

$$\tilde{\mathbf{r}}_k = \begin{bmatrix} \tilde{r}_{k,\,0} & \ldots & \tilde{r}_{k,\,L+L_h-2} \end{bmatrix}. \tag{5}$$

- For the purpose of SoF and coarse freq offset estimation for the k^{th} frame, we assume that the channel $\tilde{h}_{k,\,n}$ is known at the receiver

SoF and Coarse Freq Off Est

- Define the m^{th} $(0 \leq m \leq L_{cp} + L_d + L_h + L_{hr} - 2)$ received vector as:

(6) $$\tilde{\mathbf{r}}_{k,\,m} = \left[\begin{array}{ccc} \tilde{r}_{k,\,m} & \ldots & \tilde{r}_{k,\,m+L_p-L_{hr}} \end{array}\right].$$

- The “steady-state” preamble part of the transmitted signal appearing at the channel output can be represented by a vector:

(7) $$\tilde{\mathbf{y}}_{k,\,1} = \left[\begin{array}{ccc} \tilde{y}_{k,\,L_{hr}-1} & \ldots & \tilde{y}_{k,\,L_p-1} \end{array}\right].$$

SoF and Coarse Freq Off Est – contd

- The non-coherent maximum likelihood (ML) rule for frame detection can be stated as: Choose that time as the start of frame and that frequency $\hat{\omega}_k$, which jointly maximize the conditional pdf:

$$\max_{m,\,\hat{\omega}_k} \int_{\theta_k=0}^{2\pi} p\left(\tilde{\mathbf{r}}_{k,\,m} \middle| \tilde{\mathbf{y}}_{k,\,1}, \hat{\omega}_k, \theta_k\right) p(\theta_k)\, d\theta_k. \tag{8}$$

SoF and Coarse Freq Off Est – contd

- The final detection rule is:

$$\max_{m,\,\hat{\omega}_k} \left| \sum_{i=0}^{L_1-1} \tilde{r}_{m+i}\, \tilde{y}^{*}_{k,\,L_{hr}-1+i}\, \mathrm{e}^{-\mathrm{j}\,\hat{\omega}_k i} \right| \tag{9}$$

where $L_1 = L_p - L_{hr} + 1$

- The ideal outcome of (9) is:

$$\begin{aligned} m &= L_{hr} - 1 \\ \hat{\omega}_k &= \omega_k. \end{aligned} \tag{10}$$

SoF and Coarse Freq Off Est – conto

- In practice, the receiver has only the estimate of the channel ($\hat{h}_{k,\,n}$), hence $\tilde{y}_{k,\,n}$ must be replaced by $\hat{y}_{k,\,n}$, where

(11) $$\hat{y}_{k,\,n} = \tilde{s}_{1,\,n} \star \hat{h}_{k,\,n}.$$

- When $\hat{h}_{k,\,n}$ is not available, we propose a heuristic method of frame detection:

(12) $$\max_{m,\,\hat{\omega}_k} \left| \sum_{i=0}^{L_p-1} \tilde{r}_{m+i}\, \tilde{s}^*_{1,\,i}\, \mathrm{e}^{-\mathrm{j}\,\hat{\omega}_k i} \right|.$$

SoF and Coarse Freq Off Est – contd

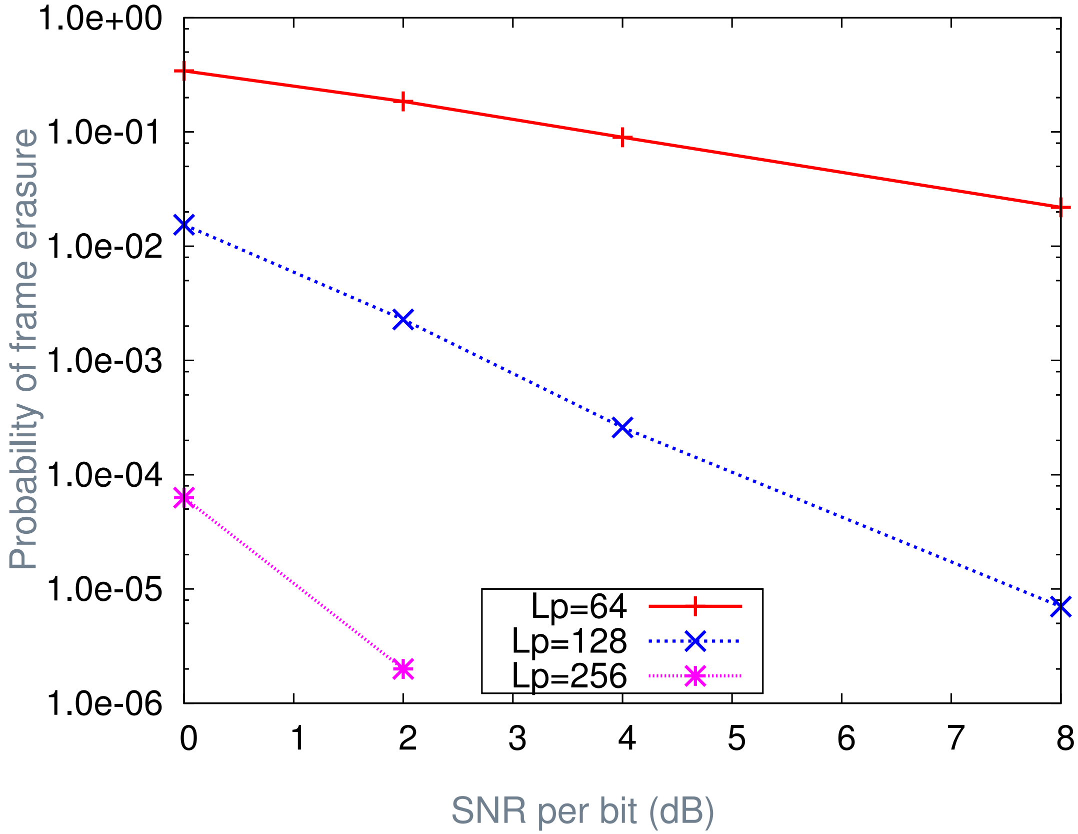
- The ideal outcome of (12) is:

$$\begin{aligned} 0 &\le m \le L_h - 1 \\ \hat{\omega}_k &= \omega_k \end{aligned} \quad (13)$$

depending on which channel coefficient has the maximum magnitude.

- When m lies outside the range in (13), the frame is declared as erased (lost).
- Coarse freq off est $\hat{\omega}_k$: obtained by dividing $[-0.04, 0.04]$ rad into B_1 freq bins and selecting that bin which maximizes (12).

SoF and Coarse Freq Off Est – contd



- For $L_p = 512$, prob of erasure is $< 10^{-6}$

Channel Estimation

- ML channel estimation is considered
- Assumptions:
  - Freq offset has been canceled
  - SoF has been detected with outcome m_0
- Define

$$m_1 = m_0 + L_h - 1. \tag{14}$$

- The steady-state, preamble part of the received signal for the k^{th} frame is:

$$\tilde{\mathbf{r}}_{k,\,m_1} = \tilde{\mathbf{s}}_1 \tilde{\mathbf{h}}_k + \tilde{\mathbf{w}}_{k,\,m_1} \tag{15}$$

Channel Estimation – contd

where

$$\tilde{\mathbf{r}}_{k,m_1} = \begin{bmatrix} \tilde{r}_{k,m_1} & \ldots & \tilde{r}_{k,m_1+L_p-L_{hr}} \end{bmatrix}^T$$

$[(L_p - L_{hr} + 1) \times 1]$ vector

$$\tilde{\mathbf{w}}_{k,m_1} = \begin{bmatrix} \tilde{w}_{k,m_1} & \ldots & \tilde{w}_{k,\,m_1+L_p-L_{hr}} \end{bmatrix}^T$$

$[(L_p - L_{hr} + 1) \times 1]$ vector

$$\tilde{\mathbf{h}}_k = \begin{bmatrix} \tilde{h}_{k,0} & \ldots & \tilde{h}_{k,L_{hr}-1} \end{bmatrix}^T$$

(16) $[L_{hr} \times 1]$ vector

Channel Estimation – contd

and

$$\tilde{\mathbf{s}}_1 = \begin{bmatrix} \tilde{s}_{1,L_{hr}-1} & \cdots & \tilde{s}_{1,0} \\ \vdots & \cdots & \vdots \\ \tilde{s}_{1,L_p-1} & \cdots & \tilde{s}_{1,L_p-L_{hr}-2} \end{bmatrix}$$

(17) $[(L_p - L_{hr} + 1) \times L_{hr}]$ matrix

Channel Estimation – contd

- The ML channel estimate is: find $\hat{\mathbf{h}}_k$ (the estimate of $\tilde{\mathbf{h}}_k$) such that:

$$\left(\tilde{\mathbf{r}}_{k,\,m_1} - \tilde{\mathbf{s}}_1\hat{\mathbf{h}}_k\right)^H \left(\tilde{\mathbf{r}}_{k,\,m_1} - \tilde{\mathbf{s}}_1\hat{\mathbf{h}}_k\right) \tag{18}$$

is minimized.

Channel Estimation – contd

- Differentiating with respect to $\hat{\mathbf{h}}_k^*$ and setting the result to zero yields:

(19) $$\hat{\mathbf{h}}_k = \left(\tilde{\mathbf{s}}_1^H \tilde{\mathbf{s}}_1\right)^{-1} \tilde{\mathbf{s}}_1^H \tilde{\mathbf{r}}_{k,\, m_1}.$$

- To see the effect of noise on the channel estimate in (19), consider

(20) $$\tilde{\mathbf{u}} = \left(\tilde{\mathbf{s}}_1^H \tilde{\mathbf{s}}_1\right)^{-1} \tilde{\mathbf{s}}_1^H \tilde{\mathbf{w}}_{k,\, m_1}.$$

- Clearly $E\left[\tilde{\mathbf{u}}\right] = 0$.

Channel Estimation – contd

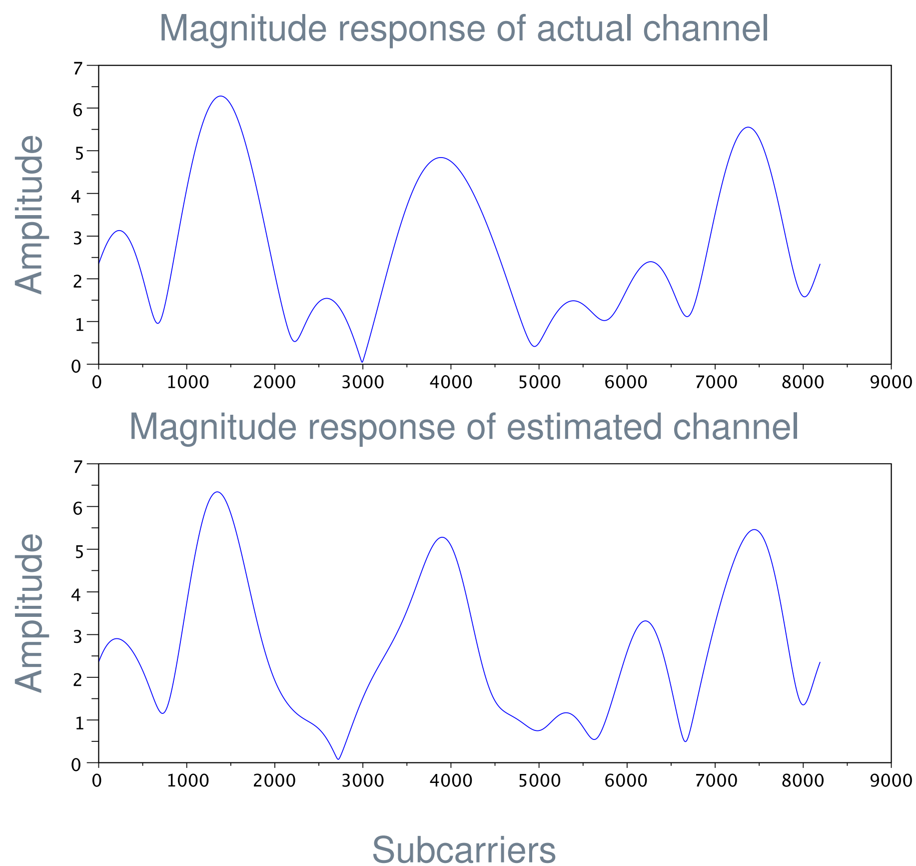
- It can be shown that

$$E\left[\tilde{\mathbf{u}}\tilde{\mathbf{u}}^H\right] = \frac{2\sigma_w^2}{L_1\sigma_s^2}\mathbf{I}_{L_{hr}} = \frac{\sigma_w^2 L_d}{L_1}\mathbf{I}_{L_{hr}} \triangleq 2\sigma_u^2\mathbf{I}_{L_{hr}}.$$

(21)

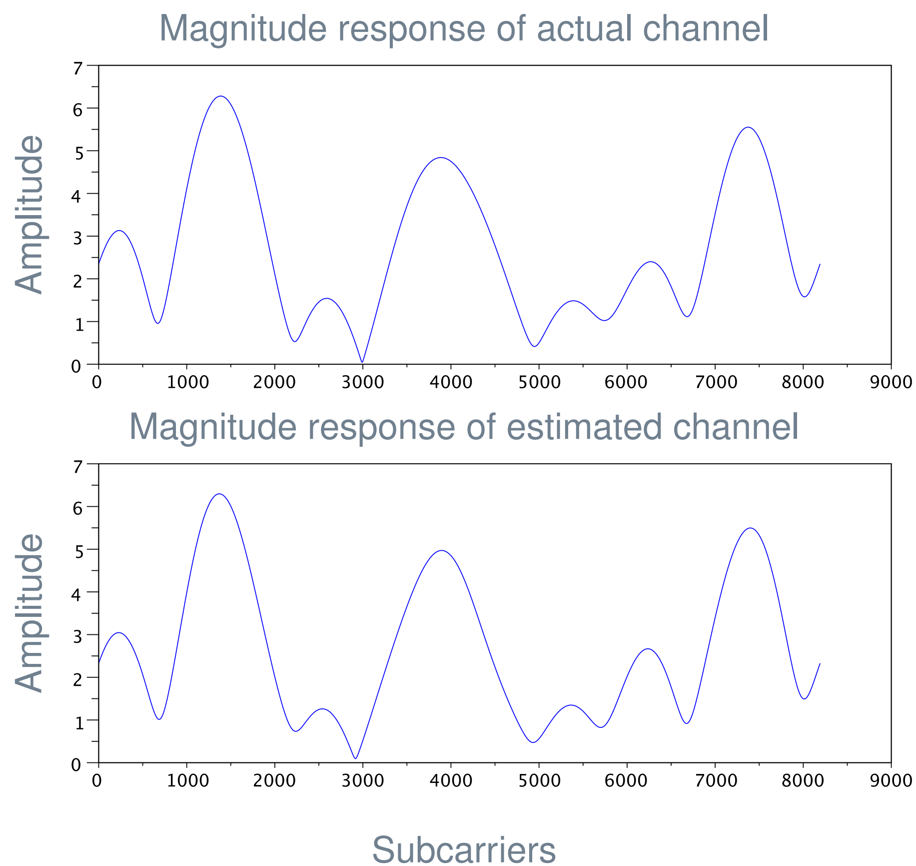
- Therefore, the variance of the ML channel estimate (σ_u^2) tends to zero as $L_1 \rightarrow \infty$ and L_d is kept fixed.

Channel Estimation – contd



- SNR per bit is 0 dB, $L_p = 512$.

Channel Estimation – contd



- SNR per bit is 10 dB, $L_p = 512$.

Fine Freq Off Est

- Estimation rule:

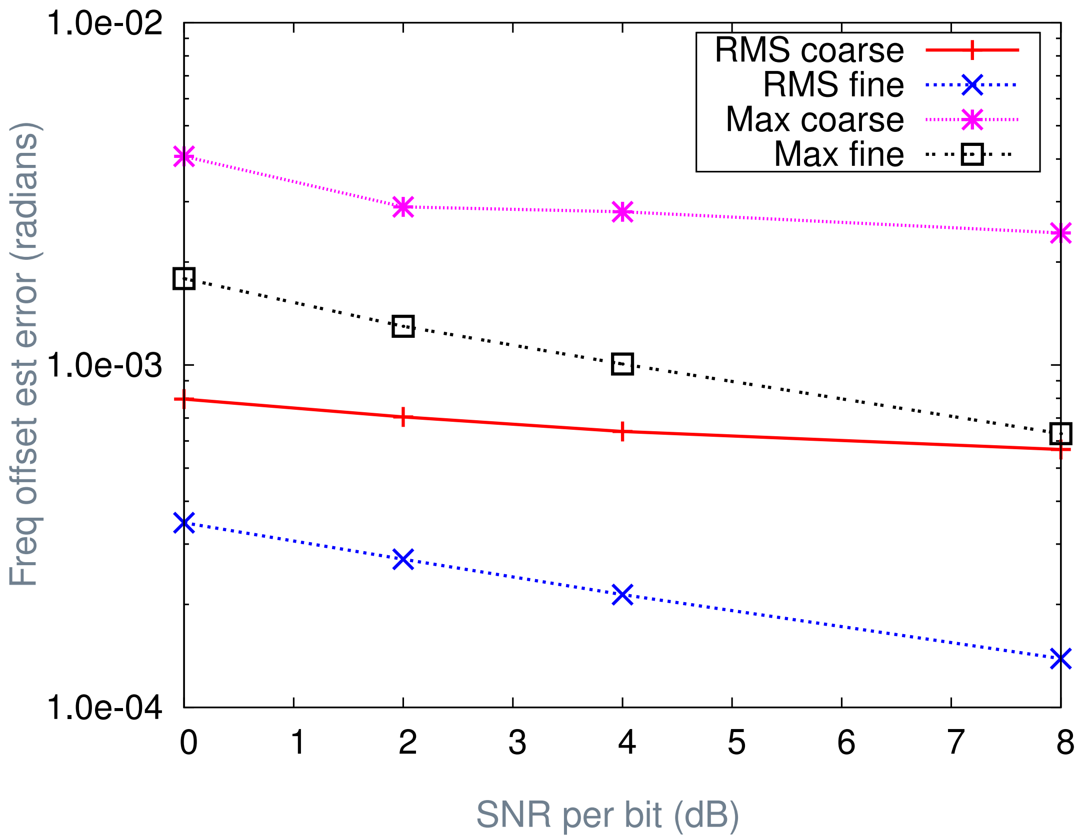
(22) $$\max_{m,\,\hat{\omega}_{k,\,f}} \left| \sum_{i=0}^{L_2-1} \tilde{r}_{m+i}\, \hat{y}_{k,\,i}^{*}\, \mathrm{e}^{-\mathrm{j}\,(\hat{\omega}_k+\hat{\omega}_{k,\,f})i} \right|$$

where $\hat{y}_{k,\,i}$ is given in (11) and:

(23) $$\begin{aligned} L_2 &= L_{hr} + L_p - 1 \\ 0 &\leq m \leq L_{hr} - 1. \end{aligned}$$

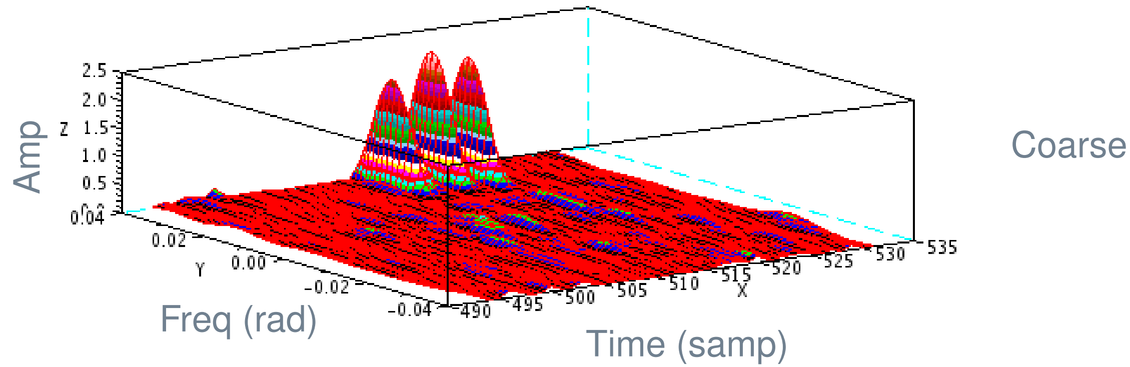
- $\hat{\omega}_{k,\,f}$ obtained by dividing $[\hat{\omega}_k - 0.005,\ \hat{\omega}_k + 0.005]$ rad into B_2 freq bins

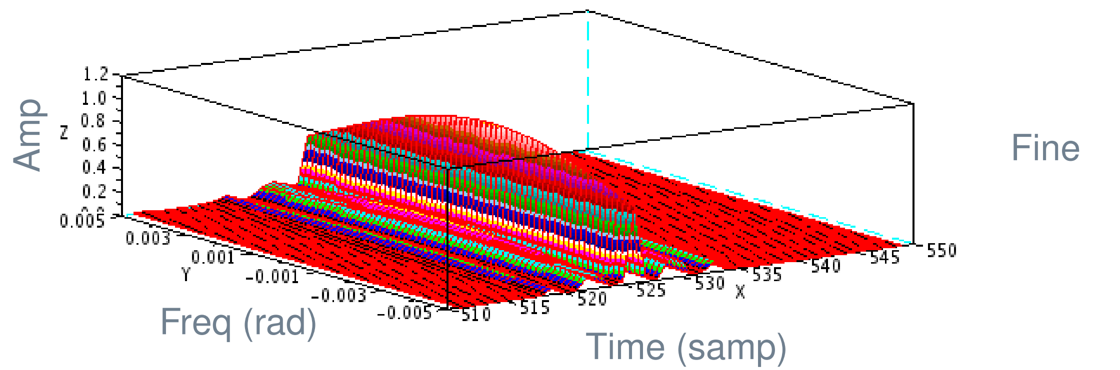
Fine Freq off est – contd



- RMS and max freq off est error for $L_p = 512$.

Fine Freq off est – contd





- SoF, coarse and fine freq off est for $L_p = 512$, SNR 0 dB, $B_1 = B_2 = 64$.

Fine Freq off est – contd

Complexity (Frequency bins)	
Single stage	Two stage
512	128

- Complexity of two-stage approach: $B_1 + B_2 = 128$.
- Resolution of two-stage approach is $2 \times 0.005/64 = 0.00015625$ radians.

Fine Freq off est – contd

- For obtaining the same resolution, the single stage approach requires $2 \times 0.04/0.00015625 = 512$ frequency bins.
- Two-stage is four times more efficient than single stage.

Noise Var Est

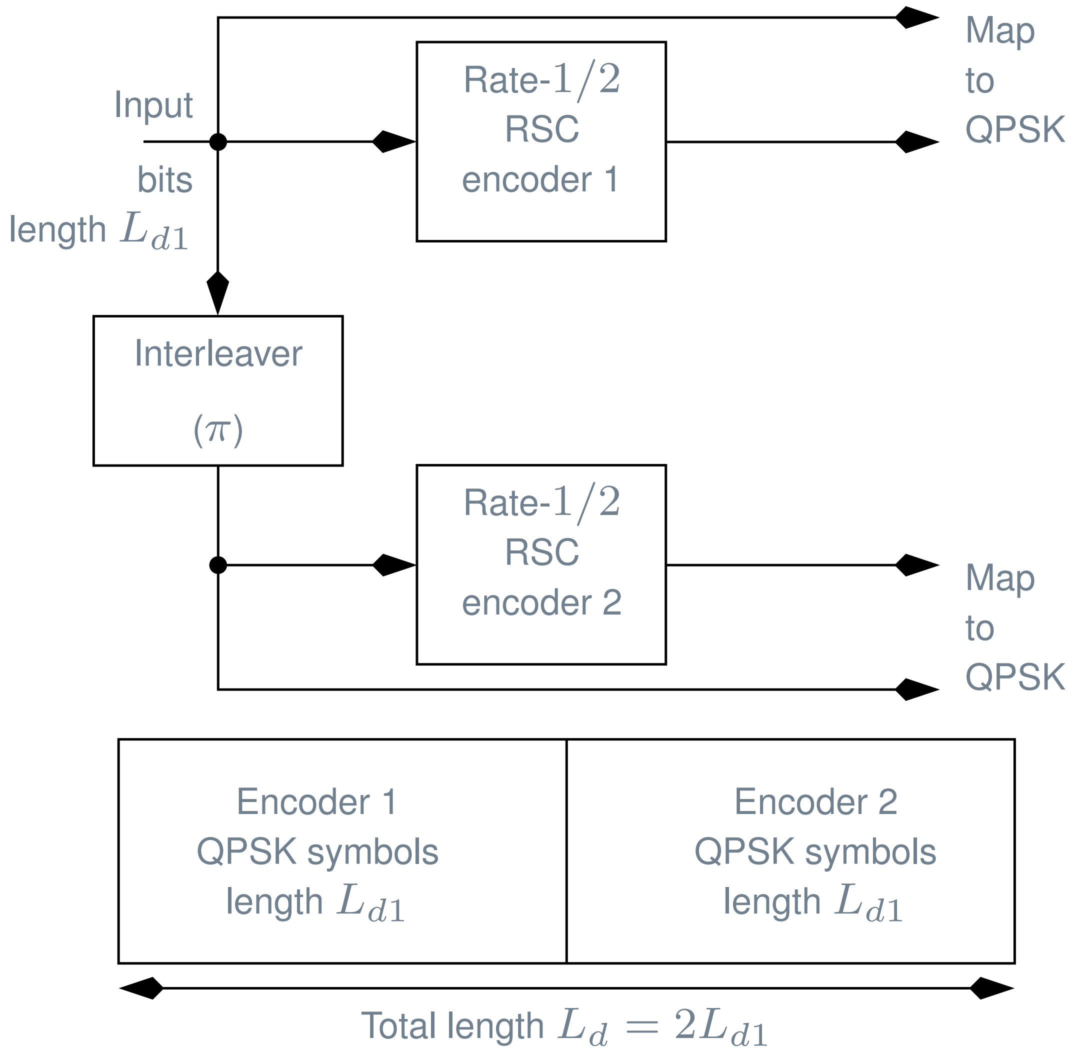
- After the channel has been estimated using (19), the noise variance is estimated as follows:

$$\hat{\sigma}_w^2 = \frac{1}{2L_1}\left(\tilde{\mathbf{r}}_{k,\,m_1} - \tilde{\mathbf{s}}_1\hat{\mathbf{h}}_k\right)^H\left(\tilde{\mathbf{r}}_{k,\,m_1} - \tilde{\mathbf{s}}_1\hat{\mathbf{h}}_k\right)$$

(24)

where $\tilde{\mathrm{s}}_1$ is defined in (17).

Turbo Decoding



Turbo Decoding – contd

- Generating matrix for each constituent encoders:

$$\mathbf{G}(D) = \begin{bmatrix} 1 & \frac{1+D^2}{1+D+D^2} \end{bmatrix}. \tag{25}$$

- Let

$$m_2 = m_1 + L_p \tag{26}$$

where m_1 is defined in (14).

Turbo Decoding – contd

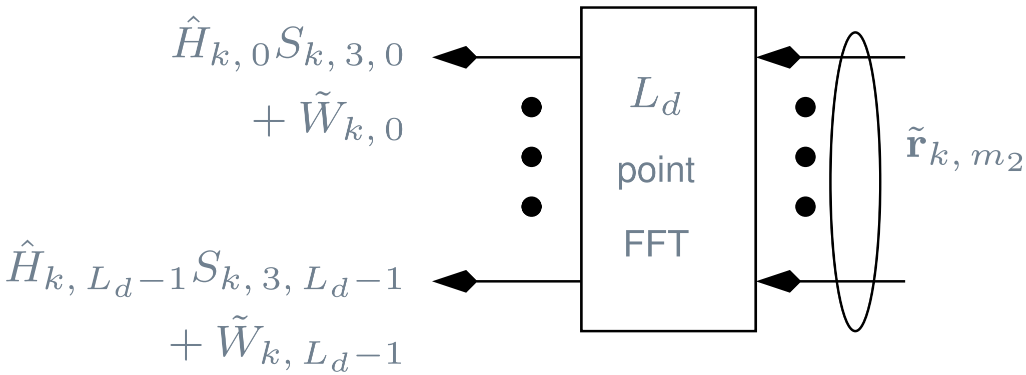
- Define

$$\tilde{\mathbf{r}}_{k,\,m_2} = \left[\begin{array}{ccc} \tilde{r}_{k,\,m_2} & \cdots & \tilde{r}_{k,\,m_2+L_d-1} \end{array}\right] \tag{27}$$

  as the data part of the received signal for the k^{th} frame.

- Assumptions:
  - SoF detected
  - Frequency offset perfectly canceled
  - Channel estimated

Turbo Decoding – contd



- OFDM receiver after synchronization.
- FFT output (for $0 \le i \le L_d - 1$):

$$\tilde{R}_{k,\,i} = \hat{H}_{k,\,i}S_{k,\,3,\,i} + \tilde{W}_{k,\,i}. \tag{28}$$

Turbo Decoding – contd

- The variance of $\tilde{W}_{k,\,i}$ is

$$\frac{1}{2}E\left[\left|\tilde{W}_{k,\,i}\right|^2\right] = L_d\sigma_w^2 \tag{29}$$

- Variance of $\hat{H}_{k,\,i}$ is (assuming perfect channel estimates, that is $\hat{H}_{k,\,i} = \tilde{H}_{k,\,i}$):

$$\frac{1}{2}E\left[\left|\tilde{H}_{k,\,i}\right|^2\right] = L_h\sigma_f^2. \tag{30}$$

Turbo Decoding – contd

- Corresponding to the transition from state m to state n, at decoder 1, for the k^{th} frame, at time i define (for $0 \le i \le L_{d1} - 1$:

$$\gamma_{1,\,k,\,i,\,m,\,n} = \exp\left[-\frac{\left(\tilde{R}_{k,\,i} - \hat{H}_{k,\,i} S_{m,\,n}\right)^2}{2L_d\hat{\sigma}_w^2}\right]$$

(31)

where $S_{m,\,n}$ denotes the QPSK symbol corresponding to the transition from state m to state n in the trellis.

Turbo Decoding – contd

- The alpha values for decoder 1 can be recursively computed as follows (forward recursion):

$$
\begin{aligned}
\alpha'_{i+1,n} &= \sum_{m \in \mathscr{C}_n} \alpha_{i,m} \gamma_{1,k,i,m,n} P\left(S_{b,i,m,n}\right) \\
\alpha_{0,n} &= 1 \qquad \text{for } 0 \leq n \leq \mathscr{S} - 1 \\
\alpha_{i+1,n} &= \alpha'_{i+1,n} \Big/ \left(\sum_{n=0}^{\mathscr{S}-1} \alpha'_{i+1,n} \right)
\end{aligned}
$$

(32)

Turbo Decoding – contd

where

$$P(S_{b,\,i,\,m,\,n}) = \begin{cases} F_{2,\,i+} & \text{if } S_{b,\,i,\,m,\,n} = +1 \\ F_{2,\,i-} & \text{if } S_{b,\,i,\,m,\,n} = -1 \end{cases}$$

(33)

denotes the *a priori* probability of the systematic bit corresponding to the transition from state m to state n, at decoder 1, at time i, obtained from the 2^{nd} decoder at time l, after deinterleaving (that is, $i = \pi^{-1}(l)$ for some $0 \leq l \leq L_{d1} - 1$.

Turbo Decoding – contd

- The recursion for beta (backward recursion) at decoder 1 can be written as:

$$\begin{aligned}
\beta'_{i,n} &= \sum_{m \in \mathscr{D}_n} \beta_{i+1,m} \gamma_{1,k,i,n,m} P\left(S_{b,i,n,m}\right) \\
\beta_{L_{d1},n} &= 1 \qquad \text{for } 0 \leq n \leq \mathscr{S} - 1 \\
\beta_{i,n} &= \beta'_{i,n} \Big/ \left(\sum_{n=0}^{\mathscr{S}-1} \beta'_{i,n} \right).
\end{aligned}$$

(34)

Turbo Decoding – contd

- Let $\rho^+(n)$ denote the state that is reached from state n when the input symbol is $+1$. Similarly let $\rho^-(n)$ denote the state that can be reached from state n when the input symbol is -1. Then (for $0 \le i \le L_{d1} - 1$)

$$
\begin{aligned}
G_{1,\,i+} &= \sum_{n=0}^{\mathscr{S}-1} \alpha_{i,\,n} \gamma_{1,\,k,\,i,\,n,\,\rho^+(n)} \beta_{i+1,\,\rho^+(n)} \\
G_{1,\,i-} &= \sum_{n=0}^{\mathscr{S}-1} \alpha_{i,\,n} \gamma_{1,\,k,\,i,\,n,\,\rho^-(n)} \beta_{i+1,\,\rho^-(n)}. \qquad (35)
\end{aligned}
$$

Turbo Decoding – contd

- The extrinsic information that is to be fed as *a priori* probabilities to the second decoder after interleaving, is computed as:

$$
\begin{aligned}
F_{1,i+} &= G_{1,i+}/(G_{1,i+} + G_{1,i-}) \\
F_{1,i-} &= G_{1,i-}/(G_{1,i+} + G_{1,i-}) \qquad (36)
\end{aligned}
$$

Simulation Results

- There are two coded QPSK symbols for every uncoded bit, the SNR per bit (over two dimensions) is defined as:

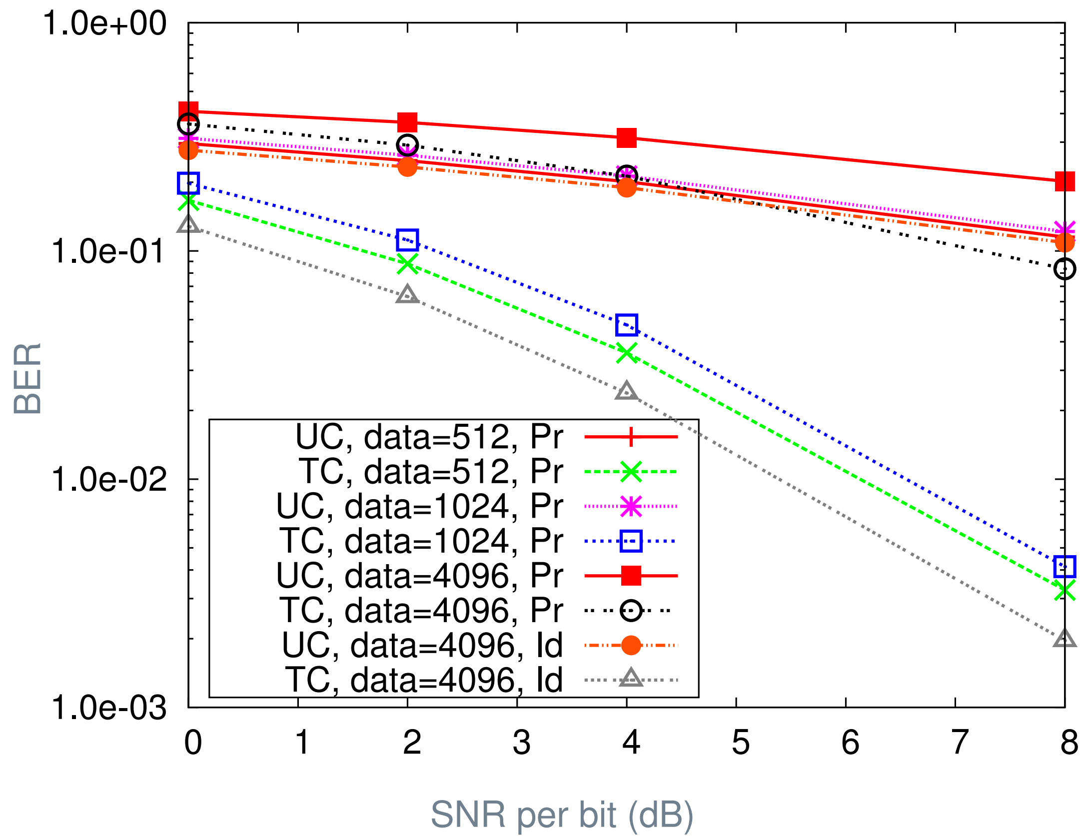
$$\text{SNR per bit} = \frac{2E\left[\left|\hat{H}_{k,\,i}S_{k,\,3,\,i}\right|^2\right]}{E\left[\left|\hat{W}_{k,\,i}\right|^2\right]}$$

$$= \frac{4L_h\sigma_f^2}{L_d\sigma_w^2}. \tag{37}$$

Simulation Results – contd

- Channel length $L_h = 10$.
- Channel length assumed by the receiver $L_{hr} = 2L_h - 1 = 19$.
- Fade variance per dimension $\sigma_f^2 = 0.5$.
- Length of preamble $L_p = 512$.

Simulation Results – contd

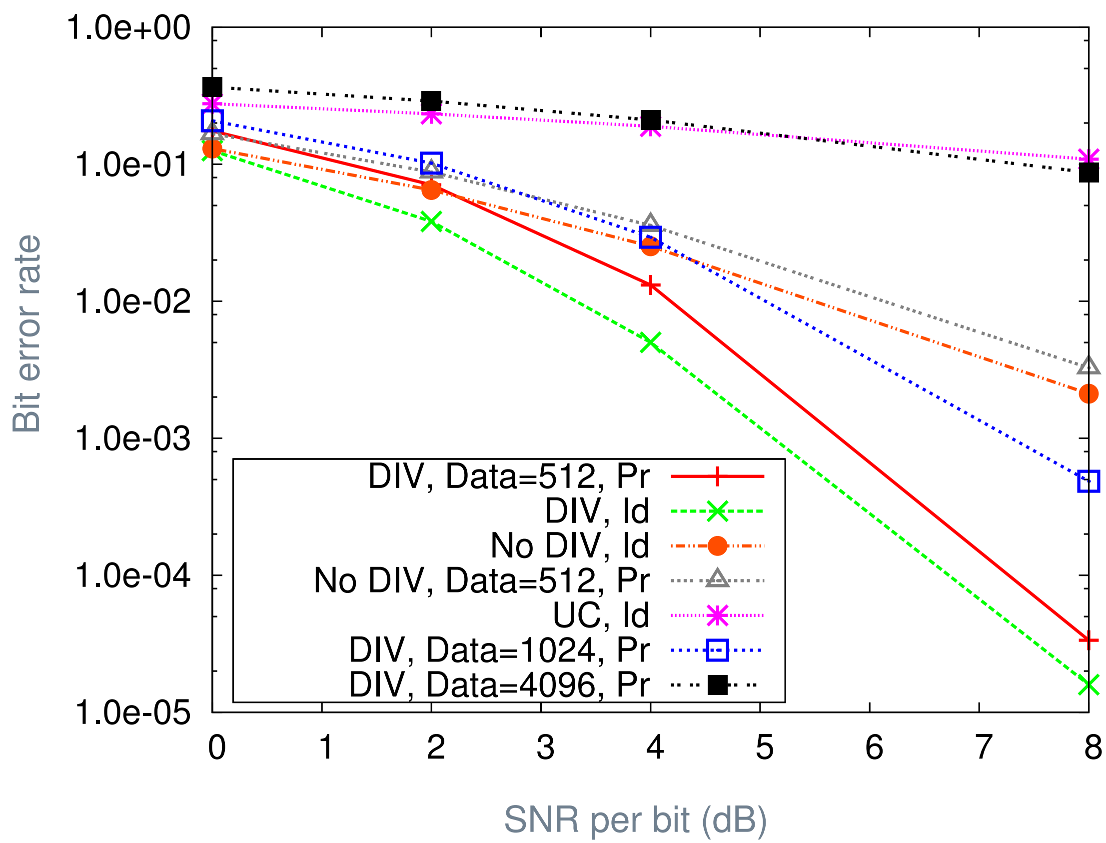


- Performance of ideal rx independent of data length

Simulation Results – contd

- The performance can be improved by noting that $\tilde{H}_{k,\,i}$ is highly correlated.
  - Interleave the coded symbols before IFFT at tx.
  - Deinterleave the coded symbols after FFT at rx.

Simulation Results – contd



- $L_{d1} = 512, L_d = 1024.$

Conclusions

- Performance of the practical rx close to ideal rx for $L_{d1} = L_p$.
- The data interleaver significantly enhances the performance of the rx.
  - BER close to 10^{-5} for 8 dB SNR per bit.

Future Work

- Improve the performance for larger data lengths.
  - Improve the accuracy of the frequency offset estimate.
- Use a prediction filter instead of a data interleaver.
  - A prediction filter exploits the correlation in $\tilde{H}_{k,\,i}$.
- Increase the overall code-rate to unity using turbo trellis coded modulation.

Acknowledgement

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Invited Talk

- This work was presented at the International Federation of Nonlinear Analysts (IFNA) World Congress, Athens, Greece, 25th June-1st July, 2012.