ISET Journal of Earthquake Technology, Paper No. 386, Vol. 35, No. 4, December 1998, pp. 263-286

A PUSH-OVER ANALYSIS METHOD FOR AXIALLY DEFORMABLE CROSS-BRACED FRAMES WITH DISSIPATIVE DEVICES

P. Colajanni and M. Papia

Dipartimento di Ingegneria Strutturale e Geotecnica
Universita di Palermo, Viale delle Scienze; I-90128, Palermo, Italy

ABSTRACT

A simple method for nonlinear analysis of cross-braced multistorey frames with hysteretic dissipative devices, subjected to monotonically increasing horizontal loads (push-over analysis) is presented. It is based on a model able to provide the approximate tangent condensed flexibility matrix of the system by closed analytical formulae. The model takes the axial deformability of the frame into account, considering that it significantly influences both the fundamental period and the corresponding mode of vibration of the structure. It enables one to evaluate separately the contributions to the first mode shape due to both the flexural deformability of the frame beams and columns (shear type behaviour) and the axial extension of the frame columns, furnishing further insight for the seismic design of the structure. The proposed simplified method is validated by comparison with the results of a push-over analysis performed by a FEM computer program for some frames. Moreover, the effectiveness of the push-over analysis in assisting the design procedure of the structure is checked by evaluation of the time history of the seismic response.

KEYWORDS: Cross-Braced Frames, Dissipative Devices, Nonlinear Seismic Response, Step-by-Step Static Analysis

INTRODUCTION

One of the most reliable methods for prediction of seismic behaviour of a structure is to perform a set of nonlinear time-history analyses of the response of the actual multi-degree of freedom structure, for different ground motion accelerograms. Nevertheless, this procedure is not practical for everyday design use, due to the great cost in terms of computational effort that it requires.

In everyday practice, either a very simple static approach (the equivalent lateral force procedure) or a simple dynamic approach (spectral modal analysis) is used. Both methods are based on the assumption of linear elastic behaviour. However, because of its straightforward nature, the inelastic response spectra-based modal analysis has been considered a potential method for inelastic structural design. Studies have concluded that this procedure may either yield unconservative designs [Haviland et al. (1976), Luyties et al. (1976)], or lead to designed structures which may, on average, be expected to resist base shears that are two or three times larger than the code design values [Housner and Jennings (1982)] without major structural damage.

Recently, several researchers have emphasized the need for changes in the existing seismic design procedure prescribed by the codes [Bertero et al. (1991), Priestley (1993), Krawinkler (1994)], aiming at a simple procedure of wide applicability, taking full advantage of presently available ground motion information and engineering knowledge. A promising method seems to be the one that has been proposed in different forms by several authors [Saiidi and Sozen (1981), Fajfar and Gaspersic (1996)]. It is based on a nonlinear static analysis of the MDOF system under a monotonically increasing lateral load (push-over analysis) and on a nonlinear dynamic analysis of an equivalent SDOF system, able to capture the main characteristics of the dynamic behaviour of the actual structure. Dynamic analysis may also be avoided if inelastic response spectra are utilized.

The nonlinear characteristics of the equivalent SDOF system are determined on the base shear top displacement relationship of the actual MDOF structure, obtained by the push-over analysis. The latter also allows one to evaluate, for a fixed value of the top displacement, local seismic demands for different response parameters (displacement, ductility, dissipated energy demands, etc.).

It is useful to remark that the method is applicable to structures oscillating predominantly in a single mode, because the most essential assumption of the method is a time-independent lateral displacement shape of the structure. Therefore, the influence of the higher modes cannot be properly taken into account. In order to predict accurately the lateral displacement shape of the structure, it is of fundamental importance that the nonlinear static analysis is performed by assuming, at each instant, a pattern of the external static load increments proportional to the mode shape corresponding to the tangent stiffness matrix of the structure.

Within this framework, in this paper an analytical model is proposed which can be used to perform, in a simplified but accurate manner, the push-over analysis for cross-braced moment resistant steel frames equipped with dissipative devices. The model takes the axial deformability of the frame into account, which is usually neglected in most of the studies on the behaviour of the structures examined [e.g. Filiatrault and Cherry (1989, 1990)]. It is shown that it significantly influences both the base shear-top displacement relationship and the fundamental mode shape of the structure. Moreover, the proposed model allows one accurately to perform the push-over analysis, easily evaluating the instantaneous shape that has to be assigned to the increments of the monotonically increasing lateral forces.

The model can be utilized as an aid for the design of the structure because it reveals the influence of the different structural elements in determining the system response. It makes it possible to calibrate the strength of the dissipative devices in such a way that they begin to operate simultaneously when a strong earthquake occurs. This condition maximizes the effectiveness of the bracing system in reducing the seismic demand of the structure.

MODELLING OF STRUCTURE

The formulation which will be proposed here is based on the simplified frame model shown in Figure 1(a), where the symbols which will be utilized are also pointed out.

The frame is assumed to have an infinitely elastic behaviour; namely, it is assumed that the diagonal braces and the dissipative devices inserted at their intersection are designed to prevent plasticisation of the structural members of the frame. The beams are assumed axially inextensible, while the axial extensibility of the columns is taken into account, because it can produce significant deck rotations influencing the horizontal displacements. Moreover, at a given storey the two columns are assumed to be made with the same steel profile and to be oriented identically in plan.

The i-th stiffening diagonal brace is assumed to behave as a slender strut hinged at the ends and affected by the critical load value $P_{co,i}$. The maximum load value in tension $P_{to,i}$, playing the role of a conventional yielding load, is actually the axial tensile force acting on the brace in the load condition at which the dissipative device begins to operate (the device obviously has to be calibrated so that the actual yielding load of the brace is greater than $P_{to,i}$). This modelling criterion is able to represent with very good approximation the behaviour of a large class of bracing systems equipped with dissipative devices, characterized by stable hysteretic "elastic-plastic" horizontal force-displacement cycles, as shown by Colajanni and Papia (1995, 1997).

Denoting as β_i the angle of inclination of the brace with respect to the horizontal direction at the i-th floor, and setting

$$S_{co,i} = P_{co,i} \cos\beta_i \tag{1a}$$

$$S_{to,i} = P_{to,i} \cos\beta_i \tag{1b}$$

The horizontal strength of the dissipative bracing systems considered is $F_{u,i} = S_{to,i} + S_{co,i}$. The lateral stiffness, on which the slope of the elastic branch of the force-displacement cycle depends, can be related to the two lateral stiffness values obtained considering the force-displacement law corresponding to the adopted modelling. This is because in the proposed model the strength $F_{u,i}$ is reached in a first phase of response, in which both diagonal braces behave elastically, followed by a second phase in which the brace in compression is buckled.

Further basic assumptions in the model shown in Figure 1(a) concern the condition of zero value of bending moment at the middle-height section of each column, and the inter-storey height, which is the same at each level. Actually, the latter assumption could be removed without special difficulties; therefore it has to be considered as a working hypothesis simplifying the formulation.

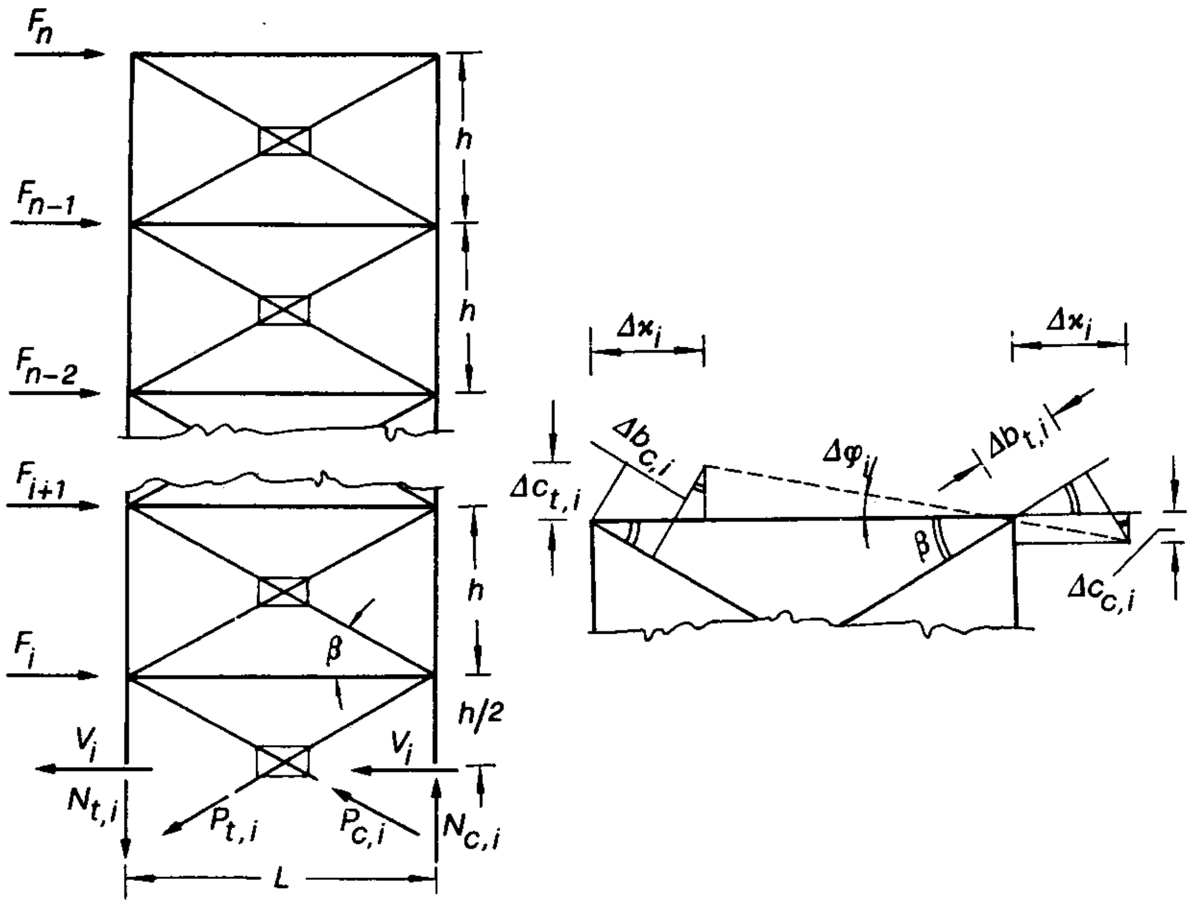

Fig.1 Modelling of structure and symbols: a) equilibrium condition; b) compatibility condition

FORMULATION

The equilibrium condition of the system in Figure 1(a) can be expressed by the three equations

$$\left(P_{c,i}+P_{t,i}\right)\cos\beta+2V_i=\sum_{j=i}^{n}F_j=T_i \tag{2}$$

$$\left(P_{t,i}-P_{c,i}\right)\sin\beta=N_{c,i}-N_{t,i} \tag{3}$$

$$N_{c,i}+N_{t,i}=2\tan\beta\sum_{j=i}^{n}F_j\left(j-i+\frac{1}{2}\right)=\tan\beta\left(T_i+2\sum_{j=i+1}^{n}T_j\right) \tag{4}$$

where n is the number of storeys, T_i is the shear value at the i-th level due to the external horizontal forces, and $\beta=\beta_i$ $(i=1,2,\ldots,n)$.

The scheme in Figure 1(b), showing the length variations in the columns and braces at the i-th storey, suggests the following compatibility conditions:

$$\frac{\Delta b_{c,i}}{\cos\beta}+\Delta c_{t,i}\tan\beta=\frac{\Delta b_{t,i}}{\cos\beta}+\Delta c_{c,i}\tan\beta=\Delta x_i \tag{5}$$

where Δx_i is the horizontal component of displacement of the i-th storey with respect to a system of reference moving jointly with the (i-1)-th storey.

Equations (2) to (5) and the force-displacement relationships referring to the single structural elements make it possible to link the relative horizontal translation Δx_i to the values of horizontal (seismic) shear T_i acting at each storey $(i = 1, 2, \ldots, n)$. This relationship at the i-th storey has to be specialized in accordance with the occurrence of one of the following work conditions of the diagonal braces:

condition I: both diagonal braces behave elastically;
condition II: the diagonal brace in compression is buckled and the one in tension is in the elastic field $(P_{c,i} = P_{co,i}$ and $P_{t,i} < P_{to,i})$;
condition III: the diagonal brace in compression is buckled and the dissipative devicė is active $(P_{c,i} = P_{co,i}$ and $P_{t,i} = P_{to,i})$;

1. Condition I: Both Diagonal Braces in Elastic Field

On the basis of the assumptions in the previous section, considering the symbols in Figure 1(a), one can set

$$P_{c,i} = P_{t,i} = P_i \tag{6a}$$

$$N_{c,i} = N_{t,i} = N_i \tag{6b}$$

Moreover, with reference to the scheme in Figure 1(b),

$$\Delta b_{c,i} = \Delta b_{t,i} = \Delta b_i \tag{7a}$$

$$\Delta c_{c,i} = \Delta c_{t,i} = \Delta c_i \tag{7b}$$

Therefore, Equations (2), (4) and (5) become respectively

$$2P_i \cos\beta + 2V_i = T_i \tag{8}$$

$$N_i = \frac{1}{2}\tan\beta\left(T_i + 2\sum_{j=i+1}^{n} T_j\right) \tag{9}$$

$$\Delta x_i = \frac{\Delta b_i}{\cos\beta} + \Delta c_i \tan\beta \tag{10}$$

Denoting as E the Young's modulus of the steel material and as $A_{d,i}$ and L_d the cross-sectional area and the length of the diagonal brace at the i-th storey, respectively, the axial stiffness of this brace, evaluated with respect to the horizontal and vertical directions, provides the expressions

$$k_{h,i}^{d} = \frac{EA_{d,i}}{L_d}\cos^2\beta \tag{11a}$$

$$k_{v,i}^{d} = \frac{EA_{d,i}}{L_d}\sin^2\beta = k_{h,i}^{d}\tan^2\beta \tag{11b}$$

By the same criterion, the lateral and vertical (axial) stiffnesses of the column at the i-th storey of the cross braced frame are expressed by

$$k_{h,i}^{c} = \frac{12EI_{c,i}}{h^3}\alpha_i \tag{12a}$$

$$k_{v,i}^{c} = \frac{EA_{c,i}}{h} \tag{12b}$$

where $A_{c,i}$ and $I_{c,i}$ are the cross-sectional area and the flexural moment of inertia of the column, and $\alpha_i < 1$ is a coefficient reducing the shear stiffness of the column, taking the elastic rotation at the top and the base of the column itself into account. The values of α_i at each storey can be determined by means of different simplified approaches. In this paper the procedure proposed by Ramasco (1985) and briefly described in Appendix is adopted. Considering that

$$\Delta b_i = \frac{P_i}{k_{h,i}^d} \cos^2 \beta \tag{13a}$$

$$\Delta c_i = \frac{N_i}{k_{v,i}^c} \tag{13b}$$

$$\Delta x_i = \frac{V_i}{k_{h,i}^c} \tag{13c}$$

by using Equations (8), (13a) and (13c), one obtains

$$\Delta b_i = \left(\frac{1}{2} \frac{T_i}{k_{h,i}^d} - \frac{k_{h,i}^c}{k_{h,i}^d} \Delta x_i \right) \cos \beta \tag{14}$$

Moreover, considering Equations (9) and (13b) gives

$$\Delta c_i = \frac{1}{2k_{v,i}^c} \left(T_i + 2 \sum_{j=i+1}^{n} T_j \right) \tan \beta \tag{15}$$

Therefore, introducing Equations (14) and (15) in Equation (10) and considering Equation (11b), one obtains

$$\Delta x_i = \frac{1}{k_{h,i}^c} \frac{1}{1+\gamma_{h,i}} \left(\frac{1+\gamma_{v,i}}{2} T_i + \gamma_{v,i} \sum_{j=i+1}^{n} T_j \right) \tag{16}$$

where,

$$\gamma_{h,i} = \frac{k_{h,i}^d}{k_{h,i}^c} \tag{17a}$$

$$\gamma_{v,i} = \frac{k_{v,i}^d}{k_{v,i}^c} \tag{17b}$$

2. Condition II: After Buckling of a Diagonal Brace in Compression

In this phase the condition $P_{c,i} = P_{co,i}$ is assumed. Although many experimental tests have shown that the load carrying capacity of a strut drops significantly following buckling, the assumption is justified on the basis of the following considerations: (i) the reduction of the load carrying capacity is appreciable only after the first buckling of the strut; (ii) during the subsequent cycles, the buckling load and the residual carrying load capacity are similar, and they define the values that have to be assigned to $P_{co,i}$ in order to characterize the strut cyclic behaviour.

Solving the system of Equations (2), (3) and (4), and remembering Equation (1a), one obtains

$$P_{t,i} = \left(T_i - 2V_i - S_{co,i} \right) \frac{1}{\cos \beta} \tag{18}$$

$$N_{c,i} = \left(T_i - \sum_{j=i+1}^{n} T_j - V_i - S_{co,i} \right) \tan \beta \tag{19}$$

$$N_{t,i} = \left(\sum_{j=i+1}^{n} T_j + V_i + S_{co,i} \right) \tan \beta \tag{20}$$

Specializing Equation (13a) for the diagonal brace in tension and Equation (13b) for the column in compression, the second of Equations (5), considering Equations (18) and (19), provides

$$\Delta x_i = \frac{1}{k^d_{h,i}}\left(T_i - 2V_i - S_{co,i}\right) + \frac{1}{k^c_{v,i}}\left(T_i + \sum_{j=i+1}^{n} T_j - V_i - S_{co,i}\right)\tan^2\beta \tag{21}$$

Therefore, deriving V_i from Equation (13c) and taking Equation (17) into account, one obtains

$$\Delta x_i = \frac{1}{k^c_{h,i}}\frac{1}{2+\gamma_{h,i}+\gamma_{v,i}}\left[\left(1+\gamma_{v,i}\right)\left(T_i - S_{co,i}\right) + \gamma_{v,i}\sum_{j=i+1}^{n} T_j\right] \tag{22}$$

3. Condition III: After Activation of Dissipative Device

Since in this case $P_{c,i} = P_{co,i}$ and $P_{t,j} = P_{to,j}$, Δx_i can be derived from Equation 2 considering Equations (1a), (1b) and (13c):

$$\Delta x_i = \frac{1}{2k^c_{h,i}}\left(T_i - S_{to,i} - S_{co,i}\right) \tag{23}$$

RELATIVE DISPLACEMENTS AND DEFORMABILITY COEFFICIENTS

Denoting as u_i the horizontal displacement of the i-th deck referred to a system of reference fixed to the ground, the relative horizontal displacement Δu_i can be related to the value of Δx_i by the relationship

$$\Delta u_i = u_i - u_{i-1} = \Delta x_i + h\sum_{j=1}^{i-1}\Delta\varphi_j \tag{24}$$

where,

$$\Delta\varphi_j = \frac{1}{L}\frac{N_{c,j}+N_{t,j}}{k^c_{v,j}} \tag{25}$$

is the rotation of the j-th deck with respect to the (j-1)-th deck, produced by the axial extensibility of the columns located between these decks.

Therefore, considering Equations (4) and (25), Equation (24) becomes

$$\Delta u_i = \Delta x_i + \tan^2\beta\sum_{j=1}^{i-1}\frac{1}{k^c_{v,j}}\left(T_j + 2\sum_{k=j+1}^{n} T_k\right) \tag{26}$$

Remembering the expressions of Δx_i in the three cases considered and using Equations (17a) and (17b), Equation (26) can be written in the general form

$$\Delta u_i = d_{1,i}T_i + d_{2,i}\sum_{j=i+1}^{n} T_j + \sum_{j=1}^{i-1} d_{3,j}\left(T_j + 2\sum_{k=j+1}^{n} T_k\right) - \left(d_{4,i}S_{co,i} + d_{5,i}S_{to,i}\right) \tag{27}$$

where the deformability factors at the i-th storey, $d_{k,i}$ $(k = 1, 2, \ldots, 5)$ can be expressed by

$$d_{k,i} = \frac{1}{k^c_{h,i}}c_{k,i} \qquad (k = 1, 2, \ldots, 5) \tag{28}$$

The dimensionless coefficients $c_{k,i}$ have to be specialized so that Equations (16), (21) or (23) are verified, in accordance with the occurrence of work condition I, II, or III, respectively. The expressions of these coefficients derived from the aforementioned equations are shown in Table 1.

Making the summations appearing on the right-hand side of Equation (27) explicit, it can easily be shown that the relationship between relative horizontal displacements and seismic shears can be expressed in matrix form by

$$\Delta \boldsymbol{u} = \boldsymbol{D}\cdot\boldsymbol{T} \tag{29}$$

where
$$\Delta \boldsymbol{u} = [\Delta u_1 \quad \Delta u_2 \quad \ldots \quad \Delta u_n]^T \tag{30a}$$
$$\boldsymbol{T} = [T_1 \quad T_2 \quad \ldots \quad T_n]^T \tag{30b}$$

and $\boldsymbol{D}$ is a square matrix containing the following terms

$$a_{ii} = d_{1,i} + 2\sum_{k=1}^{i-1} d_{3,k} \qquad (i = 1,\ 2,\ \ldots, n)$$
$$a_{ij} = d_{2,i} + 2\sum_{k=1}^{i-1} d_{3,k} \qquad (i = 1,\ 2,\ \ldots, n \text{ and } j > i) \tag{31}$$
$$a_{ij} = d_{3,j} + 2\sum_{k=1}^{j-1} d_{3,k} \qquad (i = 1,\ 2,\ \ldots, n \text{ and } i > j)$$

Table 1: Expression of Coefficients for Calculation of Deformability Factors

	Coefficients $c_{k,i} = d_{k,i} k^c_{h,i}$				
	$c_{1,i}$	$c_{2,i}$	$c_{3,i}$	$c_{4,i}$	$c_{5,i}$
Condition I	$\frac{1}{2}\frac{1+\gamma_{v,i}}{1+\gamma_{h,i}}$	$\frac{\gamma_{v,i}}{1+\gamma_{h,i}}$	$\frac{\gamma_{v,i}}{\gamma_{h,i}}$	0	0
Condition II	$\frac{1+\gamma_{v,i}}{2+\gamma_{v,i}+\gamma_{h,i}}$	$\frac{\gamma_{v,i}}{2+\gamma_{v,i}+\gamma_{h,i}}$	$\frac{\gamma_{v,i}}{\gamma_{h,i}}$	$c_{1,i}$	0
Condition III	$\frac{1}{2}$	0	$\frac{\gamma_{v,i}}{\gamma_{h,i}}$	$c_{1,i}$	$c_{1,i}$

FIELD OF APPLICATION OF DERIVED LAWS

For practical use of Equation (29), by means of which the evolution of the deformed shape under increasing seismic loads can be foreseen, a criterion for the updating of the deformability factors must be utilized.

A very simple procedure is based on the use of relationships linking at each storey the values $S_{co,i}, S_{to,i}$, which characterize the response of the dissipative bracing system, to the seismic shear distribution.

More precisely, starting from the values of $d_{k,i}$ $(k = 1,\ 2,\ \ldots, 5)$ corresponding to work condition I for all dissipative bracing systems, the transition from this condition to condition II at the i-th storey can be found considering that at this occurrence the diagonal brace in compression and the one in tension are subjected to an axial force of intensity equal to $P_{co,i}$. Therefore, taking Equations (1) and (13c) into account, Equation (8) provides

$$\Delta x_i = \frac{1}{k^c_{h,i}}\left(\frac{T_i}{2} - S_{co,i}\right) \tag{32}$$

Expressing Δx_i by means of Equation (16), Equation (32) becomes

$$S_{co,i} = \frac{1}{2}\frac{\gamma_{h,i}-\gamma_{v,i}}{1+\gamma_{h,i}}T_i - \frac{\gamma_{v,i}}{1+\gamma_{h,i}}\sum_{j=i+1}^{n}T_j \tag{33}$$

This equation relates the critical value $S_{co,i}$ characterizing the bracing system to the shear distribution at the occurrence of the transition from condition I to condition II for the i-th plane. The limit condition at which transition from condition II to the condition III occurs at the i-th storey can be found by considering the compatibility condition expressed by the second of Equations (5), where Δx_i is given by Equation (23), and

$$\frac{\Delta b_{t,i}}{\cos\beta} = \frac{S_{to,i}}{k^{d}_{h,i}} \qquad\qquad \Delta c_{c,i} = \frac{N_{c,i}}{k^{d}_{v,i}} \tag{34}$$

in which $N_{c,i}$ is expressed by Equation (19) Making these substitutions one obtains

$$\frac{S_{to,i}}{k^{d}_{h,i}} + \left(T_i + \sum_{j=i+1}^{n}T_j - V_i - S_{co,i}\right)\frac{\tan^2\beta}{k^{c}_{v,i}} = \frac{1}{2k^{c}_{h,i}}\left(T_i - S_{to,i} - S_{co,i}\right) \tag{35}$$

Expressing V_i by means of Equation (13c), in which Δx_i is given by Equation (23), Equation (35) provides the expression

$$\left(2+\gamma_{v,i}+\gamma_{h,i}\right)S_{to,i} + \left(\gamma_{h,i}-\gamma_{v,i}\right)S_{co,i} = \left(\gamma_{h,i}-\gamma_{v,i}\right)T_i - 2\gamma_{v,i}\sum_{j=i+1}^{n}T_j \tag{36}$$

linking the seismic shear distribution to the strength values $S_{co,i}, S_{to,i}$ at the occurrence of the activation of the dissipative device at the i-th storey.

If the seismic force distribution is assigned, Equations (33) and (35) can be used to design the optimal order in which the different work conditions of the bracing systems have to occur, and to calibrate activation load of the dissipative devices at each storey.

On the other hand, if an existing structure is considered, Equations (33) and (36) can be used to determine the seismic force intensity at which a diagonal brace buckles or a dissipative device begins to operate. A more convenient use of the aforementioned equations can be made by normalizing the shear T_i at the i-th level with respect to the base shear force T_1, i.e. setting

$$T_i = \tau_i T_1 \tag{37}$$

where the coefficients τ_i depend on the shear distribution law.

Carrying out a push-over analysis, more accurate results can be obtained by iteratively updating this distribution in relation to the horizontal displacements corresponding to it, whenever a variation in the work condition of a bracing system occurs. This variation is pointed out by the occurrence of the condition expressed by Equation (33) or Equation (36).

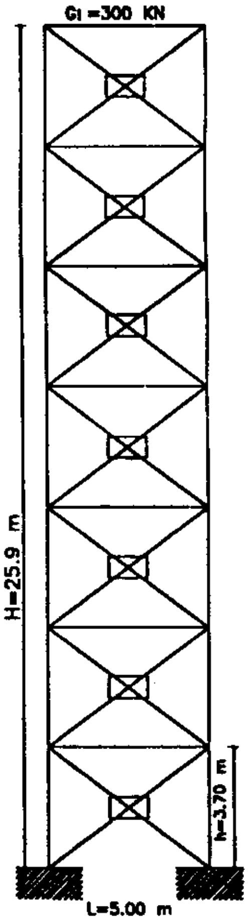

Fig. 2 Structural geometry of system examined

NUMERICAL APPLICATIONS

The numerical applications which will now be described aim at showing the reliability of the simplified model proposed and the efficacy of the push-over analysis in acquiring information about the actual seismic response of the structures considered.

Three different solutions of the same design problem are considered as test examples. The structural scheme common to these three braced moment resistant frames, equipped with dissipative devices, is shown in Figure 2, in which the following common data relating to the cases considered have to be assumed: n = 7 (number of storeys); L = 500 cm (span length); h = 370 cm (inter-storey height); G = 300 kN (seismic weight of each storey).

The three different braced frames, named structures A, B and C have been designed in accordance with design criteria described in detail in Colajanni (1995), so that they have the same fundamental period of vibration $T_b = 0.95\,\text{s}$. Structures A and B are designed in such a way as to obtain a uniform distribution of the damage along the height of the structure under seismic force distributed proportionally to the first mode shape. In structure C, according to the procedure used by Filiatrault and Cherry (1990), the unbraced moment resisting frame is designed without considering earthquake effects, and the bracing dissipative systems are designed assuming constant values of the lateral stiffness of the bracing system along the height of the structure.

Tables 2, 3 and 4 show section dimensions and geometrical characteristics of the structural elements utilized for structures A, B and C, respectively. The aforementioned tables also show the values of the critical load $P_{co,i}$ for the diagonal braces utilized. The limit strength of the dissipative bracing systems $\left(F_{u,i} = S_{to,i} + S_{co,i}\right)$ has been calibrated considering the design base shear force T_1^D derived using the response spectrum for a medium stiff soil condition, assuming a peak value of ground acceleration $A_{g,\max} = 0.35\,g$ (g = gravity acceleration) and a behaviour factor $q = 5$.

Table 2: Characteristics of Sections Utilized for Structure *A*

FLOOR LEVEL	COLUMNS			BEAMS		BRACES	
	Section	I_c (cm^4)	A_c (cm^2)	Section	I_b (cm^4)	A_d (cm^2)	Buckling load P_{co} (kN)
1	HEA 300	18263	112	HEA 400	45069	17.1	80
2	HEA 300	18263	112	HEA 400	45069	17.1	80
3	HEA 300	18263	112	HEA 400	45069	15.1	60
4	HEA 300	18263	112	HEA 360	33090	13.0	50
5	HEA 280	13673	97.3	HEA 340	27693	10.6	40
6	HEA 280	13673	97.3	HEA 300	18263	10.6	40
7	HEA 280	13673	97.3	HEA 220	5410	10.6	40

"The behaviour factor q is an approximation of the ratio of the seismic forces, that the structure would experience if its response was completely elastic to the minimum seismic forces that may be used in design with a conventional linear model, still ensuring a satisfactory response of the structure" [Eurocode 8 (Commission of European Communities (1994)]. It allows one to evaluate the minimum global value of the strength that has to be assigned to the dissipative bracing systems. The limit strengths of the dissipative bracing systems are calibrated at the different storeys in accordance with two different distribution laws. The first solution is obtained by imposing the condition that the dissipative devices are activated simultaneously for a base shear equal to T_1^D. In this case the values of $F_{u,i}$ are evaluated, once T_1^D and $S_{co,i}$ are known, by choosing a suitable shear distribution law, and by solving Equation (36) with respect to $S_{to,i}$. The second solution consists in assigning strength values $F_{u,i}$ constant for all storeys, equal to the maximum value previously calculated.

In a first phase, in order to validate the reliability of the proposed model, the response of the three structures described above to an assigned and fixed distribution of monotonically increasing horizontal forces is evaluated.The accuracy of the results is checked by comparison with results obtained by using the non-linear analysis computer code DRAIN-D2DX [Prakash et al. (1993)].

Table 3: Characteristics of Sections Utilized for Structure *B*

FLOOR LEVEL	COLUMNS			BEAMS		BRACES	
	Section	I_c (cm^4)	A_c (cm^2)	Section	I_b (cm^4)	A_d (cm^2)	Buckling load P_{co} (kN)
1	HEA 260	10455	868	HEA 300	18263	28.2	118
2	HEA 260	10455	868	HEA 300	18263	26.2	96
3	HEA 260	10455	868	HEA 300	18263	22.7	83
4	HEA 260	10455	868	HEA 280	13673	19.1	70
5	HEA 260	10455	868	HEA 260	10450	15.5	57
6	HEA 260	10455	868	HEA 260	10455	10.6	29
7	HEA 260	10455	868	HEA 220	5410	10.6	29

Table 4: Characteristics of Sections Utilized for Structure *C*

FLOOR LEVEL	COLUMNS = BEAMS		
	Section	$I_c = I_b$ (cm^4)	A_c (cm^2)
1	HEA 300	18263	112
2	HEA 300	18263	112
3	HEA 280	13673	97.3
4	HEA 260	10450	86.8
5	HEA 240	7763	76.8
6	HEA 220	5410	64.3
7	HEA 180	2510	45.3
A_d = 15.5 cm^2 P_{coi} = 57 kN (i = 1, 2, ..,7)			

In the analysis performed with the DRAIN-D2DX computer program the columns and beams of the frame are modelled by means of the "plastic hinge beam-column element (type 02)". The dissipative diagonal braces are modelled by means of the "inelastic truss bar element (type 01)", yielding both in tension and compression. The fictitious compression yield stress which is assigned to the brace is derived from the buckling stress value; the fictitious yield stress in tension is the one corresponding to the stress in the tension brace in the load condition at which the dissipative device begins to operate.

In a second set of numerical applications, a complete push-over analysis is carried out for the three structures by using the proposed formulation. These applications imply changing the distribution of the increments of the applied lateral forces, following the evolution of the first mode shape associated with the tangent lateral stiffness matrix. In this way it is shown that the proposed approach makes it possible to foresee the different seismic behaviour corresponding to the three design solutions, and consequently it can be utilized to make design choices.

Finally, the reliability of the information acquired by the push-over analysis is verified by evaluating the "exact" response of the systems considered to a given seismic excitation, performing a non-linear time-history analysis using DRAIN-D2DX.

It is useful to remark that the modelling adopted for the analyses performed with DRAIN-D2DX can accurately reproduce the behaviour of dissipative bracing systems under monotonically increasing static loads, while it tends to overestimate the dissipative capacity of the systems when step by step dynamic analyses are performed. Nevertheless, this simple model is able to furnish results with good accuracy for seismic analysis purpose, especially if strong ground excitations are considered [Filiatrault and Cherry (1986)].

1. Validation of the Proposed Model

In this set of numerical applications the strength of the dissipative bracing systems is assumed to be the same at all storeys, while the distribution of the seismic-equivalent static forces acting on the structure is assumed to increase linearly with the height. The global horizontal force, i.e. the seismic base shear T_1, for each of the three structures examined is assumed to increase monotonically from zero to the value producing the first plasticisation of a structural member of the frame, because beyond this limit condition the proposed approach is no longer valid.

The evaluation of the response by the proposed model is performed using an event-to-event strategy. An event is defined as a change in structural stiffness due to buckling of a brace in compression or to activation of a dissipative device.

The computational procedure is applied as follows:

1. The values of the coefficients α_i appearing in Equation (12a) are calculated in accordance with the distribution the share of shear borne by the frame (quantities $2V_i$ at each storey), computed at the occurrence of the last event before the current one.
2. By normalizing the shear values by means of Equation (37) and using Equation (33) or Equation (36), at each storey the base shear value $T_{1,i}$ producing buckling of the diagonal brace in compression (Equation (33)) or activation of the dissipative device (Equation (36)) – if buckling of the compressed brace has already occurred –, is calculated.
3. The lowest value of $T_{1,i}$ and the equation from which it was derived (Equation (33) or (36)) show the type of event which is occurring and the storey i which is involved; for this value of base shear the relative translations Δx_i are calculated by means of Equations (16), (22) or (23), and the shares of shear borne by the frame are derived from Equation (13c).
4. The shear distribution concerning the frame obtained by the previous step of analysis is compared with that assumed at the beginning of step 1, and if needed, the procedure described for steps 2 and 3 is iteratively repeated, updating the values of α_i up to convergence.
5. The vector $\Delta \boldsymbol{u}$ is derived from Equation (29), which allows one to calculate the absolute horizontal displacements u_i at each storey; consequently, the deformed shape of the structure corresponding to occurrence of the event found is derived.
6. The procedure is repeated starting from step 1 in order to determine the next event.

Figure 3 shows the top displacement values in the three structures in relation to the values of the design base shear multiplier T_1 / T_1^D.

The first plasticisation of a member of the frames occurs at the values $T_1 / T_1^D = 2.4$, 1.75 and 1.8 for structures A, B and C, respectively. For this limit-condition, the maximum value of the top displacement is about 25 cm for all the three cases considered. The centered symbols on the curves in Figure 3 indicate the occurrence of an event (buckling of a compressed brace or activation of a dissipative device).

Comparison with the "exact" results obtained by the DRAIN-D2DX program points out the reliability of the proposed simplified model. In particular, the simplified model provides almost exact values of the horizontal top displacement as long as activation of dissipative devices does not occur $\left(T_1 / T_1^D < 1\right)$, while the accuracy level in the results decreases as the dissipative devices begin to operate at the different storeys. This circumstance can easily be explained by considering that when the frame is subjected to increasing values of seismic shear because of loss of lateral stiffness of the bracing systems, the simplified modelling concerning its elastic response (see Section 2) and the one allowing calculation of the coefficients α_i exert a greater influence on the accuracy.

In Figure 3 the curves obtained considering that the columns of the frames are axially inextensible are also shown. These curves derived by "exact" analysis show that the aforementioned assumption is absolutely inadequate for the structural type considered here. This conclusion is confirmed by the values of fundamental period of the three structures, calculated under the assumption of shear type behaviour, they prove to be equal to 0.61 s, for structures A and B, and 0.53 s for structure C. These values are much lower than the actual value $T_b = 0.95\,s$. However, the special sensitivity of one bay frames to axial deformation of the frame columns has to be remarked. If several bay frames are considered, with bracing systems distributed in different bays, the effects of the axial deformation of the columns prove to be reduced.

The accuracy of the results obtained using the simplified model is pointed out in greater detail by the further information presented in Table 5, referring to structure *A*. The table shows the values of the seismic design base shear multipliers T_1 / T_1^D characterizing the occurrence of an event, the corresponding type of event and the storey at which it occurs, and the corresponding horizontal top displacement. These results are compared with the ones obtained by "exact" analysis.

Table 5: Results of Nonlinear Static Analysis for Structure *A*

PROPOSED MODEL				DRAIN-D2 DX			
T_1/T_1^D	Storey	Event	Top Displacement (cm)	T_1/T_1^D	Storey	Event	Top Displacement (cm)
0.444	3	B	1.99	0.429	3	B	1.99
0.489	5	B	2.21	0.494	5	B	2.23
0.606	4	B	2.79	0.606	4	B	2.78
0.632	2	B	2.92	0.632	2	B	2.92
0.657	6	B	3.06	0.647	1	B	3.00
0.691	1	B	3.25	0.667	6	B	3.11
0.969	2	A	4.86	0.980	2	A	4.92
1.07	1	A	5.60	1.01	1	A	5.14
1.10	3	A	5.94	1.10	3	A	6.02
1.178	7	B	6.78	1.158	7	B	6.57
1.240	4	A	7.50	1.251	4	A	7.59
1.504	5	A	11.06	1.153	5	A	11.22
1.977	6	A	18.48	2.028	6	A	18.43
2.4	4	C	25.95	2.4	4	C	24.33

B = Buckling of a brace in compression

A = Activation of a dissipative device

C = Yielding of a beam of the frame

Figure 4 and Figure 5 show respectively the deformed shapes of the three different structure under the design seismic force $(T_1 / T_1^D = 1)$ and under the seismic forces corresponding to the values of design base shear multiplier producing the first plasticisation on a frame member (limit condition). The curves in these figures confirm the reliability of the simplified approach and the observations concerning the results shown in Figure 3.

Finally, results of other numerical applications have shown that, when a load condition corresponding to values $(T_1 / T_1^D < 1)$ is considered, i.e. when the dissipative devices are not activated, a good level of approximation can be maintained by the proposed model even if the iterative phase described at step 4 of the procedure to determine the values of the coefficients α_i is not made. This also occurs when the lateral stiffness of the frame at the different storeys is comparable with that of the bracing system. A more general conclusion is that the iterative procedure at step 4 can be avoided when the distribution law of the portion of seismic forces borne by the frame does not exhibit significant variations during the loading process considered.

2. Push-Over Analysis

The push-over analysis is applied to the three structures considered in the previous section in order to stress the main characteristics of the expected seismic behaviour. In particular, this kind of approach proves to be useful in verifying the influence of the stiffness and strength distribution laws for the frame and the bracing systems along the height of the structures on the distribution of the response parameters determining the "seismic demand" of the structures themselves.

As was stressed above, the accuracy of the information acquired by means of the push-over analysis improves significantly when the horizontal static equivalent seismic forces are distributed proportionally to the fundamental modal shape of the system. The proposed model, at each increase in the global shear T_1 during a work phase of the bracing systems comprised between two consecutive events, allows one to distribute the corresponding increases in the static equivalent seismic forces proportionally to the increases in the horizontal displacements occurring during the analysis step considered. The iterative application of this procedure leads to a distribution of the increments of the force proportional to the first modal shape corresponding to the tangent lateral stiffness matrix.

Therefore, the procedure described in the previous section to update the coefficients α_i iteratively is applied here to calibrate the force increases too as follows: at step 1 the increases in the external horizontal forces are assumed to be proportional to the increases in the horizontal displacements detected at the occurrence of the previous event; at steps 2 and 3 the increase in the base shear ΔT_1 producing the next event is calculated; then, before the iterative procedure at step 4 is applied, step 5 is carried out in order to determine the displacement vector $\Delta \boldsymbol{u}$ and, consequently, the accuracy of the distribution of the force increases which was assumed. The procedure described for steps 2 to 5 is repeated up to convergence of the values of the coefficients α_i and of the normalized increment $\Delta\tau_i = \Delta T_i / \Delta T_1$.

It must be observed that by using this procedure and considering what was pointed out in section 6, the load activating the dissipative devices can also be calibrated, taking into account the variation in the distribution law of the external forces consequent to buckling of the braces in compression and variation in the lateral stiffness of the frame due to the variation in the share of the seismic force carried by the frame. The effectiveness of this calibration method is stressed here carrying out the push-over analysis for the three structures *A*, *B* and *C*, and assuming dissipative bracing systems with the same strength at all storeys, or with strength distribution calibrated to simultaneous activation. The symbols referring to the latter class of systems are stressed by the notation $(\ )_1$ in what follows.

The push-over analysis is applied assuming that all the structures considered reach a maximum top horizontal displacement of 10 cm. This value is calculated by means of the expression

$$u_{n,\max} = \rho\beta(T_b)\left(\frac{T_b}{2\pi}\right)A_{g,\max} \tag{38}$$

where T_b and $A_{g,\max}$ are defined in section 6, ρ is a reduction factor due to the presence of dissipative devices, and $\beta(T_b)$ is the value of the pseudo-acceleration of the normalized elastic response spectrum for a 0.05 viscous damping ratio. The coefficient ρ takes into account the fact that, for this kind of system, the inelastic response is reduced with respect to that of an elastic system having the same period and viscous damping ratio; it can be assumed to be equal to 0.8 for the values of T_b and q considered, as shown in Colajanni and Papia (1996). According the Eurocode 8 (Commission of European communities (1994)), assuming a medium stiff soil condition (spectrum type B), the values of $\beta(T_b)$ are

$$
\begin{aligned}
&\beta(T_b)=1+1.5(T_b/0.15) && 0\le T_b\le 0.15\\
&\beta(T_b)=2.5 && 0.15\le T_b\le 0.6\\
&\beta(T_b)=2.5(0.6/T_b) && 0.6\le T_b\le 3\\
&\beta(T_b)=2.5\left(0.6/\sqrt{3T_b}\right) && 3\le T_b
\end{aligned}
\tag{39}
$$

In examining the results which will now be shown, it must be observed that the limit condition of plasticisation occurring for a member of the frame and the expected damage in the non-structural elements of the building mainly depend on the share of relative translation Δx_i, (see section 3) rather than on Δu_i. This is because the share of displacement produced by the rotations $\Delta\varphi_i$ has very little influence on these phenomena. Therefore, the displacements Δx_i are assumed to be the kinematic parameters characterizing the structure seismic demand.

Table 6 shows the maximum value of Δx_i among the ones detected at all storeys, and the maximum value of the base shear, normalized with respect to the design base shear, for structures *A*, *B* and *C*. Each of them is assumed to be braced with dissipative devices calibrated by the two different criteria discussed above.

Table 6: Comparison between Push-Over and Dynamic Analysis Results

	STATIC ANALYSIS			DYNAMIC ANALYSIS		
Struct.	Top Displacement (cm)	$\Delta x_{1,max}$ (cm)	$\frac{T_{1,max}}{T_1^D}$	Top Displacement (cm)	$\Delta x_{1,max}$ (cm)	$\frac{T_{1,max}}{T_1^D}$
A	10	1.50	1.40	9.28	1.44	1.24
A_1	10	1.17	1.23	9.11	1.33	1.15
B	10	1.58	1.20	9.83	1.29	1.10
B_1	10	1.05	1.10	10.2	1.15	1.07
C	10	1.72	1.26	9.75	1.67	1.23
C_1	10	1.282	1.11	10.1	1.41	1.10

The results show that the maximum value of Δx_i, for a given law of strength distribution of the devices, is almost the same for the three structures, attaining the maximum value in structure *C* and the minimum in structure B_1. The maximum base shear occurs in structure *A* and the minimum in structure B_1 again. The values of both response parameters significantly decrease when simultaneous activation of the dissipative devices occurs. This circumstance is more evident if structures *B* and B_1 are compared.

Figure 6 shows the distributions of Δx_i along the height of the structures, while Figure 7 shows the share of total energy dissipated by the devices at each storey. These figures point out that, designing dissipative devices having the same strength at each storey, the share of relative translation Δx_i is very large for the first three storeys, and decreases significantly at the higher storeys (this is more evident for structure *B*); moreover, an analogous pattern can be observed for the dissipated energy.

By contrast, the simultaneous activation of the dissipative devices produces a reduction in the maximum value of Δx_i and a more uniform distribution of this share of relative displacement along the height, with the maximum beneficial effect for structure B_1 compared with structure *B*. Moreover, by using this more effective criterion of strength distribution, almost all the devices at the different storeys significantly contribute to the global dissipative mechanism, with reduction in the energy dissipation demand for the devices at the lower storeys.

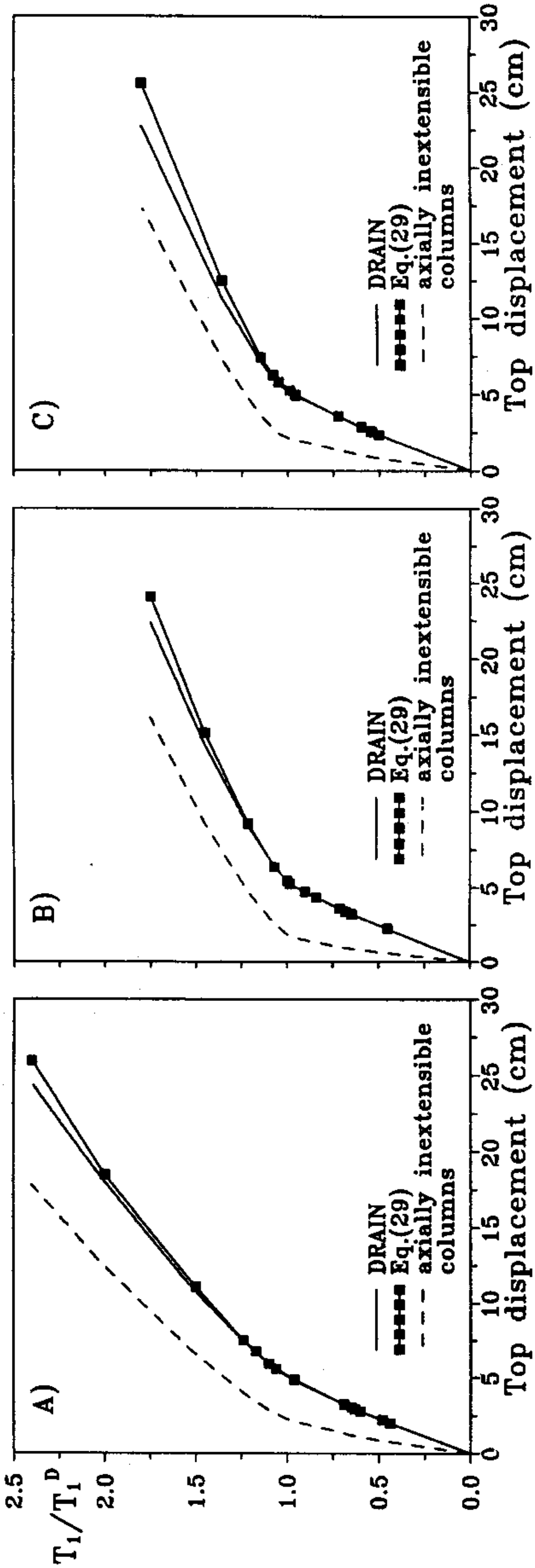

Fig. 3 Design base shear multiplier versus top displacement in structures *A*, *B* and *C*

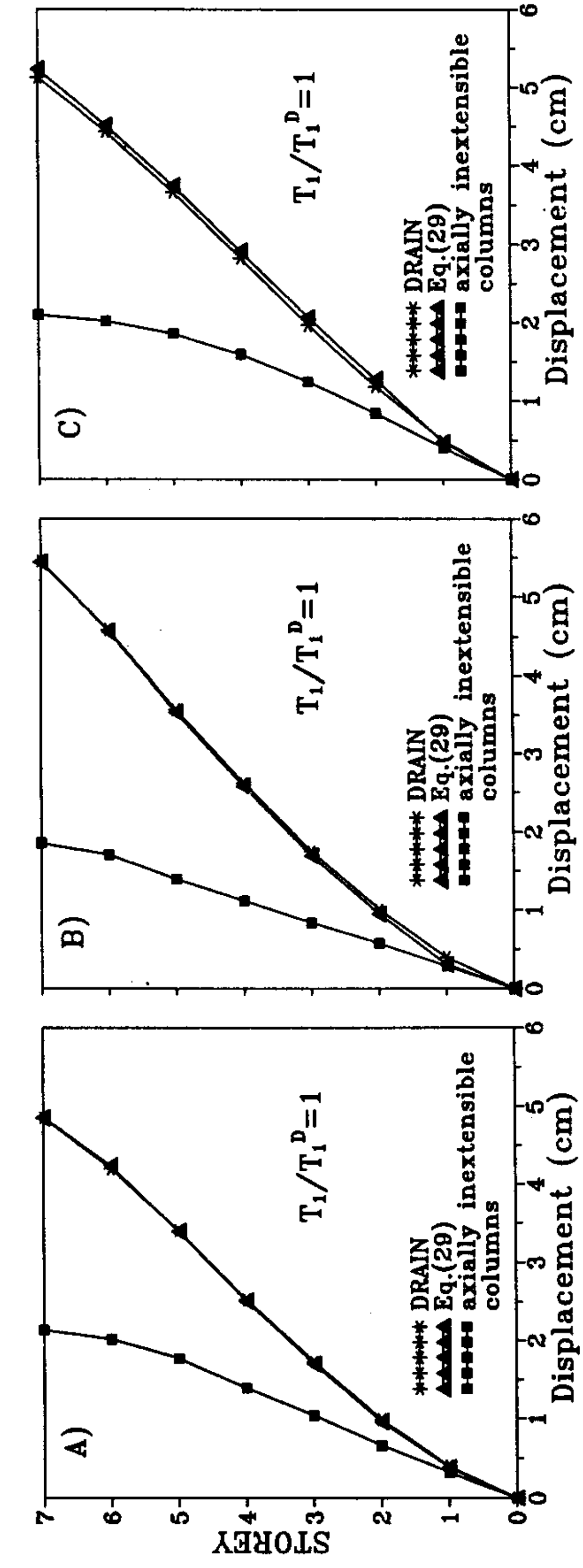

Fig. 4 Deformed shapes of structures *A*, *B* and *C* under seismic design static forces

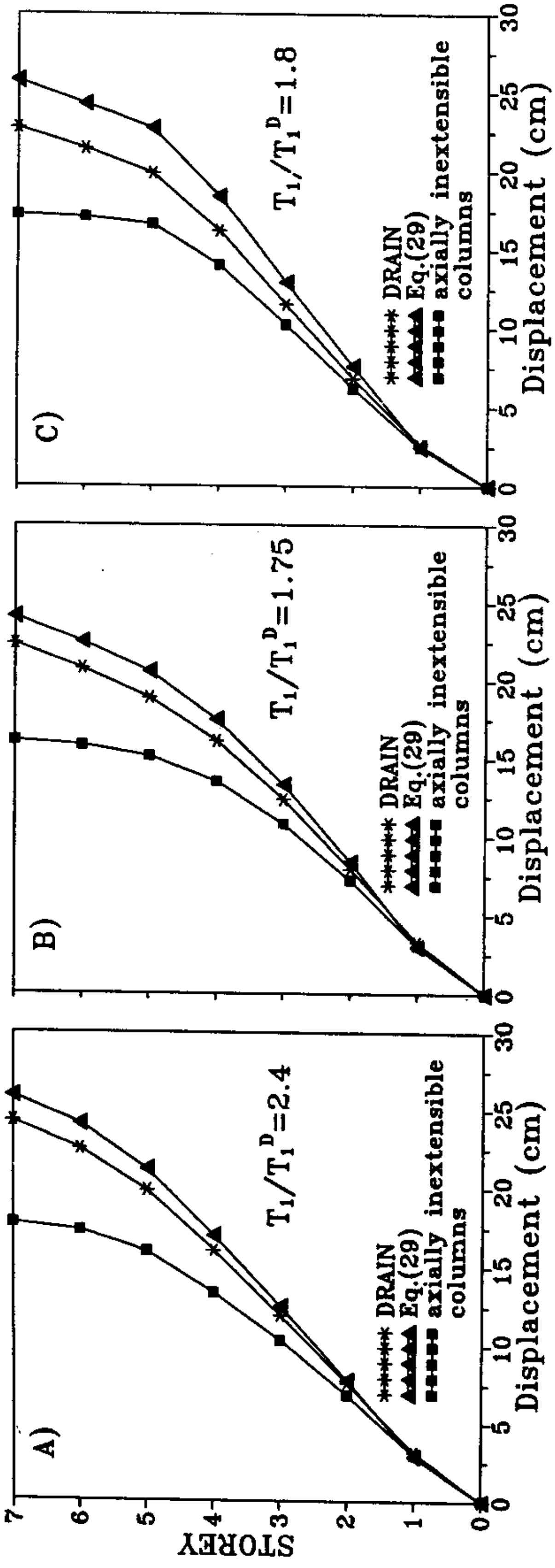

Fig. 5 Deformed shapes of structures *A*, *B* and *C* at first yielding of frame

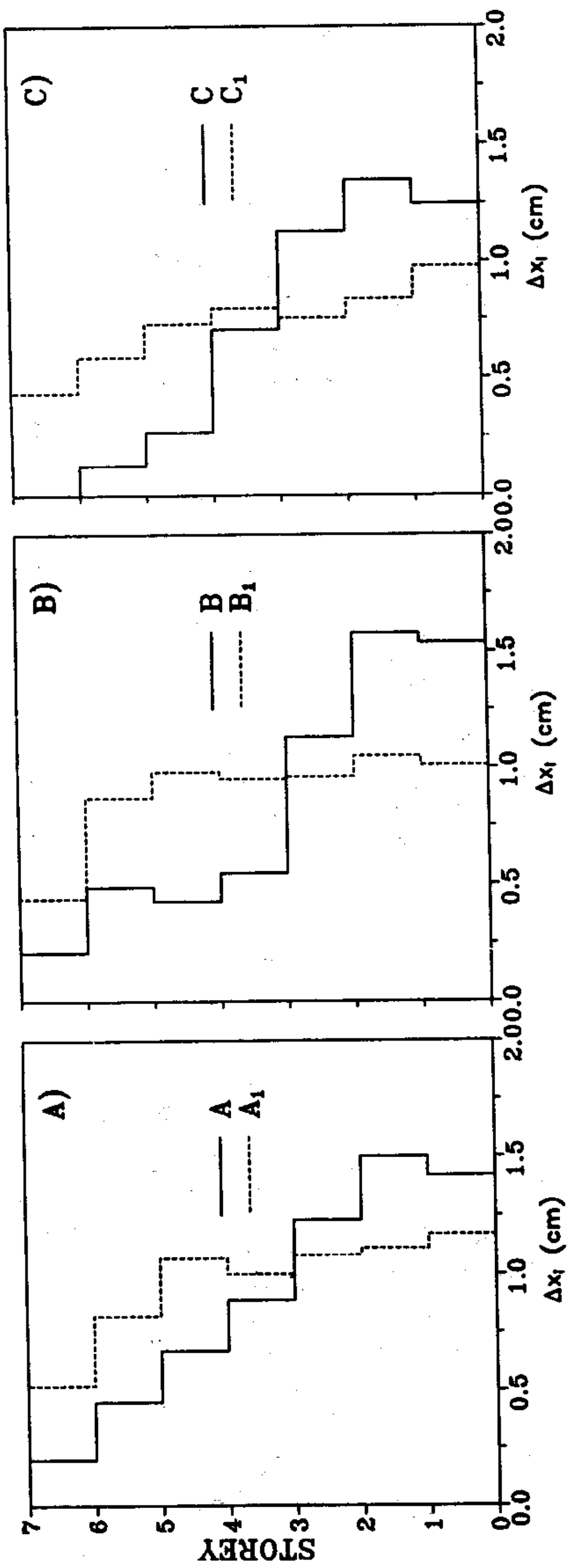

Fig. 6 Distribution of relative translations Δx_i along the height of structures *A*, *B* and *C* derived by static analysis

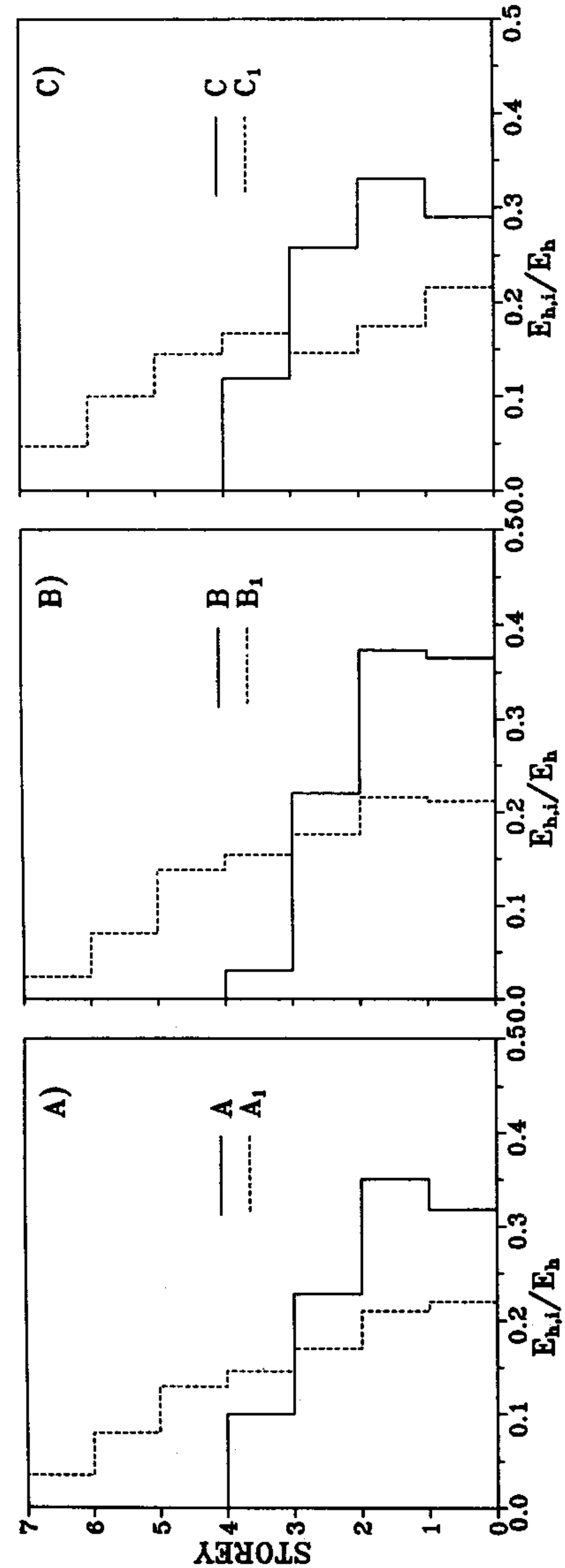

Fig. 7 Distribution of normalized dissipated energy along the height of structures *A*, *B* and *C* derived by static analysis

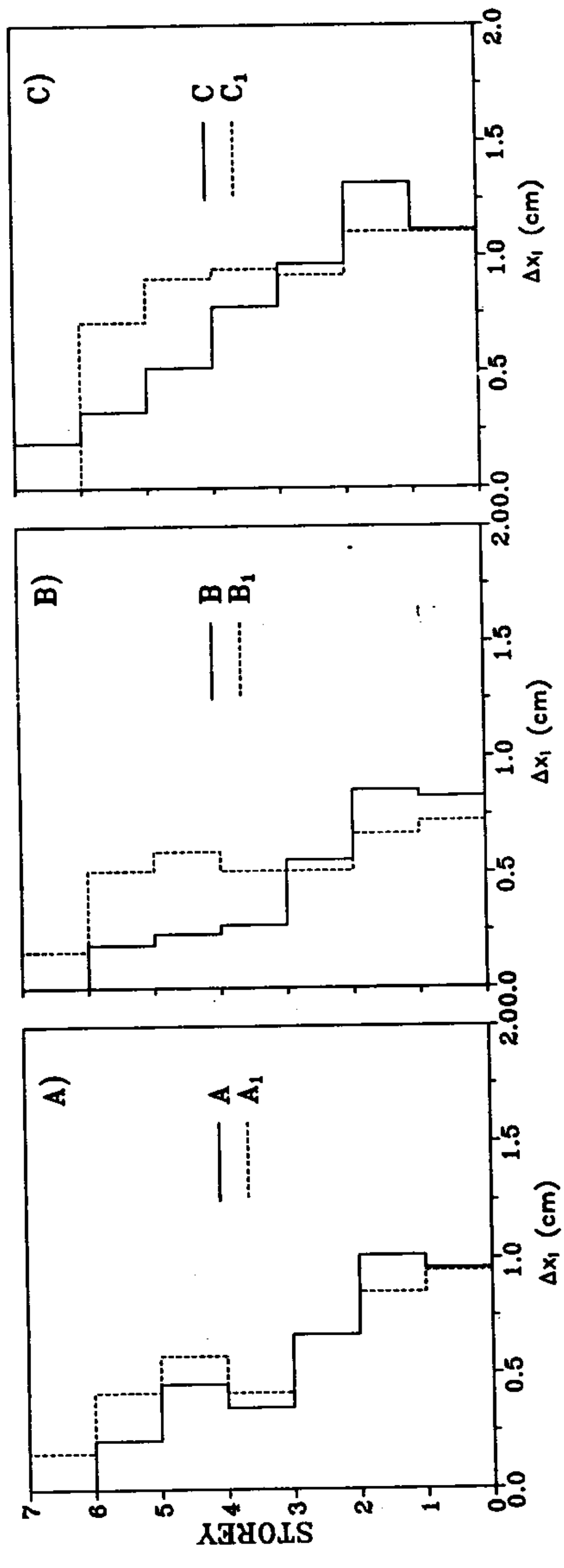

Fig. 8 Distribution of relative translations Δx_i along the height of structures *A*, *B* and *C* derived by dynamic analysis

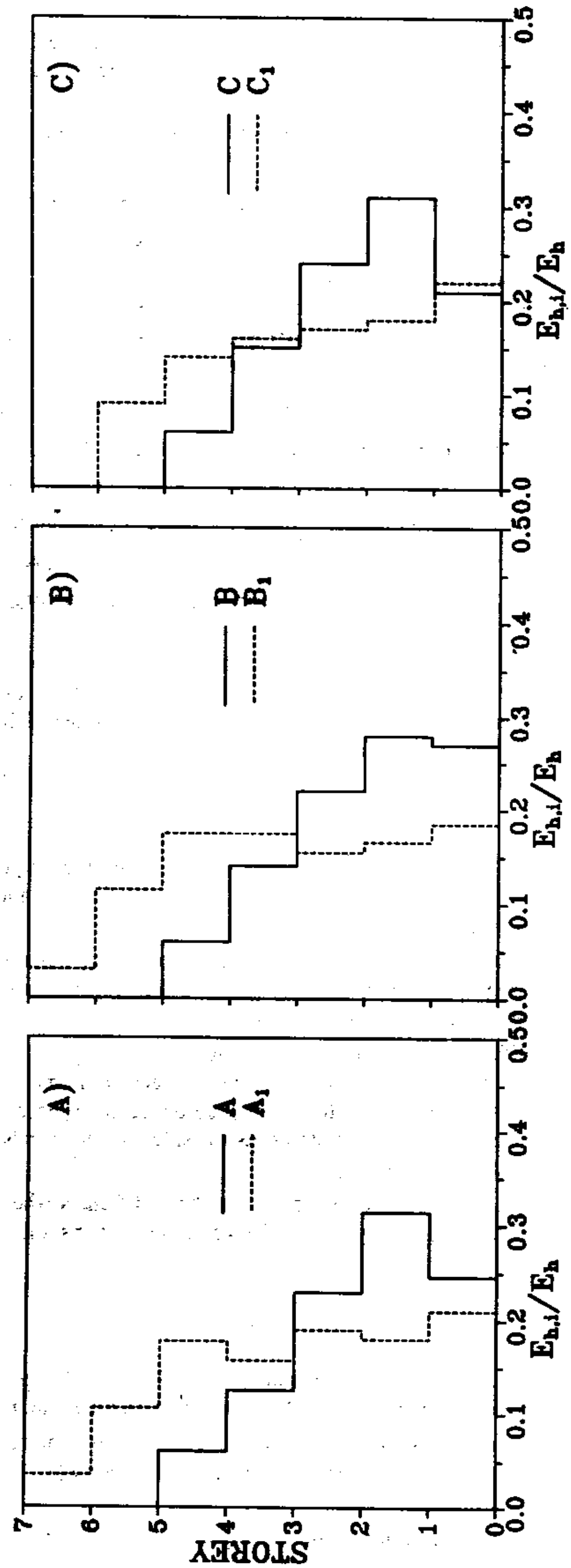

Fig. 9 Distribution of normalized dissipated energy along the height of structures A, B and C derived by dynamic analysis

3. Nonlinear Time-History Analysis

The conclusions presented in the previous section are verified in the dynamic field by carrying out a step-by-step nonlinear analysis of the seismic response of the systems considered, by means of a modified version of the DRAIN-D2DX program.

The structures are assumed to be subjected to the first 25 s of the S00E component of the El Centro (1940) earthquake.

Since the peak ground acceleration for this component is $A_{g,max} = 0.35g$ and the ordinate of the normalized elastic response spectrum of this accelerogram for $T = T_b = 0.95$ s is very close to that of the elastic design spectrum previously considered, the results of the dynamic analysis can be compared with those of the static analysis, at least qualitatively. The values of the response parameters resulting from this dynamic approach are shown in the same Table 6, commented on in the previous section. For this kind of analysis the values of the maximum horizontal top displacement are also shown.

The maximum and minimum values of Δx_i are attained in structures C and B_1, respectively. The maximum base shear occurs in structure A, and the minimum in structure B_1. The results confirm that simultaneous activation of the devices reduces the values of both these response parameters.

Figures 8 and 9 respectively show the distributions of the relative displacement share Δx_i and of the share of energy dissipated at the i-th storey $E_{h,i} / E_h$, derived using the dynamic analysis. Comparison with the corresponding Figures 6 and 7 points out that the push-over analysis allows one qualitatively to predict the distribution along the height of the response parameters.

Finally, the results of the dynamic analysis show that the assumption of the same strength of the dissipative bracing systems at all storeys is not an advantageous design solution, even though the favourable effects provided by simultaneous activation of the devices, as confirmed by the dynamic analysis, appear to be overestimated by the push-over analysis.

CONCLUSIONS

A simplified model, allowing one to predict with good precision the evolution of the deformed shape of cross-braced multistorey frames equipped with hysteretic dissipative devices and subjected to monotonically increasing horizontal forces, has been proposed.

The model takes the axial deformability of the frame into account, considering that it may significantly influence the base shear-top displacement relationship, the fundamental period of vibration and the corresponding first mode shape, as shown by some numerical examples.

The model can be used to identify the principal characteristics of the seismic behaviour of the structure to be examined by means of a push-over analysis. This is because it makes it possible to distribute the increases in the equivalent seismic static forces in accordance with the first modal shape associated with the tangent lateral stiffness matrix of the structure, considering the effects of buckling of braces in compression and activation of dissipative devices.

At this stage of the study, the model only refers to one-bay cross-braced frames where the hysteretic behaviour is concentrated in the dissipative bracing systems. Further investigations will be addressed to removing these limitations.

REFERENCES

1. Bertero, V.V., Anderson, H., Krawinkler, H. and Miranda, E. (1991). "Design Guidelines for Ductility and Drift Limits", Report No. UCB/EERC-91/15, University of California, Berkeley.
2. Colajanni, P. (1995). "Comportamento Sismico di Telai in Acciai con Controventi Dotati di Dispositivi Dissipativi", Ph.D. Thesis, Dipartimento di Ingegneria Strutturale e Geotecnica, Universita di Palermo, Italy, (in Italian).
3. Colajanni, P. and Papia, M (1995). "Seismic Response of Braced Frames with and without Friction Dampers", Engineering Structures, Vol. 17, No. 2, pp. 129-139.

4. Colajanni, P. and Papia, M.(1996). "Energy Design Approach for Braced Frames with Dissipative Devices", Proc. 11-th World Conf. Earth. Engrg., Acapulco, 1996.
5. Colajanni, P. and Papia, M (1997). "Hysteretic Behaviour Characterization of Friction Damped Braced Frames", J. Struct. Engrg., ASCE, Vol. 123, No. 8, pp. 1020-1028.
6. Commission of European Communities (1994). "Eurocode 8, Earthquake Resistant Design of Structures", Brussels, Belgium.
7. Fajfar, P. and Gaspersic, P. (1996). "The N2 Methods for the Seismic Damage Analysis of RC Buildings", Earth. and Struct. Dyn., Vol. 25, pp. 31-46,
8. Filiatrault, A. and Cherry, S. (1986). "Tests of the Behaviour of Friction Dampers in Braced Steel Frames", Proc. 8-th European Conf. Earth Engrg., Lisbon, Vol 5, pp. 8.5/41-8.5/48.
9. Filiatrault, A. and Cherry, S. (1989). "Efficient Numerical Modelling for Seismic Design of Friction Damped Braced Steel Plane Frames", Canadian J. of Civ. Engrg., Vol. 16, No. 3, pp. 211-218.
10. Filiatrault, A. and Cherry, S. (1990). "Seismic Design Spectra for Friction Damped Structures", J. Struct. Div., ASCE, Vol. 116, No. ST5, pp. 1338-1355.
11. Haviland, R.W., Biggs, J.M. and Anagnostopoulos, S.A. (1976). "Inelastic Response Spectrum Design Procedures for Steel Frames", Report R76-40, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.
12. Housner, G.W. and Jennings, P.C. (1982). "Earthquake Design Criteria", Earthquake Engrg. Res. Inst., Berkeley, California.
13. Luyties, W.H. Anagnostopoulos, S.A. and Biggs, J.M. (1976). "Studies of the Inelastic Dynamic Analysis and Design of Multistorey Frames", Report R 76-29, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.
14. Krawinkler, H. (1994). "New Trends in Seismic Design Methodology", Proc. 10th European Conf. Earth. Engrg., Vienna, Vol. 2, pp. 821-830.
15. Prakash, V., Powell, G.H. and Campbell, S. (1993). "DRAIN-D2DX, Base Program Description and User Guide", Report No. UCB / SEMM- 93/17, Dept. of Civil Engineering, University of California, Berkeley.
16. Priestley, M.J.N. (1993). "Myths and Fallacies in Earthquake Engineering - Conflicts between Design and Reality", Bull. New Zealand Nat. Soc. Earthquake Engrg., No. 26, pp. 329-341.
17. Ramasco, R. (1985). "Calcolo Degli Edifici: Analisi Statica e Dinamica", (in "Ingegneria Antisismica", a Cura di G. Sara, Liguori Editore), Napoli (in Italian).
18. Saiidi, M. and Sozen, M.A. (1981). "Simple Nonlinear Seismic Analysis of R/C Structures", J. Struct. Div., ASCE, Vol. 107, No. ST5, pp. 937-952.

APPENDIX

An approximate evaluation of the lateral stiffness of the columns at the i-th storey of the cross-braced frame can be obtained by modelling the whole unbraced frame by a single column, affected at each storey by the sum of the flexural moments of inertia of the storey columns of the frame. At each storey level, the equivalent column is provided with a rotational spring representing the flexural stiffness of the storey beams. The stiffness of the spring is evaluated by assuming that at each storey the rotation of all the nodes of the frame is the same. Moreover, in this simplified scheme the inflection point of each column is assumed at the mid-height, except for the first floor, where the position of the inflection point is assumed to be the same as for the single column scheme affected only by the first storey, for which it is determined exactly.

When the global lateral stiffness of the frame is derived at each storey, the lateral stiffness of each column can be expressed as

$$k_{h,i}^{c} = \frac{12EI_{c,i}}{h^3}\alpha_i \tag{A.1}$$

where α_i is a coefficient reducing the shear stiffness of the column, taking the elastic rotation of the top and base of the column itself into account.

The coefficients α_i are functions of the flexural moments of inertia of the columns and beams, of the aspect ratio of the frame L/h, and of the distribution law of the seismic forces. They can easily be evaluated by analyzing the simplified structure and remembering that the two columns of the one-bay frames considered in this work at each storey are made with the same steel profile and are identically oriented.

Using the notations previously introduced in the text, the coefficients α_i in this case can be expressed as follows:

$$\frac{1}{\alpha_1}=1+\frac{L}{2h}\frac{I_{c,1}}{I_{b,1}}\left(\frac{2}{2+\frac{L}{3h}\frac{I_{c,1}}{I_{b,1}}}+\frac{T_2}{T_1}\right)$$

$$\frac{1}{\alpha_i}=1+\frac{L}{2h}\left[\frac{I_{c,i}}{I_{b,i}}\left(1+\frac{T_{i+1}}{T_i}\right)+\frac{I_{c,i}}{I_{b,i-1}}\left(1+\frac{T_{i-1}}{T_i}\right)\right] \qquad 2\leq i\leq n-1 \qquad \text{(A.2)}$$

$$\frac{1}{\alpha_n}=1+\frac{L}{2h}\left[\frac{I_{c,n}}{I_{b,n}}+\frac{I_{c,n}}{I_{b,n-1}}\left(1+\frac{T_{n-1}}{T_n}\right)\right]$$

It is useful to point out that the above mentioned assumption concerning the location of the inflection point for the first storey columns have to be considered only for evaluating the coefficients α_i and not for the simplified scheme shown in Figure (1a).