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# SEISMIC SHEAR BUILDING DESIGN FOR FREQUENCY-DEPENDENT SUPPORTS VIA HYBRID INVERSE FORMULATION

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## ABSTRACT

A new stiffness design method is presented for finding the set of storey stiffnesses of the elastic shear building, supported by prescribed frequency-dependent complex springs, which exhibits a specified distribution of mean peak interstorey drifts under design earthquakes. A frequency-independent substitute model is introduced to estimate seismic responses of the original frequency-dependent model. A closed form solution to a hybrid inverse eigenmode problem is derived and the qualification conditions on the specified fundamental frequency and eigenvector are disclosed. The formula is then shown to be useful for developing the efficient seismic stiffness design method. Response analysis on the original frequency-dependent model in the frequency domain is required only for demonstrating the validity and accuracy of this design method.

**Keywords:** hybrid inverse problem, seismic stiffness design, frequency-dependent foundation, soil-structure interaction, site-dependent spectrum

## INTRODUCTION

Building structures and other civil engineering structures are supported by the ground and a better model for representing the characteristics of dynamic soil-structure interaction is a set of frequency-dependent complex springs (Thomson and Kobori (1963), Kobori et al. (1966), Veletsos and Wei (1971), Luco and Westmann (1971), Wolf (1985)).

The purpose of this paper is to develop a direct method of seismic stiffness design for finding the set of storey stiffnesses of the elastic shear building, supported by prescribed frequency-dependent complex springs, which exhibit a specified distribution of mean peak interstorey drifts to a set of design-spectrum compatible earthquakes. Prediction of the seismic response of such an elastic shear building inevitably requires response analysis in the frequency domain. It seems extremely inefficient and undesirable in the *preliminary* design stage to conduct such a response analysis for a *large number* of design earthquakes and to repeat design modification based upon the data obtained in the response analysis. In this paper, a more systematic and efficient design procedure based on the response spectrum approach is developed which adopts a shear building model (FI model) supported by frequency-independent complex springs to approximately estimate the seismic interstorey drift of a shear building model (FD model) supported by frequency-dependent complex springs. It is of theoretical significance to note that this fairly good correspondence of seismic drifts between these two models is guaranteed by the theorems (Nakamura and Takewaki (1989a)) on the correspondence of fundamental frequencies and eigenvectors. The features of the present design method are as follows; (1) as an approximate solution to the problem of seismic stiffness design of a FD model, the seismic stiffness design of the FI model is employed, (2) a new closed form solution to a hybrid inverse eigenmode problem is derived and utilized for the seismic stiffness design of the FI model.

The problem of analysis of a structure supported by a set of frequency-dependent complex springs has been investigated extensively. Kobori et al. (1964), Parmelee et al. (1968) and Jennings and Bielak (1973) investigated the harmonic and seismic response characteristics of structures supported by frequency-dependent complex springs. Bielak (1973) proposed an iterative method for finding the undamped fundamental natural frequency of a multi-storey shear building on semi-infinite visco-elastic ground. Then higher-mode natural frequencies have been evaluated by using the stiffness matrix obtained for the calculated fundamental natural frequency and an approximate procedure for evaluating damping ratios of this system has also been proposed. Warburton (1978) obtained undamped natural frequencies of a uniform tower structure on semi-infinite elastic ground using the same method as Bielak (1973) and clarified the effects of mechanical and geometrical properties of the system on the natural frequencies and harmonic responses. Gupta and Trifunac (1991) proposed a simplified response spectrum superposition method including soil-structure interaction.

To the best of the author's knowledge, it appears that almost all of the previous works are on *analysis of behaviour* of structure-foundation systems. It seems possible through a great deal of parametric analysis to reveal the seismic response characteristics of structure-foundation systems or to find a desirable design among many designs such that a certain set of response requirements is satisfied. However, parametric analysis is not necessarily a direct method and it would require a great deal of computational tasks especially for structures with huge degrees of freedom or with many design variables. While a direct method of seismic stiffness design of shear buildings supported by frequency-independent complex springs has been proposed in a previous preliminary paper (Nakamura and Takewaki (1989b)), no direct method of seismic stiffness design has ever been proposed for structures supported by frequency-dependent complex springs. Nonlinear eigenvalue problems have to be solved for structures supported by frequency-dependent springs. Here, the *direct* method implies that the member stiffnesses and/or strengths of a structure satisfying design constraints are found without design-sensitivity analysis or parametric analysis as stated just above. The principal significance of proposing a direct design method for structure-foundation systems is to facilitate the achievement of a structural design with well-balanced stiffnesses or margins to elastic limits of a structure and its foundation in the *preliminary* design stage where a great deal of computational tasks is not desired and allowed.

Although simplified design earthquakes are employed in this paper, it is due to the fact that the principal objective of this paper is to develop a direct design method for shear buildings with frequency-dependent complex springs for the first time and to present the basic concept of the method. It is expected that the fundamental concept of the proposed design method can be extended straightforwardly to a problem of stiffness design for other class of structures, e.g. shear-flexural building models and moment-resisting building frames (Takewaki et al.(1998)), supported by frequency-dependent foundations.

## PROBLEM OF SEISMIC STIFFNESS DESIGN FOR FREQUENCY-DEPENDENT MODELS

Consider an $f$-storey elastic shear building, as shown in Fig.1, supported by prescribed frequency-dependent complex springs. The $i$-th floor from the bottom except the ground floor will be called 'floor $i$'. Let $m_i$, $I_{Ri}$ and $h_i$ denote the mass of floor $i$, the moment of inertia of floor $i$ around its centroid and the height of the $i$-th storey, respectively. It is assumed that, while the mass of each floor includes the effect of masses of all the structural elements adjacent to the floor, $m_i$ and $I_{Ri}$ will not be changed in the process of design modification. Let $k_i$ and $c_i$ denote the stiffness of the $i$-th storey structural elements with respect to the relative horizontal displacement between floor $i$-1 and floor $i$ without rocking component and the damping coefficient with respect to the corresponding relative horizontal velocity between floor $i$-1 and floor $i$. A set of stiffnesses $\mathbf{k}=\{k_1 ... k_f\}^T$ of the shear building is chosen as the design variable vector and will be called 'design $\mathbf{k}$' where $(\ )^T$ indicates transpose of a vector.

A frequency-dependent complex spring is a proper model for representing dynamic interaction between soil and a foundation. The complex stiffnesses $K_H(\omega)$ and $K_R(\omega)$ of the horizontal and rotational springs, respectively, supporting the base are expressed in terms of real and imaginary parts as

$$K_H(\omega) = k_H(\omega) + \mathrm{i}\omega c_H(\omega) \qquad (1)$$

$$K_R(\omega) = k_R(\omega) + \mathrm{i}\omega c_R(\omega), \tag{2}$$

where i and $\omega$ denote the imaginary unit and excitation frequency, respectively. To simply show the essential feature of the proposed formulation, coupling between the horizontal and rocking components is neglected in the present formulation (see, for example, Bielak (1978)). When the coupling is not negligible, the present formulation must be modified. An augmented procedure will be shown later (after eqn (15)). It is sufficient that $k_H(\omega), k_R(\omega), c_H(\omega), c_R(\omega)$ are given numerically. In this paper the only case is dealt with where $k_H(\omega)$ and $k_R(\omega)$ are single-valued non-increasing positive functions of frequency.

If the kinematic interaction effect is negligible (rigid slab footing of negligible thickness) and the rocking component of the ground acceleration is negligible (see, for example, Gupta and Trifunac (1991)), the equations of motion of the model shown in Fig.1 in the frequency domain to the horizontal ground acceleration $\ddot{u}_g(t)$ may be expressed as

$$[\mathbf{K}_B(\mathbf{k}) + \mathbf{K}_F(k_H(\omega), k_R(\omega)) + \mathrm{i}\omega\{\mathbf{C}_B + \mathbf{C}_F(c_H(\omega), c_R(\omega))\} - \omega^2\mathbf{M}]\mathbf{U}(\omega) = -\mathbf{M}\mathbf{r}\ddot{U}_g(\omega) \tag{3}$$

where $\ddot{U}_g(\omega)$ is the Fourier transform of $\ddot{u}_g(t)$ and $\mathbf{K}_B(\mathbf{k})$, $\mathbf{K}_F(k_H(\omega), k_R(\omega))$, $\mathbf{M}$, $\mathbf{U}(\omega)$ and $\mathbf{r}$ indicate the following matrices or vectors.

$$\mathbf{K}_B(\mathbf{k}) = \begin{bmatrix} k_1 + k_2 & -k_2 & & & & & & & \\ & \ddots & \ddots & & & & & \mathbf{0} & \\ & & \ddots & -k_i & & & & & \\ & & & k_i + k_{i+1} & \ddots & & & & \\ & & & & \ddots & -k_{f-1} & & & \\ & & & & & k_{f-1} + k_f & -k_f & & \\ & & & & & & k_f & 0 & \\ & sym. & & & & & & 0 & 0 \\ & & & & & & & & 0 \end{bmatrix},$$

$$\mathbf{K}_F(k_H(\omega), k_R(\omega)) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & k_H(\omega) & 0 \\ sym. & & k_R(\omega) \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{M}_B & \mathbf{M}_{BS} & \mathbf{M}_{BR} \\ & E_1 & E_2 \\ sym. & & E_3 \end{bmatrix},$$

$$\mathbf{M}_B = \mathrm{diag}(m_1 \cdots m_f), \quad \mathbf{M}_{BS} = \{m_1 \cdots m_f\}^T, \quad \mathbf{M}_{BR} = \{m_1 H_1 \cdots m_f H_f\}^T,$$

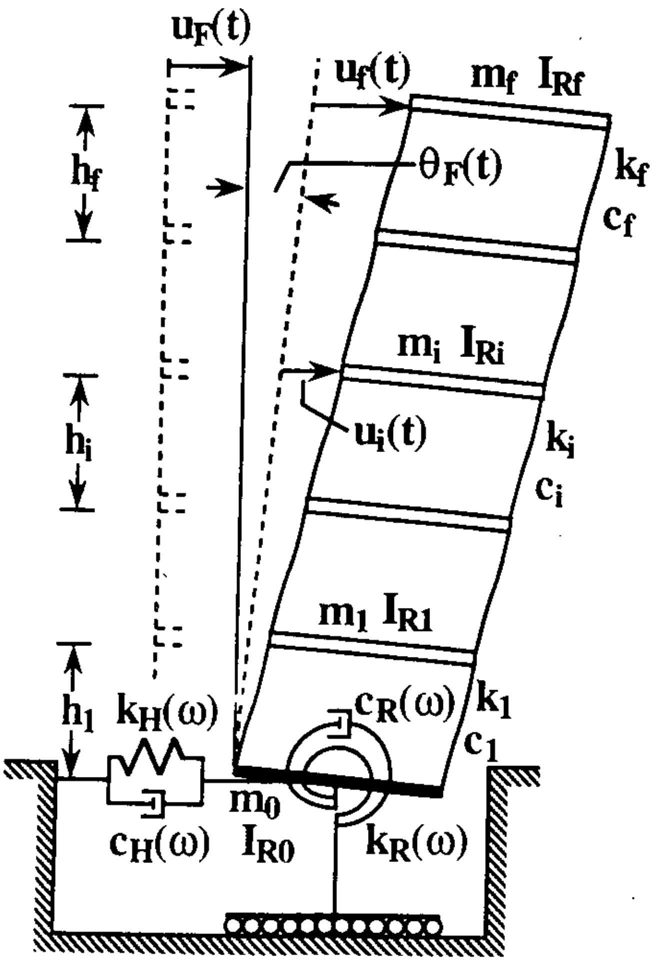


Fig.1. Elastic shear building supported by frequency-dependent complex springs (FD2 model)

$$E_1 = \sum_{i=0}^{f} m_i,\ E_2 = \sum_{i=1}^{f} m_i H_i,\ E_3 = \sum_{i=1}^{f} m_i H_i^2 + \sum_{i=0}^{f} I_{Ri},\ H_i = \sum_{j=1}^{i} h_j,$$

$$\mathbf{U}(\omega) = \{U_1(\omega)\cdots U_i(\omega)\cdots U_f(\omega)\ \ U_F(\omega)\ \ \Theta_F(\omega)\}^T,$$

$$\mathbf{r} = \{0\cdots 0\ \ 1\ \ 0\}^T \tag{4}$$

The damping matrix $\mathbf{C}_B$ for the super-structure can be derived by replacing $\{k_j\}$ by $\{c_j\}$ and $\mathbf{C}_F(c_H(\omega), c_R(\omega))$ can be derived by replacing $k_H(\omega), k_R(\omega)$ by $c_H(\omega), c_R(\omega)$. Although the kinematic interaction effect is neglected and only a horizontal ground motion is considered in the present paper, the extension of the present formulation *in the context of design* to more general cases is open to further research.

The model shown in Fig.1 will be called an 'FD2 model' in order to distinguish it from three other auxiliary models shown later. The purpose of this paper is to find the set of storey stiffnesses of the FD2 model which exhibits a specified distribution of mean peak interstorey drifts to design earthquakes compatible with a site-dependent design response spectrum (Mostaghel and Ahmadi (1979)) (see Appendix 1). The design earthquakes are defined for $\ddot{u}_g(t)$ at the free ground surface level. In this paper the relative displacement between adjacent floors without the component due to rocking will be called the 'interstorey drift' for the sake of simplicity of expression. The relative displacement between adjacent floors without the component due to rocking just corresponds to the actual deformation of the storey and sometimes utilized as an index for functionality checking. Let $\delta_{j\max}$ and $\overline{\delta}_j$ denote the mean peak interstorey drift of the $j$-th storey and its specified value. The corresponding problem of seismic Stiffness Design for Frequency-Dependent models may be stated as:

**[Problem SDFD]**

Find the design $\tilde{\mathbf{k}}$ of the FD2 model with prescribed frequency-dependent complex springs $K_H(\omega)$ and $K_R(\omega)$, for which the following constraints on mean peak interstorey drifts are satisfied when subjected to design earthquakes compatible with a specified design response spectrum.

$$\delta_{j\max} = \overline{\delta}_j \quad (j = 1, \cdots, f) \tag{5}$$

The response analysis in the frequency domain is required to obtain a time-history response of a structure supported by frequency-dependent memebrs. Since this response analysis would require a great deal of computational task for a *large number* of design earthquake motions, it is neither convenient nor efficient to conduct it at the preliminary design stage. This difficulty can be avoided if the mean peak interstorey drifts of the shear building of design **k** supported by frequency-dependent complex springs are approximately estimated with the use of the following three auxiliary models. The shear building of design **k** supported by $\{\overline{k}_H, \overline{k}_R\}$, that supported by $\{\overline{k}_H, \overline{k}_R, \overline{c}_H, \overline{c}_R\}$ and that supported by $\{k_H(\omega), k_R(\omega)\}$ will be referred to as the FI1 model of design **k**, the FI2 model of design **k** and the FD1 model of design **k**, respectively, where $\overline{k}_H = k_H(\overline{\omega}_1)$, $\overline{k}_R = k_R(\overline{\omega}_1)$, $\overline{c}_H = c_H(\overline{\omega}_1)$, $\overline{c}_R = c_R(\overline{\omega}_1)$ (see Fig.2). Here $\overline{\omega}_1$ denotes the fundamental natural circular frequency of the FD1 model of design **k**.

## PROBLEM OF SEISMIC STIFFNESS DESIGN FOR FREQUENCY-INDEPENDENT MODELS

Consider the following problem of seismic Stiffness Design for Frequency-Independent models (FI2 models). The solution to this problem will be adopted as an approximate solution to Problem SDFD later.

**[Problem SDFI]**

Find the design $\tilde{\mathbf{k}}$ of the FI2 model with $k_H(\overline{\omega}_1), k_R(\overline{\omega}_1)$ and with $c_H(\overline{\omega}_1), c_R(\overline{\omega}_1)$ such that the following constraints on mean peak interstorey drifts are satisfied when subjected to design earthquakes compatible with a specified design response spectrum.

$$\delta_{j\max} = \overline{\delta}_j \quad (j = 1, \cdots, f) \tag{6}$$

Note that $\overline{\omega}_1$ is the fundamental frequency of the FD1 model of design $\tilde{\mathbf{k}}$. The mean peak interstorey drift $\delta_{j\max}$ of the FI2 model of design $\tilde{\mathbf{k}}$ is evaluated approximately by the following well-known procedure.

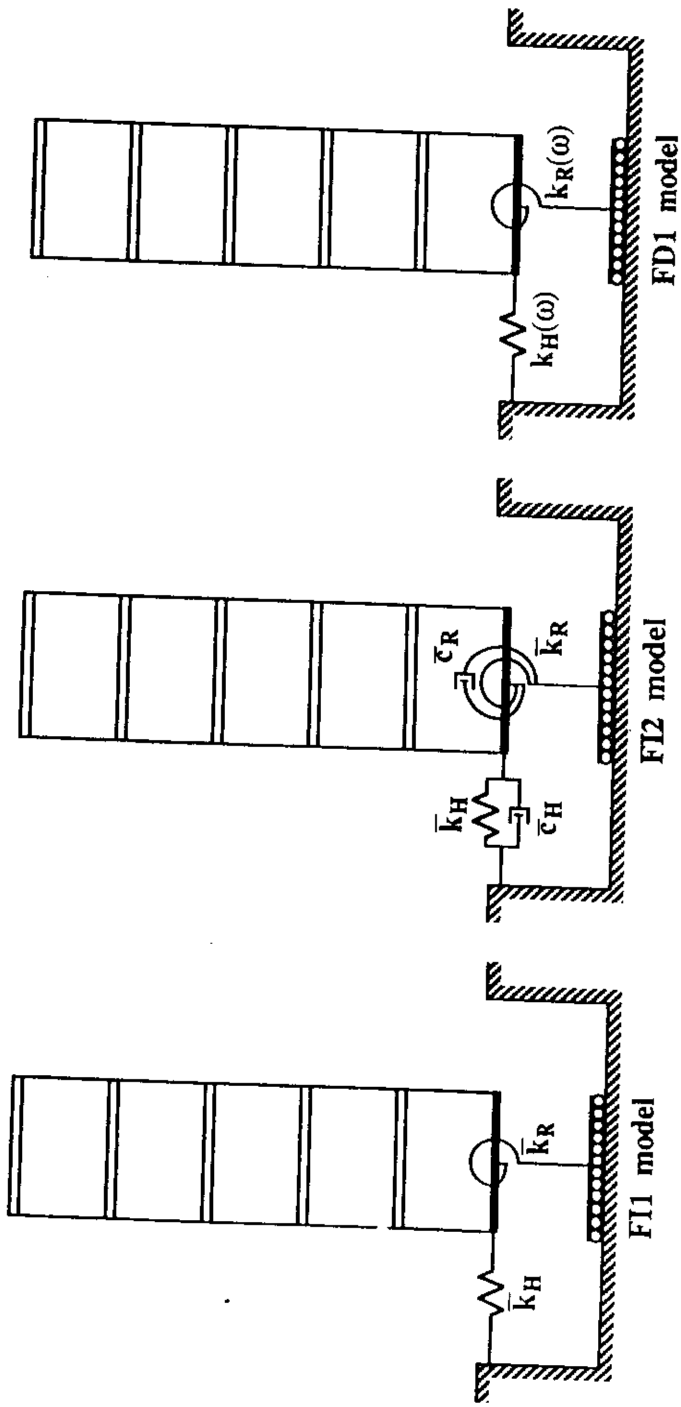


Fig.2. Three auxiliary shear building models with various supporting systems

The damping matrix $\mathbf{C}_B$ defined as a matrix proportional to the stiffness matrix can be obtained by specifying the damping ratio $h_s$ for the fundamental mode of the shear building with a fixed base which has the fundamental frequency $\omega_s$

$$\mathbf{C}_B = \frac{2h_s}{\omega_s}\mathbf{K}_B(\tilde{\mathbf{k}}) \tag{7}$$

The damping matrix $\mathbf{C}_F(c_H(\overline{\omega}_1), c_R(\overline{\omega}_1))$ of the foundation is such that $c_H(\omega)$ and $c_R(\omega)$ in $\mathbf{C}_F(c_H(\omega), c_R(\omega))$ are replaced by $c_H(\overline{\omega}_1)$ and $c_R(\overline{\omega}_1)$, respectively. Since the FI2 model has a non-classical damping matrix, classical normal modes do not exist. In this paper, an approximate procedure of ignoring off-diagonal components in the process of modal decomposition is adopted so that classical modal superposition is applicable. Then the $r$-th damping ratio $h^{(r)}$ of the FI2 model of design $\tilde{\mathbf{k}}$ may be evaluated by

$$h^{(r)} = \frac{1}{2\omega_r}\frac{\mathbf{\Phi}^{(r)T}[\mathbf{C}_B + \mathbf{C}_F(c_H(\overline{\omega}_1), c_R(\overline{\omega}_1))]\mathbf{\Phi}^{(r)}}{\mathbf{\Phi}^{(r)T}\mathbf{M}\mathbf{\Phi}^{(r)}}, \tag{8}$$

where $\omega_r$ and $\mathbf{\Phi}^{(r)}$ denote the $r$-th natural circular frequency and the $r$-th eigenvector, respectively, of the FI1 model of design $\tilde{\mathbf{k}}$. It is shown later through time-history response analysis that this treatment is appropriate for the present model (FI2 model). Although Bielak (1976) proposed another procedure for evaluating damping ratios such that the frequency dependence of damping coefficients is taken into account, the basic procedure due to Thomson et al.(1974) is employed here so as to be consistent with the procedure for evaluating the natural frequencies of the FI1 model.

With the use of the damping ratios of the FI2 model, the mean peak interstorey drifts of the FI2 model can be evaluated by the SRSS method as

$$\delta_{j\max} = \sqrt{\sum_{r=1}^{N_T}\left\{\gamma^{(r)}\left(\Phi_j^{(r)} - \Phi_{j-1}^{(r)}\right)S_D(T_r; h^{(r)})\right\}^2} \quad (j = 1, \cdots, f), \tag{9}$$

where $\gamma^{(r)}(= \mathbf{\Phi}^{(r)T}\mathbf{Mr} / \mathbf{\Phi}^{(r)T}\mathbf{M}\mathbf{\Phi}^{(r)})$, $\Phi_j^{(r)}$, $T_r$ and $N_T$ denote the $r$-th participation factor, the $j$-th component of the $r$-th eigenvector, the $r$-th natural period of the FI1 model of design $\tilde{\mathbf{k}}$ and the number of adopted modes in the SRSS method, respectively.

$S_D(T_r;h^{(r)})$ in eqn (9) is the design displacement response spectrum and is expressed as $S_D(T;h) = S_V(T;h)T/(2\pi)$ in terms of the design velocity response spectrum given in Appendix 1. Note that eqn (9) will also be used later for evaluating the mean peak interstorey drifts of a FI2 model of any design $\mathbf{k}$.

In order to find the solution to Problem SDFI, an efficient method will be developed which utilizes a closed form solution of stiffness design of elastically supported shear buildings for a specified fundamental frequency and a specified set of lowest-mode interstorey drifts based upon a new concept of *hybrid inverse formulation.*

## HYBRID INVERSE PROBLEM FOR FI1 MODELS

While all the components of a fundamental eigenvector can be specified in the theory due to Nakamura and Yamane (1986) for shear buildings with fixed bases, only those components of the fundamental eigenvector excluding horizontal and rotational displacements of a base can be specified in the present case. This is due to the fact that finite stiffnesses of horizontal and rotational springs supporting the base are prescribed in advance. Then the horizontal and rotational displacements are to be found through an analysis procedure. Since the design problem for finding $\mathbf{k}$ and the analysis problem for finding some of the components in the fundamental eigenvector are mixed in the present problem, it may be appropriate to call it a *hybrid inverse problem.* When the horizontal and rotational spring stiffnesses are prescribed as $\bar{k}_H = k_H(\bar{\omega}_1), \bar{k}_R = k_R(\bar{\omega}_1)$, a Hybrid Inverse Problem in lowest eigenvibration for Frequency-Independent models (FI1 models) may be stated as follows.

**[Problem HIPFI]**

Find the design $\mathbf{k}$ of the FI1 model which has the fundamental frequency $\bar{\omega}_1$ and the interstorey drift components $\{\Delta_1 \cdots \Delta_f\}$ in the fundamental eigenvector.

The governing equations of lowest eigenvibration of the FI1 model with the fundamental frequency $\bar{\omega}_1$ and the fundamental eigenvector $\mathbf{\Phi}^{(1)} = \{U_1 \cdots U_f \;\; U_F \;\; \Theta_F\}^T$ may be written as;

$$[\mathbf{K}_B(\mathbf{k})+\mathbf{K}_F(\bar{k}_H,\bar{k}_R)-\overline{\Omega}_1\mathbf{M}]\mathbf{\Phi}^{(1)}=\mathbf{0}, \tag{10}$$

where $\overline{\Omega}_1=\overline{\omega}_1{}^2$. Since the lowest-mode interstorey drifts $\{\Delta_i\}$ are related to $\{U_i\}$ by $U_i-U_{i-1}=\Delta_i\ (i=1,\cdots,f;\ U_0=0)$, $U_i$ can be expressed in terms of $\{\Delta_i\}$.

$$U_i=\sum_{j=1}^{i}\Delta_j \quad (i=1,\cdots,f) \tag{11}$$

Substitution of eqn (11) into the last two equations of (10) provides

$$\begin{bmatrix} D_1 & D_2 \\ D_2 & D_4 \end{bmatrix}\begin{Bmatrix} U_F \\ \Theta_F \end{Bmatrix}=-\begin{Bmatrix} D_3 \\ D_5 \end{Bmatrix}, \tag{12}$$

where $D_1$, ..., $D_5$ represent the following quantities.

$$D_1=E_1-\frac{\bar{k}_H}{\overline{\Omega}_1},\ D_2=E_2,\ D_3=\sum_{i=1}^{f}m_i\sum_{j=1}^{i}\Delta_j,\ \ D_4=E_3-\frac{\bar{k}_R}{\overline{\Omega}_1},\ D_5=\sum_{i=1}^{f}m_iH_i\sum_{j=1}^{i}\Delta_j \tag{13}$$

It should be remarked here that eqn (12) does not include the design variables **k** and constitutes a set of simultaneous linear equations for $U_F,\Theta_F$.

The specified fundamental frequency $\overline{\omega}_1$ must satisfy the inequality condition $\overline{\omega}_1<\hat{\omega}_1$ where $\hat{\omega}_1$ denotes the fundamental frequency of the elastically supported rigid model (rigid FD1 model). It is important to note that the specified fundamental frequency $\overline{\omega}_1$ in PROBLEM HIPFI is also the fundamental frequency of a FD1 model and this condition $\overline{\omega}_1<\hat{\omega}_1$ is completely different from the lemma stated in the previous paper (Nakamura and Takewaki (1989b)) for structures with frequency-independent supporting stiffnesses. This condition $\overline{\omega}_1<\hat{\omega}_1$ and the regularity of the coefficient matrix in eqn (12) in the range of $\overline{\omega}_1<\hat{\omega}_1$ can be proved by means of the Rayleigh's principle. $U_F$ and $\Theta_F$ are then obtained from eqn (12).

$$U_F=\frac{D_2D_5-D_3D_4}{D_1D_4-D_2{}^2},\quad \Theta_F=\frac{D_2D_3-D_1D_5}{D_1D_4-D_2{}^2} \tag{14a, b}$$

Note that $U_F$ and $\Theta_F$ are functions of the specified fundamental frequency $\overline{\omega}_1$ and the specified lowest-mode interstorey drift vector $\mathbf{\Delta}=\{\Delta_1\cdots\Delta_f\}^T$ in Problem HIPFI. $U_F$ and $\Theta_F$ will hereafter be denoted by $U_F(\overline{\omega}_1,\mathbf{\Delta})$ and $\Theta_F(\overline{\omega}_1,\mathbf{\Delta})$, respectively.

Substitution of eqns (11) into the first $f$ equations of (10) and summation of the resulting equations from the $j$-th through the $f$-th provide the solution to Problem HIPFI.

$$k_j = \frac{\overline{\Omega}_1}{\Delta_j} \sum_{i=j}^{f} m_i \left\{ U_F(\overline{\omega}_1, \boldsymbol{\Delta}) + \Theta_F(\overline{\omega}_1, \boldsymbol{\Delta}) H_i + \sum_{l=1}^{i} \Delta_l \right\} \quad (j = 1, \cdots, f) \tag{15}$$

If the coupling term of ground stiffness is not negligible, the present formulation can be extended straightforwardly simply by modifying the term $D_2$ in eqn (13) to $D_2^* = E_2 - (\bar{k}_{HR} / \overline{\Omega}_1)$ where $\bar{k}_{HR} = k_{HR}(\overline{\omega}_1)$ and $k_{HR}(\omega)$ is the coupling term of ground stiffness in eqns (1, 2). This modification is based on the exact governing equation of undamped lowest eigenvibration.

It is of theoretical and practical significance to disclose the qualification conditions which the specified frequency and eigenvector have to satisfy.

**Property 1**

Let all the masses and moments of inertia of floors be prescribed and the supporting spring stiffnesses be prescribed. The sufficient conditions for the specified eigenvalue and the corresponding specified eigenmode to provide the positive storey stiffnesses are;

(i) $\overline{\omega}_1 < \hat{\omega}_1$,

(ii) $\{\Delta_1 \cdots \Delta_f\}$ $(\Delta_1 \neq 0, \cdots, \Delta_f \neq 0)$ have the same sign.

The specified fundamental frequency $\overline{\omega}_1$ in PROBLEM HIPFI and Property 1 is also the fundamental frequency of a FD1 model and Property 1 is completely different from the property stated for structures with frequency-independent supporting stiffnesses in Nakamura and Takewaki (1989b). The difference will be shown clearly in the sequel.

It is possible to show that $-D_1, -D_4$ given by eqn (13) and $D_1 D_4 - D_2^2$ in eqn (14) are positive functions of $\overline{\omega}_1$ in the range of $\overline{\omega}_1 < \hat{\omega}_1$. The proof is shown briefly.

$D_1 D_4 - D_2^2$ in eqn (14) can be expressed as

$$D_1 D_4 - D_2^2 = \frac{1}{\overline{\omega}_1^4} \left\{ \alpha_1 \overline{\omega}_1^4 - \alpha_2(\overline{\omega}_1) \overline{\omega}_1^2 + \alpha_3(\overline{\omega}_1) \right\} \tag{16}$$

Since $\alpha_3(0) = k_H(0) k_R(0) > 0$ and $\hat{\omega}_1$ is the positive minimum solution (eigenvalue) of eqn (12) in case of $D_3 = D_5 = 0$, $\alpha_1 \overline{\omega}_1^4 - \alpha_2(\overline{\omega}_1) \overline{\omega}_1^2 + \alpha_3(\overline{\omega}_1) > 0$ $(0 < \overline{\omega}_1 < \hat{\omega}_1)$. This leads to $D_1 D_4 - D_2^2 > 0 \ (0 < \overline{\omega}_1 < \hat{\omega}_1)$. If $\overline{\omega}_1^*$ denotes the positive minimum solution of $D_1 = 0$, $\overline{\omega}_1^*$ is the fundamental frequency in the horizontal vibration of the rigid body with a mass of $E_1$ supported by $k_H(\omega)$ and a rotational spring with infinite stiffness. In this vibration, the horizontal spring stiffness attains $k_H(\overline{\omega}_1^*)$. If the inequality $\overline{\omega}_1^* < \hat{\omega}_1$ were to hold, the fundamental frequency would become smaller in spite of the fact that both of the spring stiffnesses become larger or remain the same, i.e. $k_H(\overline{\omega}_1^*) \geq k_H(\hat{\omega}_1)$ and the rotational spring stiffness = $\infty > k_R(\hat{\omega}_1)$. The relation of $k_H(\overline{\omega}_1^*) \geq k_H(\hat{\omega}_1)$ is due to the non-increasing characteristic of $k_H(\omega)$. This conclusion contradicts the well-known characteristic shown in Courant and Hilbert (1953) and hence it has to be rejected. It is therefore concluded that $\overline{\omega}_1^* \geq \hat{\omega}_1$. Since $D_1 \to -\infty$ as $\overline{\omega}_1 \to 0$ and $\overline{\omega}_1^*$ is the positive minimum solution of $D_1 = 0$, the relation of $D_1 < 0 \ (0 < \overline{\omega}_1 < \hat{\omega}_1)$ holds. The same proof is also applicable to the rotational vibration and the relation of $D_4 < 0 \ (0 < \overline{\omega}_1 < \hat{\omega}_1)$ holds.

If all the interstorey drift components in the fundamental eigenvector have the same sign and the condition $\overline{\omega}_1 < \hat{\omega}_1$ is satisfied, $U_F(\overline{\omega}_1, \Delta)$ and $\Theta_F(\overline{\omega}_1, \Delta)$ given by eqn (14) also have the same sign as the former. In view of the form of eqn (15), this leads to the conclusion that the conditions that all the interstorey drift components in the fundamental eigenvector have the same sign and $\overline{\omega}_1 < \hat{\omega}_1$ are the sufficient conditions for providing positive storey stiffnesses. It is assured in most cases that the mode derived under these qualification conditions is certainly the lowest eigenmode. If the correspondence of the fundamental natural periods of the frequency dependent model and the frequency-independent model disappears, the responses of both models could exhibit much different values. Property 1 guarantees this correspondence.

## PROCEDURE FOR FINDING THE SEISMIC STIFFNESS DESIGN FOR FI2 MODELS

A procedure for finding the solution to Problem SDFI is developed in this section. The feature of this design procedure is to regard the fundamental frequency and fundamental eigenvector of a FI1 model as the principal parameters for adjustment of mean peak interstorey drifts. Note that iterative calculation of the determinant of the matrix representing the eigenvalue problem for a FD1 model is required in order to determine the fundamental frequency $\overline{\omega}_1$ of the FD1 model the stiffnesses **k** of which are prescribed. It is often the case that the stiffnesses **k** are assumed at least in the first iteration cycle in a usual design procedure including design-sensitivity analysis. On the other hand, no such an iterative calculation is required in the hybrid inverse formulation described in the previous section where the stiffnesses **k** are obtained through the closed form solution for a set of the specified fundamental frequency $\overline{\omega}_1$ and specified interstorey drift vector $\mathbf{\Delta}$ in the fundamental eigenvector. Since the storey stiffnesses **k** are functions of $\overline{\omega}_1$ and $\mathbf{\Delta}$ in problem HIPFI, the mean peak interstorey drifts $\delta_{j\max}$ given by eqn (9) are also functions of $\overline{\omega}_1$ and $\mathbf{\Delta}$ so long as only the solution stiffnesses to Problem HIPFI are employed.

The participation factor for the fundamental mode can be expressed as follows.

$$\gamma^{(1)}(\overline{\omega}_1,\mathbf{\Delta}) = \frac{N}{D} \tag{17a}$$

where

$$N = m_0 U_F(\overline{\omega}_1,\mathbf{\Delta}) + \sum_{i=1}^{f} m_i \left\{ U_F(\overline{\omega}_1,\mathbf{\Delta}) + \Theta_F(\overline{\omega}_1,\mathbf{\Delta}) H_i + \sum_{j=1}^{i} \Delta_j \right\} \tag{17b}$$

$$D = m_0 U_F(\overline{\omega}_1,\mathbf{\Delta})^2 + \sum_{i=1}^{f} m_i \left\{ U_F(\overline{\omega}_1,\mathbf{\Delta}) + \Theta_F(\overline{\omega}_1,\mathbf{\Delta}) H_i + \sum_{j=1}^{i} \Delta_j \right\}^2 + \sum_{i=0}^{f} I_{Ri} \Theta_F(\overline{\omega}_1,\mathbf{\Delta})^2 \tag{17c}$$

The damping ratio for the fundamental mode given by eqn (8) is also a function of $\overline{\omega}_1$ and $\mathbf{\Delta}$ in the hybrid inverse formulation. It is therefore denoted by $h^{(1)}(\overline{\omega}_1,\mathbf{\Delta})$.

The procedure for finding the solution to Problem SDFI is shown in Fig.3 where the superscripts c1, c2 denote the number of iteration cycles. Note that an initial fundamental frequency $\overline{\omega}_1{}^{c1} = 2\pi / \overline{T}_1^{c1}$ of a FI1 model can be found by the following equation by specifying an initial set of interstorey drift components in the fundamental eigenvector by $\Delta_j{}^{c1} = \overline{\delta}_j / \overline{\delta}_1$ $(j = 1,\cdots,f)$ and assuming that the mean peak interstorey

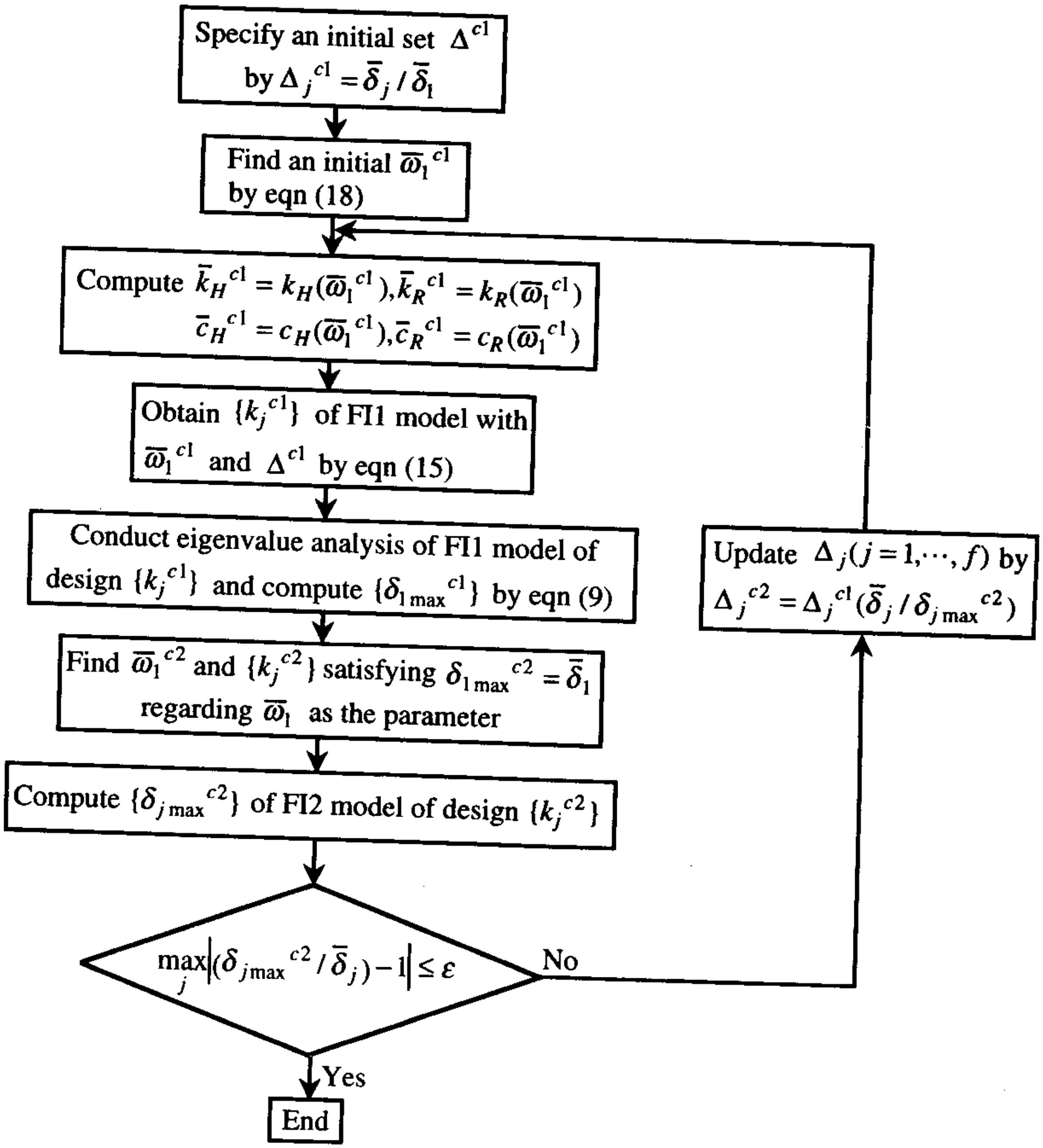


Fig.3 Flowchart for finding the seismic-drift constrained design

drift in the first storey can be expressed by the fundamental-mode component alone in the SRSS estimate.

$$\gamma^{(1)}(\overline{\omega}_1{}^{c1},\Delta^{c1})S_D(\overline{T}_1{}^{c1};h^{(1)}(\overline{\omega}_1{}^{c1},\Delta^{c1}))=\overline{\delta}_1 \tag{18}$$

Higher mode effects are taken into account in the refinement of the lowest-mode component by $\Delta_j{}^{c2}=\Delta_j{}^{c1}(\overline{\delta}_j/\delta_{j\max}{}^{c2})$. If the participation of higher modes is large, the corresponding lowest-mode component is suppressed by this refinement equation. This procedure provides an efficient algorithm with rapid convergence. This property results from the fact that the fundamental-mode component is predominant in this shear building and the procedure of utilizing the fundamental frequency and eigenvector as the principal parameters for adjustment of mean peak interstorey drifts is employed here.

Even if the FI1 model of design $\{k_j{}^{c1}\}$ obtained in the fourth step in Fig.3 has $\overline{\omega}_1{}^{c1}$ as the fundamental frequency, the FD1 model of design $\{k_j{}^{c1}\}$ does not necessarily have it as the fundamental frequency (Nakamura and Takewaki (1989a)). In the case where both of $k_H(\omega)$ and $k_R(\omega)$ are single-valued non-increasing positive functions of frequency, it can be proved that both models have the same set of the fundamental frequency and the fundamental eigenvector (Nakamura and Takewaki (1989a)). This characteristic guarantees good accuracy of prediction of the seismic drift of a FD2 model by means of a FI2 model so far as the fundamental-mode component is dominant in the SRSS estimate of the seismic drift.

Although there is a limitation in this paper that spring stiffnesses have to satisfy the condition of single-valued non-increasing positive functions of frequency, the procedure described above may be applicable to the case where not both of $k_H(\omega)$ and $k_R(\omega)$ are single-valued non-increasing positive functions of frequency. However, it is required to confirm the correspondence of fundamental frequencies of a FI1 model and a FD1 model. If such a correspondence does not exist, a different procedure should be devised.

## EXAMPLES OF SEISMIC STIFFNESS DESIGN

Examples of seismic stiffness design for ten- and twenty-storey shear buildings are presented which are calculated for Problem SDFI using the design procedure described in the previous section. As described before, the shear building obtained is to be adopted as an approximate solution to Problem SDFD later.

Floor masses, moments of inertia of floor masses and storey heights are assumed to be $m_0$=720(x$10^3$kg), $m_1$=...=$m_f$=240(x$10^3$kg), $I_{R0}$=2.4x$10^7$(kg·m$^2$), $I_{R1}$=...=$I_{Rf}$= 8.0x$10^7$ (kg·m$^2$), $h_1$=...=$h_f$=3.50(m). A rigid square base mat of 20(m)x20(m) is assumed to be on a semi-infinite elastic ground. The complex spring constants of the base mat are evaluated by an approximate formula due to Veletsos and Verbic (1974) for a circular base mat of radius $r_o$=11.28(m) with the same area.

$$K_H(\omega) = k_H(\omega) + \mathrm{i}\omega c_H(\omega) = \frac{8Gr_o}{2-\nu}\left[K_{11}\left(\frac{\omega r_o}{V_S}\right) + \mathrm{i}\frac{\omega r_o}{V_S}C_{11}\left(\frac{\omega r_o}{V_S}\right)\right] \qquad (19a)$$

$$K_R(\omega) = k_R(\omega) + \mathrm{i}\omega c_R(\omega) = \frac{8Gr_o^3}{3(1-\nu)}\left[K_{22}\left(\frac{\omega r_o}{V_S}\right) + \mathrm{i}\frac{\omega r_o}{V_S}C_{22}\left(\frac{\omega r_o}{V_S}\right)\right], \qquad (19b)$$

where $G$, $V_S$ and ν denote the elastic shear modulus, the velocity of shear wave and Poisson's ratio of soil, respectively. $K_{11}(\ )$, $K_{22}(\ )$, $C_{11}(\ )$ and $C_{22}(\ )$ are non-dimensional quantities of which approximate expressions have been proposed by Veletsos and Verbic (1974). In this paper those expressions are adopted. Although the complex spring constants due to Veletsos and Verbic (1974) are utilized here as a simple example, more rigorous expressions may be used.

A site-dependent response spectrum (Mostaghel and Ahmadi (1979)) is adopted as the design response spectrum. The soil with the predominant period $T_c$=0.4(s) (shear wave velocity $V_S$=200(m/s)) is considered here. The density and Poisson's ratio of the soil are assumed to be ρ=1800(kg/m$^3$) and ν=1/3, respectively. In this case, $K_{11}$ and $C_{11}$ in eqn (19a) are frequency-independent and $K_{11}$=1.0, $C_{11}$=0.65 are employed here. On the other hand, $K_{22}$ and $C_{22}$ in eqn (19b) are frequency-dependent. The plots of the functions $K_{22}$ and $C_{22}$ with respect to dimensionless frequency are shown in Figs. 4(a)

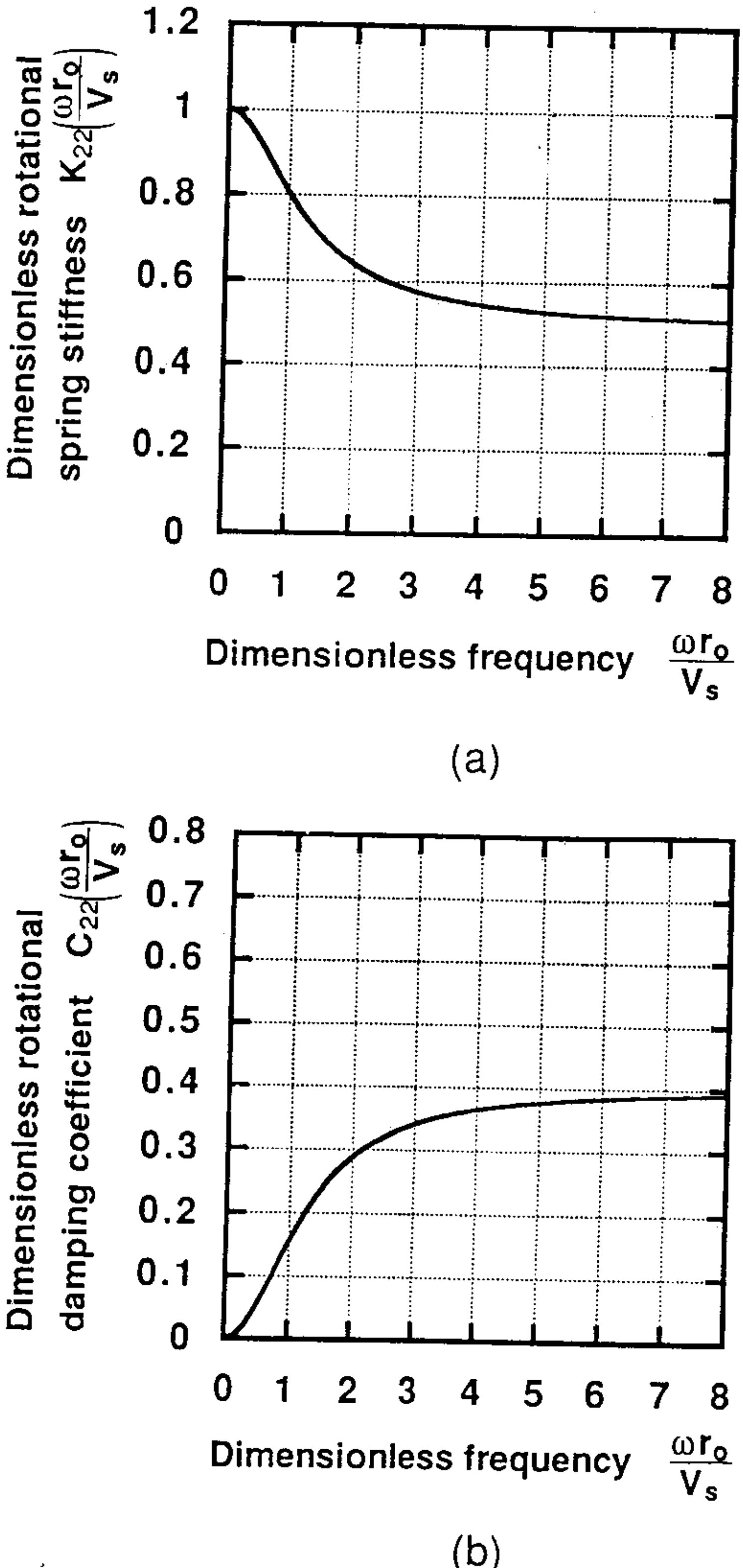


Fig.4. (a) Dimensionless rotational spring stiffness with respect to dimensionless frequency; (b) Dimensionless rotational damping coefficient with respect to dimensionless frequency (Poisson's ratio $\nu=1/3$)

and 4(b), respectively. The maximum acceleration at the free ground surface level is to have the value of 0.205(g) where g denotes acceleration of gravity. The damping ratio for the fundamental mode of each shear building designed for three interstorey drift levels stated afterwards with a fixed base is 0.02. All modes are taken into account in the SRSS estimate, i.e. $N_T = f+2$.

Since the seismic interstorey drifts in few storeys near the topmost and the lowest are often suppressed so as to attain smaller values than those in middle storeys in the usual structural design practice, the following specification of mean peak interstorey drifts is employed here, i.e. $\bar{\delta}_4 = \cdots = \bar{\delta}_{f-3} = \bar{\delta}$, $\bar{\delta}_1 = 0.75\bar{\delta}$, $\bar{\delta}_f = 0.5\bar{\delta}$ and

$$\bar{\delta}_2 = \bar{\delta}\left\{1 - 0.25\left(\frac{2}{3}\right)^2\right\},\ \bar{\delta}_3 = \bar{\delta}\left\{1 - 0.25\left(\frac{1}{3}\right)^2\right\}$$

$$\bar{\delta}_{f-1} = \bar{\delta}\left\{1 - 0.5\left(\frac{2}{3}\right)^2\right\},\ \bar{\delta}_{f-2} = \bar{\delta}\left\{1 - 0.5\left(\frac{1}{3}\right)^2\right\}$$

Three different levels of the mean peak interstorey drift $\bar{\delta}$ in the middle storeys are specified so as to attain 0.75, 1.0, 1.5($\times 10^{-2}$m).

Solid lines in Fig.5 show the storey stiffnesses of ten- and twenty-storey shear buildings designed with the present method for these three specified distributions of mean peak interstorey drifts. The storey stiffnesses of shear buildings with fixed bases designed with the method due to Nakamura and Yamane (1986) for the same distributions of mean peak interstorey drifts are also shown in Fig.5 by dashed lines. The fundamental natural periods of the designed shear building models (interaction models) are 0.781, 1.01 and 1.49(s) for the ten-storey buildings with $\bar{\delta}$=0.75, 1.0, 1.5 ($\times 10^{-2}$m), respectively, and 1.57, 2.06 and 3.04(s) for the twenty-storey buildings with $\bar{\delta}$=0.75, 1.0, 1.5($\times 10^{-2}$m), respectively. The fundamental natural periods of the shear building models with fixed bases are 0.758, 0.991 and 1.47(s) for the ten-storey buildings with $\bar{\delta}$=0.75, 1.0, 1.5($\times 10^{-2}$m), respectively, and 1.52, 2.02 and 3.02(s) for the twenty-storey buildings with $\bar{\delta}$=0.75, 1.0, 1.5($\times 10^{-2}$m), respectively. It can be observed from Fig.5 that the shear buildings with fixed bases require greater storey

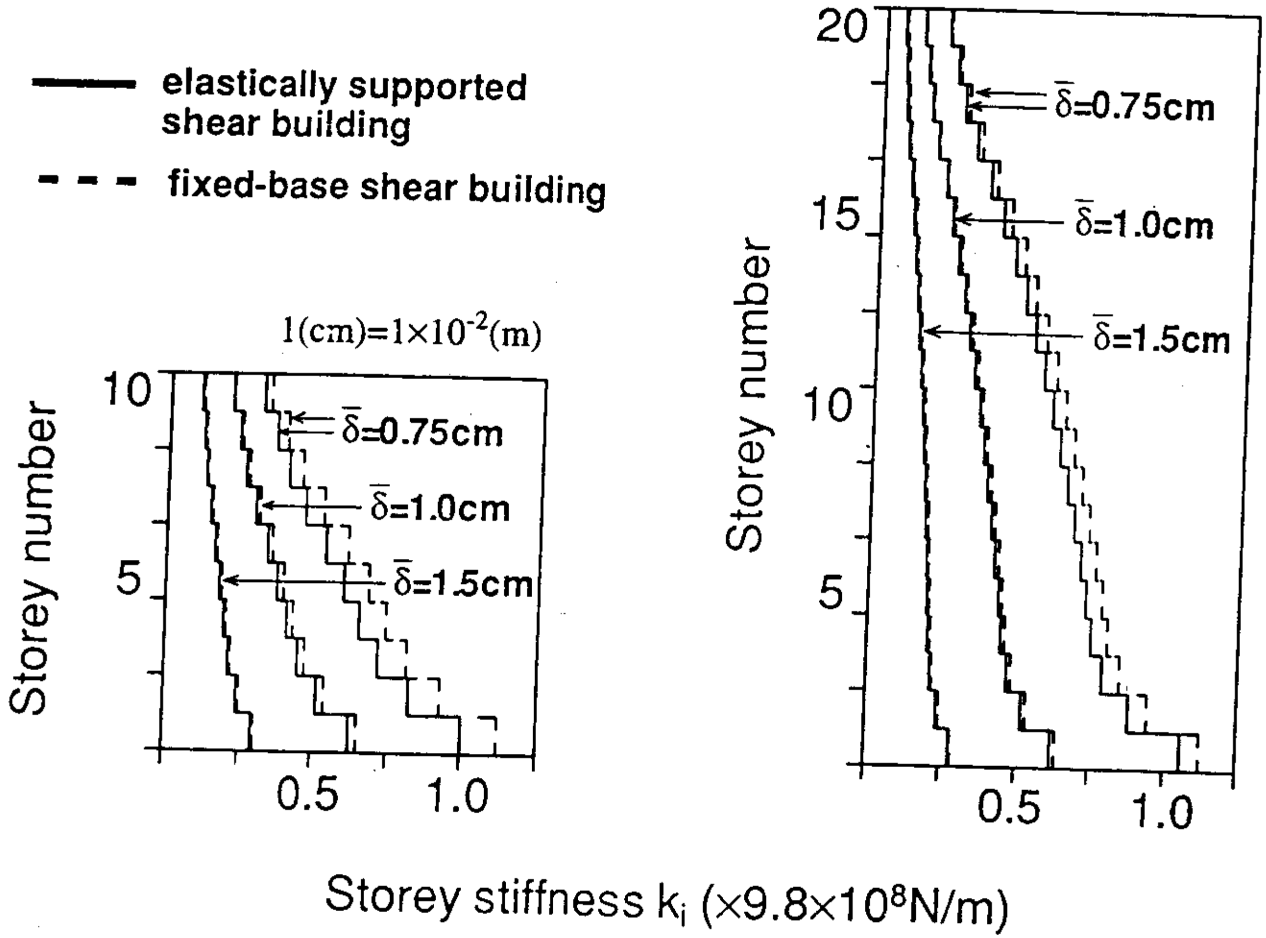


Fig.5. Storey stiffnesses of ten- and twenty-storey elastically supported shear buildings of seismic-drift constrained design and those of fixed-base shear buildings with common specified mean peak interstorey drifts $\bar{\delta}$=0.75, 1.0, 1.5(x10$^{-2}$m)

stiffnesses than those supported by complex springs exhibiting the same distribution of mean peak interstorey drifts.

## VERIFICATION BY RESPONSE ANALYSIS ON FI2 MODELS

Time-history response analysis has been performed first on FI2 models to demonstrate the validity of the present design method developed for Problem SDFI and to examine its accuracy. Twenty artificial ground motions compatible with the design response spectrum have been generated (see Appendix 2). It has been assured that increase of the number of ground motions over twenty does not affect much the response statistics. FI2 models designed for $\bar{\delta}$=0.75, 1.0, 1.5($\times 10^{-2}$m) by means of the present design method have been subjected to these artificial ground motions. The damping matrix $\mathbf{C}_B$ of the shear building has been obtained by specifying the damping ratio 0.02 for the fundamental mode of the shear building with a fixed base. Numerical integration has been performed in matrix form by means of the Newmark-β method with a constant acceleration scheme to maintain the non-classical damping characteristics. The interval of 0.01(s) has been adopted for each time step. The accuracy of this program of time-history response analysis has been investigated through the comparison with that of the response analysis in the frequency domain. It has been assured that the peak interstorey drift in every storey due to the present response analysis program coincides with that due to the program utilizing the frequency-domain analysis within the accuracy of 1%. This result indicates the validity of both the response analysis program in the time domain and that in the frequency domain.

Figure 6(a) shows the mean values and mean ± one standard deviation lines of peak interstorey drifts of the ten-storey shear building (FI2 model) designed for $\bar{\delta}$=1.0 ($\times 10^{-2}$m) subjected to twenty artificial ground motions. Figure 6(a) indicates that the mean peak interstorey drifts of the ten-storey shear building (FI2 model) are within ±5% lines of the specified values. The result for the twenty-storey shear building is shown in Fig.6(b). It can be observed from Fig.6(b) that the mean peak interstorey drifts of the twenty-storey shear building (FI2 model) are mostly within ±10% lines of the specified values.

In order to investigate the effect of off-diagonal terms appearing in the modal decomposition on the total response, additional time-history response analysis has been performed where a FI2 model is employed and those off-diagonal terms are ignored. It has been observed that the deviations of the mean peak interstorey drifts obtained by this time-history response analysis from those shown in Fig.6 are almost within 2% of the latter. The degree of compatibility of the mean response spectrum of twenty artificial ground motions with the design response spectrum has then been evaluated. It has been found that the deviations of the former corresponding to the fundamental frequency from the latter are within 2% of the latter and that the effects of these deviations on the total response are 3% at most. It can be inferred from these facts that the principal factor causing those deviations shown in Fig.6(b) is the inaccuracy of the SRSS method for the elastically supported shear building model where modal decomposition is achieved exactly for a FI2 model by ignoring off-diagonal terms appearing in the modal decomposition. It should be remarked that the SRSS method can be justified only when the following basic assumptions are satisfied (Der Kiureghian (1980)), i.e. (i) the ground motion under consideration is a stationary Gaussian process with a wide-band power spectral density, (ii) the response of the structure is a stationary process and (iii) modes are well separated. In order to investigate the effect of the second assumption of stationary process, another set of twenty artificial ground motions have been generated where only the interval of strong phase has been changed from 19(sec) to 38(sec). It has been observed from the time-history response analysis to this set of twenty artificial ground motions that the maximum deviation of the mean peak interstorey drifts of the twenty-storey shear building designed for $\bar{\delta}=1.0(\times 10^{-2}\mathrm{m})$ has been reduced from 11% to 7%.

## VERIFICATION BY RESPONSE ANALYSIS ON FD2 MODELS

Response analysis in the frequency domain has been performed to demonstrate the validity of the present design method for Problem SDFD and to examine its accuracy.

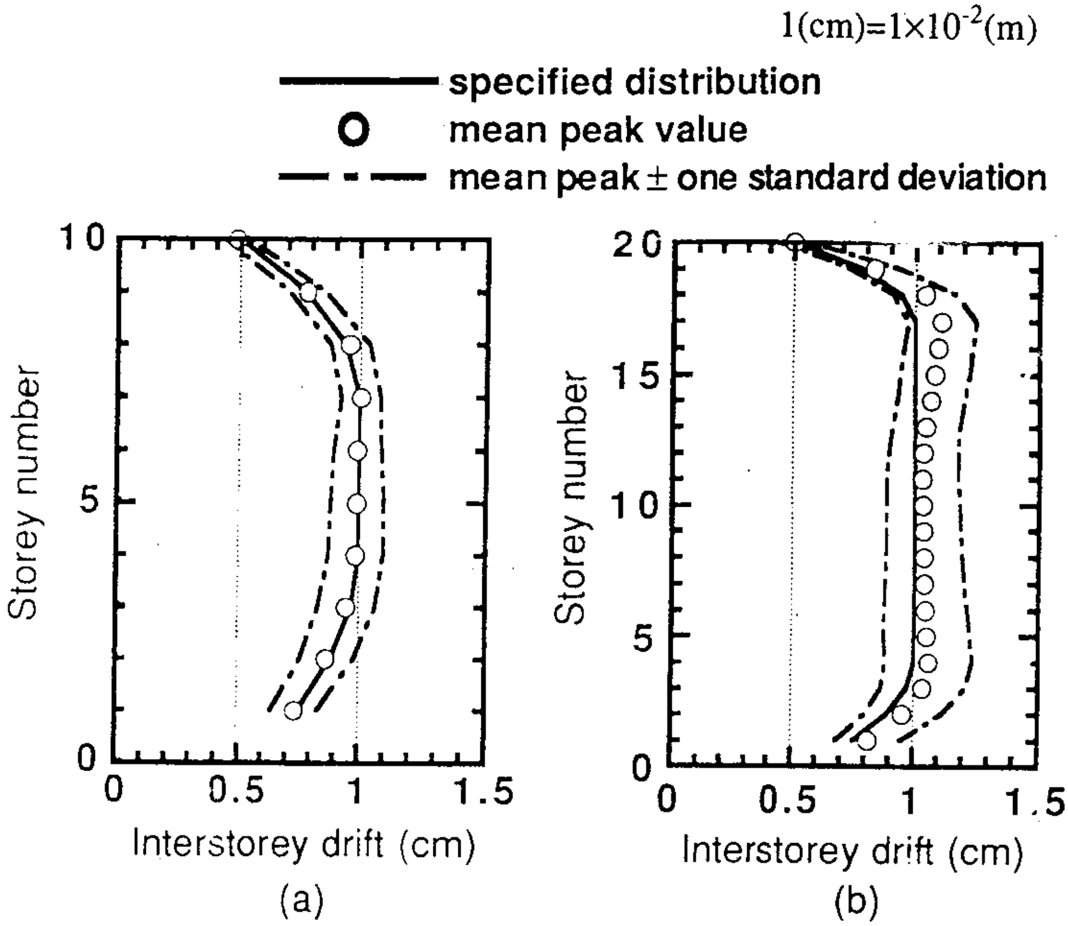


Fig.6. Distributions of mean peak interstorey drifts and ± one standard deviation lines of ten- and twenty-storey shear buildings (FI2 model) designed for $\bar{\delta}=1.0(\times 10^{-2}m)$ subjected to twenty spectrum-compatible artificial earthquakes

The shear buildings designed for $\bar{\delta}$=1.0(x$10^{-2}$m) by means of the present design method using FI2 models have been supported by frequency-dependent complex springs and have been subjected to the same set of artificial ground motions as described above (strong motion duration=19(sec)). The Fourier transforms of response displacements can be calculated from eqn (3) as

$$\mathbf{U}(\omega) = -[\mathbf{K}_B(\mathbf{k}) + \mathbf{K}_F(k_H(\omega), k_R(\omega)) + \mathrm{i}\omega\{\mathbf{C}_B + \mathbf{C}_F(c_H(\omega), c_R(\omega))\} - \omega^2\mathbf{M}]^{-1}\mathbf{Mr}\ddot{U}_g(\omega) \tag{20}$$

The corresponding time-history response $\mathbf{u}(t) = \{u_1(t) \cdots u_f(t) \;\; u_F(t) \;\; \theta_F(t)\}^T$ may be obtained by the Fourier inverse transformation of eqn (20).

$$\mathbf{u}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{U}(\omega)\exp(\mathrm{i}\omega t)\mathrm{d}\omega \tag{21}$$

Computation of $\ddot{U}_g(\omega)$ in eqn (20) and $\mathbf{u}(t)$ in eqn (21) can be performed by using FFT (Liu, and Fagel (1971), Fagel and Liu (1972), Liu and Fagel (1973)). Nyquist frequency has been chosen as 50(Hz) for all the cases.

Figure 7(a) shows the mean values and mean ± one standard deviation lines of peak interstorey drifts of the ten-storey shear building (FD2 model) designed for $\bar{\delta}$=1.0 (x$10^{-2}$m) subjected to twenty artificial ground motions. The result for the twenty-storey shear building is shown in Fig.7(b). It can be observed from Fig.7 that the plots of the mean peak interstorey drifts are mostly within ±10% lines of the specified mean peak interstorey drifts. Furthermore, it can be observed from Figs. 6 and 7 that the distribution of the mean peak interstorey drifts of the FD2 model indicates almost the same distribution as that of the FI2 model.

In order to investigate the validity of utilizing a FI2 model for estimating the seismic response of a FD2 model from another point of view, transfer functions of 1st, 5th and 10th interstorey drifts are plotted in Fig.8 for the ten-storey shear building (FI2 model) and that (FD2 model) designed for $\bar{\delta}$=1.0(x$10^{-2}$m). A similar approach can be found in Wu and Smith (1995). The transfer function indicates the ratio of the interstorey drift to the free-field ground acceleration in the frequency domain. It can be observed from Fig.8

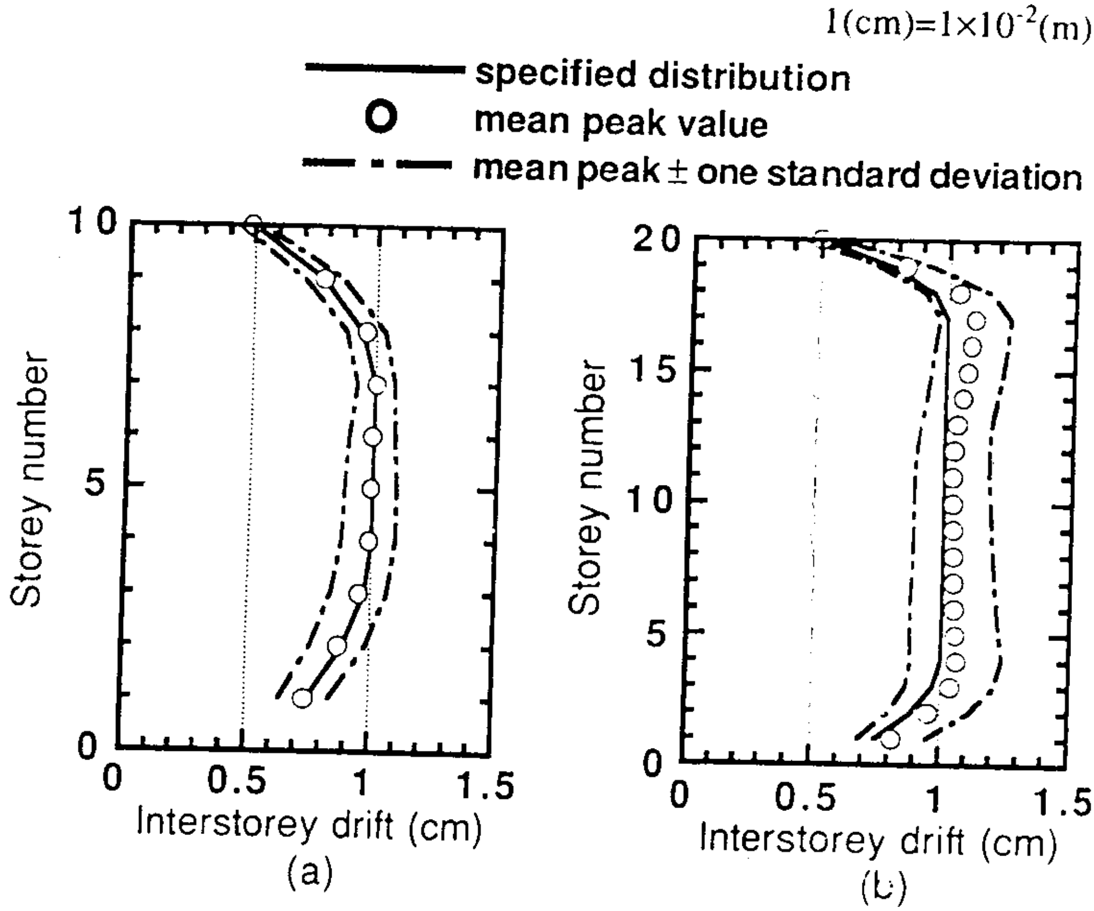


Fig.7. Distributions of mean peak interstorey drifts and ± one standard deviation lines of ten- and twenty-storey shear buildings (FD2 model) designed for $\delta=1.0(\times 10^{-2}\text{m})$ subjected to twenty spectrum-compatible artificial earthquakes

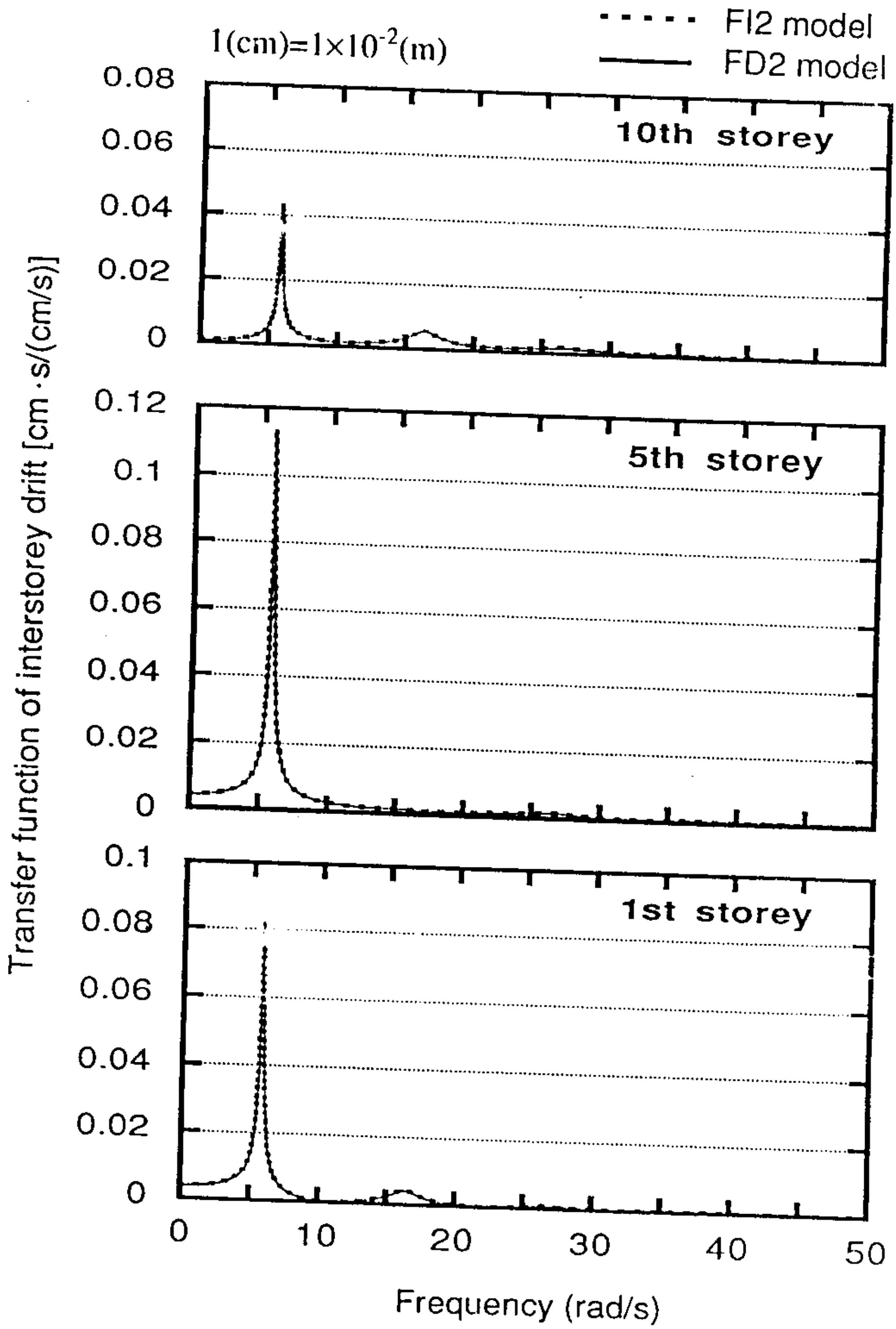


Fig.8 Transfer functions of 1st, 5th and 10th interstorey drifts of the ten-storey shear building (FI2 model) and that (FD2 model) designed for $\bar{\delta}=1.0(\times 10^{-2}\text{m})$

that there does not exist a significant difference between these two models. Almost the same characteristics have been observed from the result for the twenty-storey shear building designed for $\bar{\delta}=1.0(\times 10^{-2}m)$. Figure 8 indicates that the FI2 model is a good model for estimating the seismic drift of the FD2 model from the point of view of characteristics in the frequency domain. Contributions from higher modes can also be observed from Fig.8 and it can be understood that their effects are small in ten-storey buildings except few storeys near the top.

It is interesting to note in view of Figs. 4(a) and (b) that the FI2 model is defined so that rotational spring stiffnesses corresponding to higher natural frequencies are overestimated compared to the original model and rotational damping coefficients corresponding to higher natural frequencies are underestimated in this case. This fact guarantees good accuracy of estimation of the earthquake response of the FD2 model through that of the FI2 model together with the fact that spring stiffnesses and damping coefficients of both models corresponding to the fundamental natural frequency just coincide. It may be concluded from Figs. 6, 7 and 8 that the accuracy of this design method depends mainly on the accuracy of prediction of the mean peak seismic drift of a FI2 model.

## CONCLUSIONS

A direct method of seismic stiffness design has been developed for finding the set of storey stiffnesses of the shear building, supported by frequency-dependent complex springs, which exhibits a specified distribution of mean peak interstorey drifts to a set of design-spectrum compatible earthquakes.

The seismic drift of the shear building model (FD model) supported by frequency-dependent complex springs has been estimated approximately through that of a shear building model (FI model) supported by the corresponding frequency-independent complex springs. It has been shown that this fairly good correspondence of seismic drifts between these two models is guaranteed by the theorems (Nakamura and Takewaki (1989a)). As an approximate solution to the problem of seismic stiffness design of a FD model, the seismic stiffness design of the FI model has been employed. A closed form solution to a hybrid inverse eigenmode problem has been derived and the qualification conditions on the specified fundamental frequency and eigenvector have been disclosed for providing positive storey stiffnesses. The formula has then been shown to be useful for developing the direct and efficient method of seismic stiffness design of the FI model. It has been pointed out that a fundamental frequency and a fundamental eigenvector can play a role of the principal parameters for adjustment of mean peak seismic drifts in the design procedure and that a rapidly convergent algorithm can be devised due to the predominant role of the lowest eigenvibration.

Response analysis on the original model in the frequency domain has been performed only for evaluating accuracy of this design method. It has been disclosed that the shear building designed with the present design method exhibits the specified distribution of mean peak interstorey drifts within a reasonable accuracy.

It may be concluded from these results that this paper has indeed enabled one to find without design-sensitivity analysis nor parametric analysis a set of storey stiffnesses of the FD model which exhibits a specified distribution of mean peak interstorey drifts to design earthquakes within a reasonable accuracy. It is expected that this paper plays a role for providing a guideline for developing a direct design method for more realistic structure-foundation systems (see, for example, Todorovska and Trifunac (1992), Todorovska (1993)).

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## Appendix 1: Design response spectrum

The velocity response spectrum $S_V(T; h)$ (Mostaghel and Ahmadi (1979)) is expressed as follows.

$$T \le T_L: \quad S_V(T; h) = \ddot{u}_{gmax} \frac{T}{2\pi}$$

$$T_c/10 \le T \le T_c/3: \quad S_V(T; h) = \ddot{u}_{gmax} N \frac{T}{2\pi}$$

$$T_c \le T \le T_d: \quad S_V(T; h) = \frac{T_c}{2\pi} \ddot{u}_{gmax} N \sqrt{(1 - e^{-100h})}$$

$$3T_d \le T \le T_{dU}: \quad S_V(T; h) = \frac{T_c}{2\pi} \ddot{u}_{gmax} N \frac{T_d}{T}, \qquad \text{(A1a-d)}$$

where $T$, $h$, $T_c$ and $\ddot{u}_{gmax}$ denote a natural period, a damping ratio, the predominant period of soil and the maximum value of ground acceleration, respectively, and $T_L$, $T_U$, $T_d$, $N$, $\bar{h}$ indicate the following quantities; $T_L$=0.03(s), $T_U$= a number greater than 60(s), $T_d$ =4/$T_c$, $N = (1+2h)\sqrt{(\bar{h}/h)}$, $\bar{h}$=0.20.

Linear interpolation is employed in the regions of $T_L \le T \le T_c/10$, $T_c/3 \le T \le T_c$ and $T_d \le T \le 3T_d$ with respect to double logarithmic axes (log$T$, log$S_V(T; h)$).

## Appendix 2: Generation of artificial earthquake ground motions

Twenty artificial ground motions have been generated by using SIMQKE program (Gasparini and Vanmarcke (1976)). The target spectrum has been chosen from the velocity response spectrum with damping ratio of 0.02. The control points of the target spectra are as follows. ($T$(s), $S_V$(m/s))=(0.03, 0.00960), (0.04, 0.0421), (0.133, 0.14), (0.40, 0.391), (10.0, 0.391).

The following envelope function has been employed.

$$\zeta(t) = 0.205\text{g}\,(t/3)^2 \quad (0 \le t \le 3)$$

$$\zeta(t) = 0.205\text{g} \quad (3 \le t \le 22)$$

$$\zeta(t) = 0.205\text{g} \cdot \exp[-0.24\,(t-22)] \quad (22 \le t \le 34.5) \qquad \text{(A2a-c)}$$

The duration of each artificial ground motion is 34.5(s) and uniform random numbers have been adopted for the characteristics of phase angles.