Turbo Coded Single User Massive MIMO

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Introduction

- Massive MIMO (MMIMO) - large number of antennas at transmitter and receiver(s).
- Massive MIMO can be classified as
  - Multi user (MU) - user has a single antenna. Large number of users.
  - Single user (SU) - user has large number of antennas.
- In both cases, base station has large number of antennas.

Introduction

Table 1.1: MU- & SU-MMIMO.

MU-MIMO	SU-MMIMO
Beamforming possible in downlink	Beamforming possible in uplink & downlink
Spatial multiplexing not possible	Spatial multiplexing possible in uplink & downlink
Low spectral efficiency per user	High spectral efficiency per user
High directivity in downlink	High directivity in uplink & downlink

Introduction

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- High directivity at the expense of spectral efficiency per user.
- Same signal is transmitted from each antenna element.
- Spectral efficiency per user can be improved by increasing the constellation size.
  - High peak-to-average power ratio (PAPR).

Introduction

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- No directivity.
- Independent signals are transmitted from each antenna element.
- Rich scattering channel essential for effective operation.
  - Smart “reflectors” could be used.
- High spectral efficiency per user.
- Constellations with low PAPR e.g. QPSK, can be transmitted from each antenna element.

Introduction

- Beamforming and spatial multiplexing are conflicting operations.
- It may not be possible to use beamforming and spatial multiplexing simultaneously.

Introduction

Table 1.2: QPSK vs M-ary.

Spectral efficiency η (bits/transmission)	QPSK		M-ary	
	Transmit antennas N_t	Total avg transmit power	M	$(N_t = 1)$ Avg transmit power
4	2	4	16-QAM	10
6	3	6	64-QAM	42

- Uncoded signalling.
- All constellations have minimum distance 2 (similar SER).
- Significant power savings using QPSK - good for mobile handsets.
- Easy to turbo code QPSK, hard for M-ary constellations.

Introduction

Table 1.3: QPSK vs M-ary.

Spectral efficiency η (bits/transmission)	QPSK		M-ary	
	Transmit antennas N_t	PAPR (dB)	M	PAPR (dB)
4	2	0	16-QAM	2.5
6	3	0	64-QAM	3.7

- QPSK has 0 dB PAPR - good for RF amplifiers.
- M-ary signalling not possible for $\eta \sim$ 100, 1000.

System Model - Transmitter

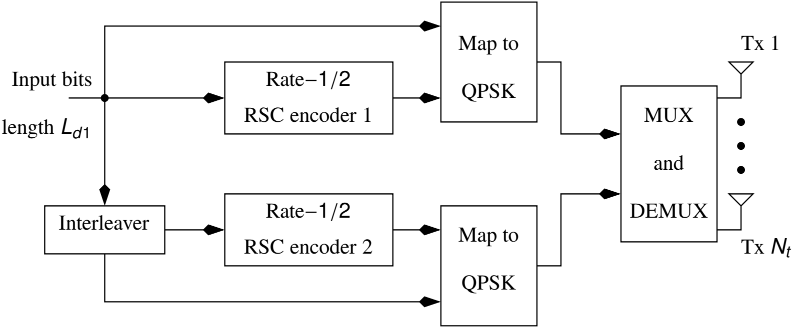

Figure 2.1: Transmitter.

System Model - Frame Structure

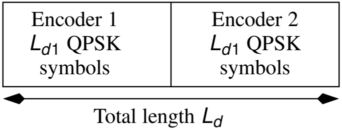

Figure 2.2: Frame structure.

System Model - Received Signal

- The received signal is (for k^{th} re-transmission, $1 \leq k \leq N_{rt}$)

$$\tilde{\mathbf{R}}_k = \tilde{\mathbf{H}}_k \mathbf{S} + \tilde{\mathbf{W}}_k. \tag{2.1}$$

  - $\tilde{\mathbf{R}}_k \in \mathbb{C}^{N_r \times 1}$.
  - $\tilde{\mathbf{H}}_k \in \mathbb{C}^{N_r \times N_t}$.
  - $\tilde{\mathbf{S}} \in \mathbb{C}^{N_t \times 1}$.
  - $\tilde{\mathbf{W}}_k \in \mathbb{C}^{N_r \times 1}$.

System Model - Received Signal

- Total number of antennas at transmitter & receiver

$$N_{\text{tot}} = N_t + N_r. \tag{2.2}$$

- We have

$$\begin{aligned} \frac{1}{2} E\left[\tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_l\right] &= N_r \sigma_H^2 \delta_K(k-l) \mathbf{I}_N \\ \frac{1}{2} E\left[\tilde{\mathbf{W}}_k \tilde{\mathbf{W}}_l^H\right] &= \sigma_W^2 \delta_K(k-l) \mathbf{I}_N. \end{aligned} \tag{2.3}$$

- Replace ensemble average by time average.

System Model - Received Signal

- The symbols are uncorrelated

$$E\left[S_j S_l^*\right] = P_{\mathrm{av}}\delta_K(j-l)$$
$$= 2\delta_K(j-l). \quad (2.4)$$

System Model - Receiver

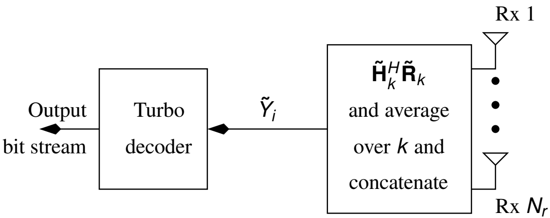

Figure 2.3: Receiver.

Analysis

- The i^{th} element of $\tilde{\mathbf{H}}_k^H \tilde{\mathbf{R}}_k$ is

$$\tilde{Y}_{k,i} = \tilde{F}_{k,i,i} S_i + \tilde{I}_{k,i} + \tilde{V}_{k,i} \quad \text{for } 1 \le i \le N_t \tag{3.1}$$

where

$$\tilde{V}_{k,i} = \sum_{j=1}^{N_r} \tilde{H}^*_{k,j,i} \tilde{W}_{k,j}$$

$$\tilde{I}_{k,i} = \sum_{\substack{j=1 \\ j \neq i}}^{N_t} \tilde{F}_{k,i,j} S_j$$

$$\tilde{F}_{k,i,j} = \sum_{l=1}^{N_r} \tilde{H}^*_{k,l,i} \tilde{H}_{k,l,j}. \tag{3.2}$$

Analysis

- It can be shown that

$$E\left[\tilde{F}_{k,i,i}^{2}\right] = 4\sigma_H^4 N_r(N_r+1). \tag{3.3}$$

Analysis

- We also have

$$E\left[\tilde{F}_{k,i,i}\right] = 2\sigma_H^2 N_r \tag{3.4}$$

and

$$\begin{aligned} E\left[\left|\tilde{I}_{k,i}\right|^2\right] &= 8\sigma_H^4 N_r(N_t - 1) \\ E\left[\left|\tilde{V}_{k,i}\right|^2\right] &= 4\sigma_W^2 \sigma_H^2 N_r. \end{aligned} \tag{3.5}$$

Analysis

- The total power of interference plus noise is

$$
\begin{aligned}
E\left[\left|\tilde{I}_{k,i} + \tilde{V}_{k,i}\right|^2\right] &= E\left[\left|\tilde{I}_{k,i}\right|^2\right] + E\left[\left|\tilde{V}_{k,i}\right|^2\right] \\
&= 8\sigma_H^4 N_r(N_t - 1) \\
&\quad + 4\sigma_W^2 \sigma_H^2 N_r \\
&= \sigma_{U'}^2 \qquad \text{(say)}.
\end{aligned}
\tag{3.6}
$$

Analysis

- The total information emitted by N_t antennas per transmission is $N_t/(2N_{rt})$ bits.
- The information contained in $\tilde{Y}_{k,\,i}$ in (3.1) is $N_t/(2N_{rt}N_t)$ bits.
- Hence, the average SINR per bit is

$$\begin{aligned} \mathrm{SINR}_{\mathrm{av},\,b} &= \frac{E\left[\left|\tilde{F}_{k,\,i,\,i}S_i\right|^2\right] \times 2N_{rt}}{E\left[\left|\tilde{I}_{k,\,i} + \tilde{W}_{k,\,i}\right|^2\right]} \\ &= \frac{8\sigma_H^4(N_r+1) \times 2N_{rt}}{8\sigma_H^4(N_t-1) + 4\sigma_W^2\sigma_H^2}. \end{aligned} \tag{3.7}$$

Analysis

- For $\sigma_W^2 = 0$ we get the upper bound on SINR per bit as

$$\text{SINR}_{\text{av}, b, \text{UB}} = \frac{(N_r + 1) \times 2N_{rt}}{N_t - 1}. \tag{3.8}$$

- The spectral efficiency is

$$\eta = \frac{N_t}{2N_{rt}} \quad \text{bits per transmission.} \tag{3.9}$$

Analysis

- Define

$$f(N_t) = \mathrm{SINR}_{\mathrm{av},\,b,\,\mathrm{UB}} + \eta = \frac{2N_{rt}(N_r+1)}{N_t-1} + \frac{N_t}{2N_{rt}} = \frac{2N_{rt}(N_{\mathrm{tot}}-N_t+1)}{N_t-1} + \frac{N_t}{2N_{rt}} \tag{3.10}$$

- The value of N_t that minimizes $f(N_t)$ in (3.10) is

$$N_t = 2N_{rt}\sqrt{N_{\mathrm{tot}}} + 1. \tag{3.11}$$

- Avoid the minimum.

Analysis

- Compute the average of $\tilde{Y}_{k,\,i}$ over all re-transmissions

$$\begin{aligned}\tilde{Y}_i &= \frac{1}{N_{rt}} \sum_{k=0}^{N_{rt}-1} \tilde{Y}_{k,\,i} \\ &= \frac{1}{N_{rt}} \sum_{k=0}^{N_{rt}-1} \left(\tilde{F}_{k,\,i,\,i} S_i + \tilde{I}_{k,\,i} + \tilde{V}_{k,\,i} \right) \\ &= F_i S_i + \tilde{U}_i \qquad \text{for } 1 \leq i \leq N_t \end{aligned} \tag{3.12}$$

where $\tilde{Y}_{k,\,i}$ is given by (3.1).

Analysis

- Note that

$$
\begin{aligned}
F_i &= \frac{1}{N_{rt}} \sum_{k=0}^{N_{rt}-1} \tilde{F}_{k,\,i,\,i} \\
\tilde{U}_i &= \frac{1}{N_{rt}} \sum_{k=0}^{N_{rt}-1} \left(\tilde{I}_{k,\,i} + \tilde{V}_{k,\,i}\right) \\
&= \frac{1}{N_{rt}} \sum_{k=0}^{N_{rt}-1} \tilde{U}'_{k,\,i} \qquad \text{(say).}
\end{aligned} \tag{3.13}
$$

Analysis

- Since $\tilde{F}_{k,i,i}$ and $\tilde{U}'_{k,i}$ are independent over re-transmissions (k)

$$E\left[F_i^2\right] = \frac{1}{N_{rt}^2} E\left[\sum_{k=0}^{N_{rt}-1} \tilde{F}_{k,i,i} \sum_{n=0}^{N_{rt}-1} \tilde{F}_{n,i,i}\right]$$
$$= \frac{4\sigma_H^4 N_r \left[N_r + 1 + N_r(N_{rt}-1)\right]}{N_{rt}}$$
$$= \frac{4\sigma_H^4 N_r (N_r N_{rt} + 1)}{N_{rt}}$$
$$E\left[\left|\tilde{U}_i\right|^2\right] = \frac{\sigma_{U'}^2}{N_{rt}}$$
$$= \frac{8\sigma_H^4 N_r (N_t - 1) + 4\sigma_W^2 \sigma_H^2 N_r}{N_{rt}}. \quad (3.14)$$

Analysis

- Note that

$$E\left[\tilde{U}'_{k,i}\right] = 0 \tag{3.15}$$

where $\tilde{U}'_{k,i}$ is defined in (3.13).

Analysis

- The average information content of $\tilde{Y}_i$ in (3.12) is $1/2$ bit.
- The average SINR per bit of $\tilde{Y}_i$ is

$$\begin{aligned} \text{SINR}_{\text{av}, b, C} &= \frac{E\left[\left|F_i S_i\right|^2\right] \times 2}{E\left[\left|\tilde{U}_i\right|^2\right]} \\ &= \frac{8\sigma_H^2 (N_r N_{rt} + 1) \times 2}{8\sigma_H^2 (N_t - 1) + 4\sigma_W^2} \end{aligned} \qquad (3.16)$$

where the subscript "C" denotes "after combining" and we have used (2.4) and (3.14).

Analysis

- When $\sigma_W^2 = 0$ and $N_r N_{rt} \gg 1$, we get the upper bound as

$$\begin{aligned}\text{SINR}_{\text{av},\,b,\,C,\,\text{UB}} &= \frac{8\sigma_H^2(N_r N_{rt}+1)\times 2}{8\sigma_H^2(N_t-1)} \\ &\approx \frac{2N_{rt}N_r}{N_t-1} \\ &\approx \text{SINR}_{\text{av},\,b,\,\text{UB}}\end{aligned} \tag{3.17}$$

for $N_r \gg 1$.

Analysis

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Re-transmissions increases the upper bound on the average SINR per bit, it does not improve the BER.

Turbo Decoding

- Forward recursion for decoder 1 ($(0 \leq i \leq L_{d1} - 2)$)

$$
\begin{aligned}
\alpha'_{i+1,n} &= \sum_{m \in \mathscr{C}_n} \alpha_{i,m} \gamma_{1,i,m,n} P(S_{b,i,m,n}) \\
\alpha_{0,n} &= 1 \qquad \text{for } 0 \leq n \leq \mathscr{S} - 1 \\
\alpha_{i+1,n} &= \alpha'_{i+1,n} \Bigg/ \left(\sum_{n=0}^{\mathscr{S}-1} \alpha'_{i+1,n} \right).
\end{aligned} \tag{4.1}
$$

Turbo Decoding

- In (4.1)

$$\gamma_{1,\,i,\,m,\,n} = \exp\left[-\frac{\left|\tilde{Y}_i - F_i S_{m,\,n}\right|^2}{2\sigma_U^2}\right]. \tag{4.2}$$

Turbo Decoding

- The backward recursion for decoder 1 $(1 \leq i \leq L_{d1} - 1)$

$$\begin{aligned} \beta'_{i,n} &= \sum_{m \in \mathscr{D}_n} \beta_{i+1,m} \gamma_{1,i,n,m} P\left(S_{b,i,n,m}\right) \\ \beta_{L_{d1},n} &= 1 \qquad \text{for } 0 \leq n \leq \mathscr{S} - 1 \\ \beta_{i,n} &= \beta'_{i,n} \Bigg/ \left(\sum_{n=0}^{\mathscr{S}-1} \beta'_{i,n} \right). \end{aligned} \tag{4.3}$$

Turbo Decoding

- Extrinsic information from decoder 1 to 2 ($0 \le i \le L_{d1} - 1$)

$$\begin{aligned} G_{\mathrm{norm},\, i+} &= \sum_{n=0}^{\mathscr{S}-1} \alpha_{i,n} \gamma_{1,i,n,\rho^{+}(n)} \beta_{i+1,\rho^{+}(n)} \\ G_{\mathrm{norm},\, i-} &= \sum_{n=0}^{\mathscr{S}-1} \alpha_{i,n} \gamma_{1,i,n,\rho^{-}(n)} \beta_{i+1,\rho^{-}(n)}. \end{aligned} \tag{4.4}$$

Results

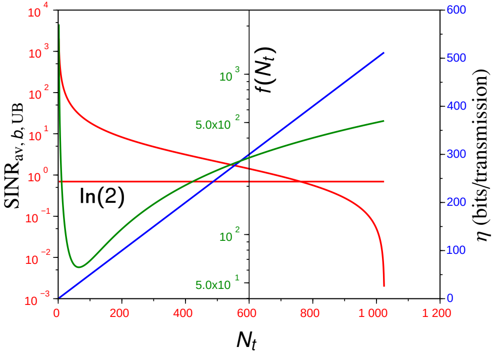

Figure 5.1: SINR upper bound & spectral efficiency for $N_{\text{tot}} = 1024$, $N_{rt} = 1$.

Simulation Results

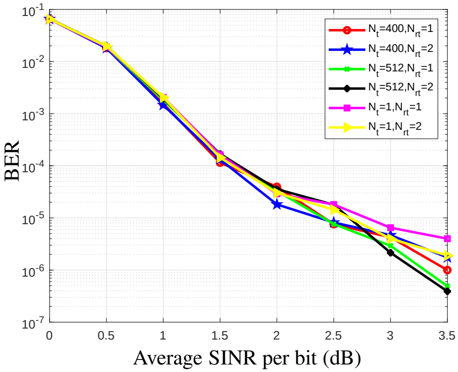

Figure 5.2: BER simulation results for $N_{\text{tot}} = 1024$.

Conclusions

- The bit-error-rate (BER) appears to be independent of re-transmissions (N_{rt}), for a rich scattering channel.
- Large scope for parallel processing.
- The BER may get affected by channel correlations.
- Difficult to get such low BER with higher order constellations.

References

- K. Vasudevan, "Digital Communications and Signal Processing", Universities Press, Second edition 2010.
- K. Vasudevan, K. Madhu, Shivani Singh, "Data Detection in Single User Massive MIMO Using Re-Transmissions", The Open Signal Processing Journal, vol. 6, pp. 15–26, March 2019.
- K. Vasudevan, Shivani Singh and A. Phani Kumar Reddy, "Coherent Receiver for Turbo Coded Single-User Massive MIMO-OFDM with Retransmissions", IntechOpen, April 2019.
- K. Vasudevan, A. Phani Kumar Reddy, Gyanesh Kumar Pathak, Mahmoud Albreem, "Turbo Coded Single User Massive MIMO", submitted. Also available at: http://arxiv.org/abs/2107.02437