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STATIC DEFORMATION OF A UNIFORM HALF-SPACE DUE TO A VERY LONG TENSILE FAULT

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ABSTRACT

The Airy stress function for a long tensile fault of arbitrary dip and finite width buried in a homogeneous, isotropic, perfectly elastic half-space is obtained. This Airy stress function is used to derive closed-form analytical expressions for the displacements and stresses at an arbitrary point of the half-space caused by a long vertical tensile fault of finite width. The variation of the displacement and stress fields with distance from the fault and with depth is studied numerically. Contour maps showing the displacement and stress fields around a long vertical tensile fault in a half-space are also presented.

KEYWORDS: Half-Space, Long Tensile Fault, Static Deformation

INTRODUCTION

Since dislocation theory was first introduced in the field of seismology by Steketee (1958), numerous theoretical formulations describing the deformation of an isotropic, homogeneous, semi-infinite medium have been developed (Okada, 1992). In contrast to the progress that has been made in the modelling of the deformation fields due to a shear fault, the studies related to a tensile fault are scarce. The main reason for this is the importance that has been attached to model the static field changes associated with earthquake occurrence. Tensile fault representation also has several very important geophysical applications, such as modelling of the deformation fields due to a dyke injection in the volcanic region, mine collapse and fluid-driven cracks. Recent studies have shown that a large number of earthquake sources cannot be represented by the double-couple source mechanism which models a shear fault. According to Sipkin (1986), the non-double-couple mechanism might be due to tensile failure under high fluid pressure.

Maruyama (1964) obtained surface displacements due to vertical and horizontal rectangular tensile faults in a semi-infinite Poisson solid. Davis (1983) modelled the crustal deformation associated with hydrofracture by a dipping rectangular tensile fault beneath the surface of an elastic half-space. Yang and Davis (1986) obtained closed analytical expressions for the displacements, strains and stresses due to a rectangular inclined tensile fault in an elastic half-space.

Singh and Garg (1986) obtained integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional sources. Beginning with these results, Rani et al. (1991) obtained closed-form analytical expressions for the Airy stress function, and displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space due to an arbitrary line source. By integration over the width of the fault, Rani and Singh (1992) obtained the expressions for the Airy stress function, and displacements and stresses in a uniform half-space due to a long dip-slip fault of finite width.

The aim of the present paper is to study the two-dimensional deformation of a uniform half-space caused by a long tensile fault of finite width. The corresponding problem of a long dip-slip fault has been discussed by Freund and Barnett (1976) and Rani and Singh (1992). Although two-dimensional approximation is an oversimplification of the physical system, it is very useful in gaining insight into the relationship among various fault parameters and in improving understanding of the deformation (see, e.g., Savage, 1987). Moreover, there are faults which are sufficiently long and shallow that the two-dimensional approximation may be used. The two-dimensional solution obtained here is useful because of its considerable simplicity as compared to the three-dimensional solution given by Yang and Davis (1986). We begin with the closed-form expression for the Airy stress function for an arbitrary line source in a uniform half-space given by Rani et al. (1991). Analytic integration over the width of the fault yields the Airy stress function for a long tensile fault of arbitrary dip and finite width. The expressions for the displacements and stresses at any point of the half-space caused by a long vertical tensile fault follow immediately.

THEORY

Let the Cartesian coordinates be denoted by (x_1, x_2, x_3) with the x_3-axis vertically downwards. Consider a two-dimensional approximation in which the displacement components u_1, u_2 and u_3 are independent of x_1 so that $\partial/\partial x_1 \equiv 0$. Under this assumption, the plane strain problem $(u_1 = 0)$ can be solved in terms of the Airy stress function U such that

$$\tau_{22} = \frac{\partial^2 U}{\partial x_3^2},\ \tau_{33} = \frac{\partial^2 U}{\partial x_2^2},\ \tau_{23} = -\frac{\partial^2 U}{\partial x_2 \partial x_3} \tag{1}$$

where τ_{ij} are the components of stress. Following Rani et al. (1991), the Airy stress function for an arbitrary line source parallel to the x_1-axis acting at the point $(0, 0, h)$ in a uniform half-space $x_3 \geq 0$ is given by

$$\begin{aligned} U = {} & L_0 \tan^{-1}\left(\frac{x_2}{|x_3 - h|}\right) + M_0 \frac{x_2 |x_3 - h|}{R^2} - P_0 \log R + Q_0 \frac{(x_3 - h)^2}{R^2} \\ & - L^- \left[\tan^{-1}\left(\frac{x_2}{x_3 + h}\right) + \frac{2 x_2 x_3}{S^2}\right] + \frac{M^- x_2}{S^2}\left[(x_3 - h) - \frac{4 h x_3 (x_3 + h)}{S^2}\right] \\ & + P^- \left[\log S - \frac{2 x_3 (x_3 + h)}{S^2}\right] + \frac{Q^-}{S^2}\left[(x_3^2 - h^2 + 2 h x_3) - \frac{4 h x_3 (x_3 + h)^2}{S^2}\right] \end{aligned} \tag{2}$$

where $R^2 = x_2^2 + (x_3 - h)^2,\ \ S^2 = x_2^2 + (x_3 + h)^2,\ \ x_3 \neq h.$

In Equation (2), L_0, M_0, P_0, Q_0 are the source coefficients and L^-, M^-, P^-, Q^- are the values of these coefficients valid for $x_3 < h$. Singh and Garg (1986) and Singh and Rani (1991) have given these source coefficients for various seismic sources. For ready reference, the relevant source coefficients are given in the Appendix.

Let U_i denote the Airy stress function at an arbitrary point $P(x_2, x_3)$ for a unit concentrated force acting at the point $Q(y_2, y_3)$ in the x_i-direction. Then, the Airy stress function for a long fault can be expressed as a line integral (Maruyama).

$$U = \int_L \Delta u_i\, U_{ij}\, n_j\, ds \tag{3}$$

where the summation convention has been used (the suffixes can assume the values 2 and 3 only). In Equation (3), Δu_i is the displacement dislocation vector; n_j is the unit normal to the fault section L; λ, μ are the Lamē constants; and

$$U_{ij} = U_{ji} = \lambda\, \delta_{ij} \frac{\partial}{\partial y_k} U_k + \mu\left(\frac{\partial}{\partial y_i} U_j + \frac{\partial}{\partial y_j} U_i\right)$$

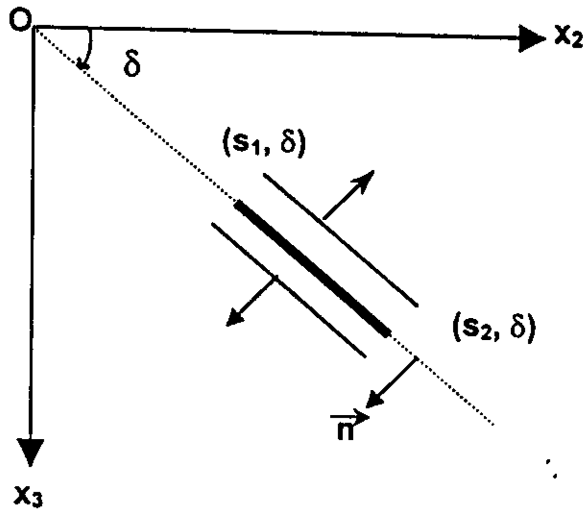

Fig. 1 Geometry of a long tensile fault of width $L = s_2 - s_1$ (the Cartesian coordinates of a point on the fault are (y_2, y_3) and its polar coordinates (s, δ), where δ is the dip angle and $s_1 \le s \le s_2$)

For a tensile fault, vector Δu_i is parallel to the normal n_i to the fault. Therefore, if b is the magnitude of Δu_i and δ is the dip angle (Figure 1), we have

$$\Delta u_2 = -b \sin\delta, \qquad \Delta u_3 = b\cos\delta, \tag{4}$$

$$n_2 = -\sin\delta, \qquad n_3 = \cos\delta \tag{5}$$

Using Equations (4) and (5) in Equation (3), we get the following expression for the Airy stress function for a line source

$$U = bds\left[U_{22}\sin^2\delta - U_{23}\sin 2\delta + U_{33}\cos^2\delta\right] \tag{6}$$

where ds is the width of the line fault. Therefore, the Airy stress function for a long tensile fault of arbitrary dip can be expressed as a linear combination of

(i) $bdsU_{22}$, the Airy stress function for a vertical tensile fault $(\delta = 90^\circ)$ with dislocation in the x_2-direction;

(ii) $bdsU_{33}$, the Airy stress function for a horizontal tensile fault $(\delta = 0^\circ)$ with dislocation in the x_3-direction; and

(iii) $bdsU_{23}$, the Airy stress function for a vertical dip-slip fault.

Using the values of the source coefficients L_0, M_0, P_0, Q_0 given in the Appendix, Equations (2) and (6) yield the Airy stress function due to a long tensile fault of arbitrary dip located at the point (y_2, y_3) in the form

$$U=\frac{\mu b ds}{2\pi(1-\sigma)}\left[\log(S/R)-\frac{2x_3(x_3+y_3)}{S^2}+\cos 2\delta\left\{\frac{(x_3-y_3)^2}{R^2}+\frac{x_3^2-y_3^2+2x_3y_3}{S^2}-\frac{4x_3y_3(x_3+y_3)^2}{S^4}\right\}\right.$$

$$\left.+\sin 2\delta\left\{(x_2-y_2)(x_3-y_3)\left(1/R^2-1/S^2\right)+\frac{4x_3y_3(x_2-y_2)(x_3+y_3)}{S^4}\right\}\right] \tag{7}$$

where σ =Poisson's ratio, μ = rigidity,

$$R^2=(x_2-y_2)^2+(x_3-y_3)^2,\quad S^2=(x_2-y_2)^2+(x_3+y_3)^2$$

From Figure 1, we put $y_2=s\cos\delta,\quad y_3=s\sin\delta$ into Equation (7) and integrate over s between the limits $(s_1,\, s_2)$. We thus obtain the following expression for the Airy stress function for a long tensile fault of finite width $L=s_2-s_1$;

$$U=\frac{\mu b}{2\pi(1-\sigma)}\left[(x_2\cos\delta+x_3\sin\delta-s)\log(R/S)+2x_3\sin\delta\left(x_2^2+x_3^2\right)\left(1/S^2\right)\right.$$

$$\left.+2x_3\sin\delta(x_3\sin\delta-x_2\cos\delta)\left(s/S^2\right)\right]\Big|_{s_1}^{s_2} \tag{8}$$

where now

$$R^2=(x_2-s\cos\delta)^2+(x_3-s\sin\delta)^2,$$
$$S^2=(x_2-s\cos\delta)^2+(x_3+s\sin\delta)^2,$$
$$f(s)\Big|_{s_1}^{s_2}=f(s_2)-f(s_1)$$

From Equations (1) and (8), we get the following expressions for stress components due to a vertical tensile fault $\left(\delta=90^0\right)$:

$$\tau_{22}=\frac{\mu b}{2\pi(1-\sigma)}\left[(x_3-s)\left(1/R^2\right)-(3x_3+s)\left(1/S^2\right)+2x_2^2(x_3-s)\left(1/R^4\right)\right.$$
$$\left.+2\left\{\left(x_3^2+s^2\right)(x_3+s)-2sx_2^2\right\}\left(1/S^4\right)+16x_2^2x_3(x_3+s)\left(s/S^6\right)\right]\Big|_{s_1}^{s_2} \tag{9}$$

$$\tau_{23}=-\frac{\mu b x_2}{2\pi(1-\sigma)}\left[-\left(1/R^2+1/S^2\right)+2x_2^2\left(1/R^4\right)+2\left(x_3^2+s^2-4sx_3\right)\left(1/S^4\right)+16x_2^2x_3\left(s/S^6\right)\right]\Big|_{s_1}^{s_2} \tag{10}$$

$$\tau_{33}=\frac{\mu b}{2\pi(1-\sigma)}\left[(x_3-s)\left(1/R^2-1/S^2\right)-2x_2^2(x_3-s)\left(1/R^4-1/S^4\right)+4x_3s(x_3+s)\left(1/S^4\right)\right.$$
$$\left.-16x_2^2x_3(x_3+s)\left(s/S^6\right)\right]\Big|_{s_1}^{s_2} \tag{11}$$

Corresponding to stresses given by Equations (9) to (11), the displacements are found to be

$$u_2 = \frac{b}{4\pi(1-\sigma)}\left[2(1-\sigma)\left\{\frac{2x_2 s}{S^2} + \tan^{-1}\left(\frac{x_2}{x_3 - s}\right) - \tan^{-1}\left(\frac{x_2}{x_3 + s}\right)\right\} - \left\{x_2(x_3 - s)(1/R^2 - 1/S^2) + 4x_2x_3s(x_3+s)(1/S^4)\right\}\right] \Big|_{s_1}^{s_2} \quad (12)$$

$$u_3 = \frac{b}{4\pi(1-\sigma)}\left[2(1-\sigma)\left\{\log(R/S) - \frac{2s(x_3+s)}{S^2}\right\} - \left\{\log(R/S) - \frac{x_2^2}{R^2} - \frac{(x_3^2+s^2)}{S^2} - \frac{4sx_2^2x_3}{S^4}\right\}\right] \Big|_{s_1}^{s_2} \quad (13)$$

NUMERICAL RESULTS

We wish to study the two-dimensional displacement and stress fields around a long vertical tensile fault of finite width L in a uniform half-space. We, therefore, put $s_1 = 0$, $s_2 = L$, and assume $\sigma = 0.25$. For numerical calculations, we define the following dimensionless quantities

$$Y = x_2/L,\ \ Z = x_3/L,\ \ D_i = \frac{\pi}{b}u_i,\ \ P_{ij} = \frac{\pi L}{\mu b}\tau_{ij} \quad (14)$$

Thus, Y is the dimensionless distance from the fault trace; Z is the dimensionless depth; D_2 and D_3 are the dimensionless displacements and P_{22}, P_{23} and P_{33} are the dimensionless stresses.

From Equations (9) to (14), we obtain

$$P_{22} = (2/3)\left[\frac{(Z-1)}{A^2} - \frac{(3Z+1)}{B^2} + \frac{2Y^2(Z-1)}{A^4} + \frac{2(Z^3+Z^2+Z+1-2Y^2)}{B^4} + \frac{16Y^2Z(Z+1)}{B^6}\right] \quad (15)$$

$$P_{23} = (3Y/2)\left[\frac{1}{A^2} + \frac{1}{B^2} - \frac{2Y^2}{A^4} - \frac{2(Z^2-4Z+1)}{B^4} - \frac{16Y^2Z}{B^6}\right] \quad (16)$$

$$P_{33} = (2/3)\left[(Z-1)\left(\frac{1}{A^2} - \frac{1}{B^2}\right) - 2Y^2(Z-1)\left(\frac{1}{A^4} - \frac{1}{B^4}\right) + \frac{4Z(Z+1)}{B^4} - \frac{16Y^2Z(Z+1)}{B^6}\right] \quad (17)$$

$$D_2 = (1/2)\left[\frac{2Y}{B^2} + \tan^{-1}\left(\frac{Y}{Z-1}\right) - \tan^{-1}\left(\frac{Y}{Z+1}\right) - \frac{2Y(Z-1)}{3}\left(\frac{1}{A^2} - \frac{1}{B^2}\right) - \frac{8YZ(Z+1)}{3B^4}\right] \quad (18)$$

$$D_3 = (1/3)\left[(1/2)\log(A/B) + \frac{Y^2}{A^2} + \frac{(Z^2-3Z-2)}{B^2} + \frac{4Y^2Z}{B^4} - 1\right] \quad (19)$$

where

$$A^2 = Y^2 + (Z-1)^2,\ \ B^2 = Y^2 + (Z+1)^2$$

Figure 2 shows the variation of the dimensionless normal stress P_{22} with the dimensionless distance from the fault at $x_3 = 0,\ L/2,\ 3L/2$. P_{22} is zero at $x_2 = 0$ for $x_3 = 0$; it is non-zero for other values of

x_3. Normal stress P_{22} is negative (compressive stress) at $x_2 = 0$ for $0 < x_3 < L$; for $x_3 > L$, it is positive (tensile stress). P_{22} tends to zero as x_2 tends to infinity. Moreover, as the depth x_3 increases, P_{22} converges to zero faster.

Figure 3 shows the variation of the dimensionless normal and shear stresses with depth at three locations: $x_2 = L/10, L/2, L$. The variation of P_{22} is smooth at $x_2 = L$, but for $x_2 = L/10$, P_{22} varies strongly with depth. We notice that near the fault, the variation of P_{23} with depth is noteworthy; however, as we move away from the fault, this variation becomes smooth.

Figure 4(a) shows the variation of the dimensionless horizontal displacement D_2 with the dimensionless distance from the fault for $x_3 = 0, L/2, 3L/2$. From Equation (18), we note that at $x_2 = 0$, $D_2 = 0$ for $x_3 > L$ and has the value $\pi/2 = 1.571$ for $x_3 < L$. Figure 4(b) shows the variation of the vertical displacement D_3 with distance from the fault for $x_3 = 0, L/2, 3L/2$. The pattern of variation of D_3 is similar for the three values of x_3 considered. Figure 5 (a, b) shows the variation of the displacements with depth for $x_2 = L/10, L/2, L$.

The contour maps in vertical planes perpendicular to the length of the fault for the stresses P_{22} and P_{23} are given in Figures 6(a, b) and for the displacement components D_2 and D_3 in Figures 7 (a, b). In these maps, the horizontal axis is the distance from the fault and the vertical axis is the depth, both measured in the units of the fault width L. The contour values for the isolines are indicated. The stresses are measured in the units of $\mu b/\pi L$ and the displacements in the units of b/π. These maps exhibit the variation of the elastic field around the fault.

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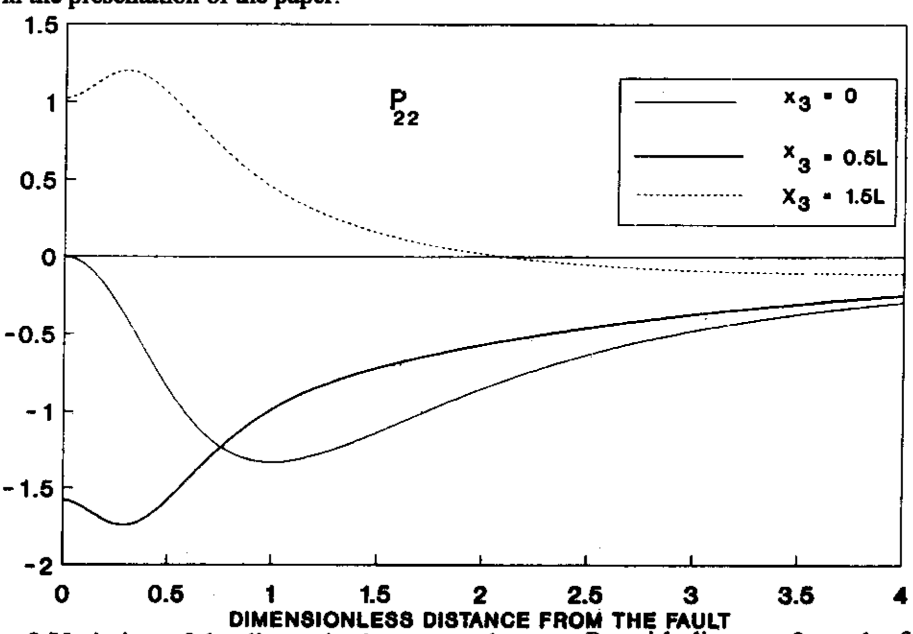
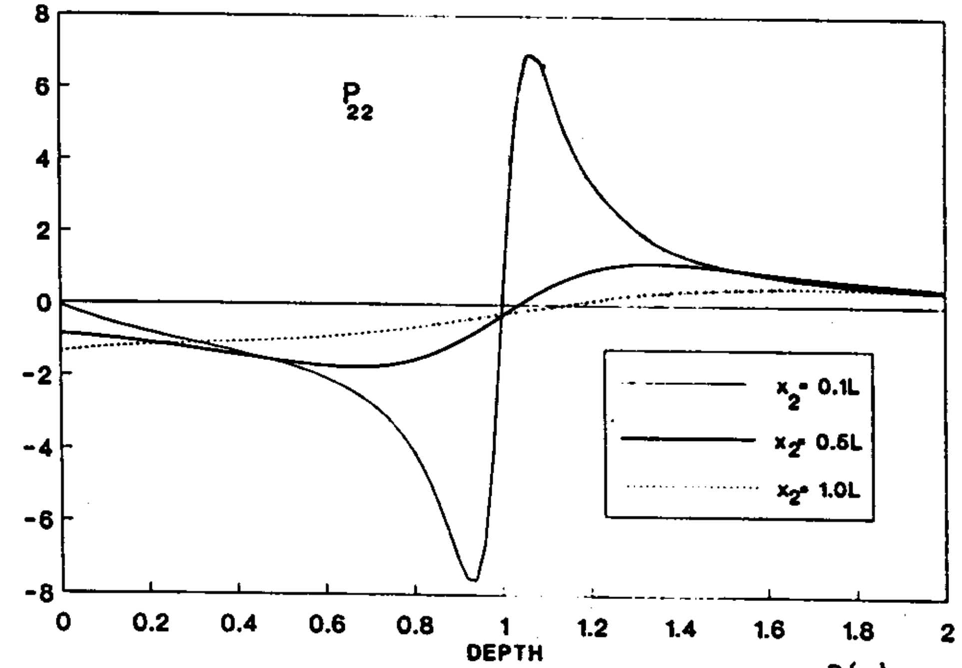
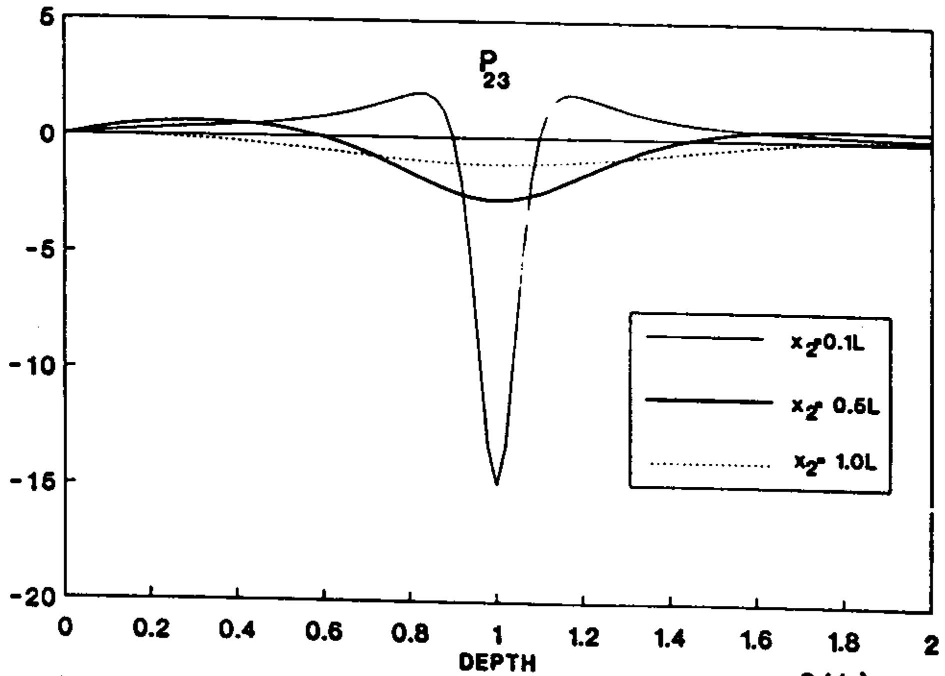


Fig. 2 Variation of the dimensionless normal stress P_{22} with distance from the fault

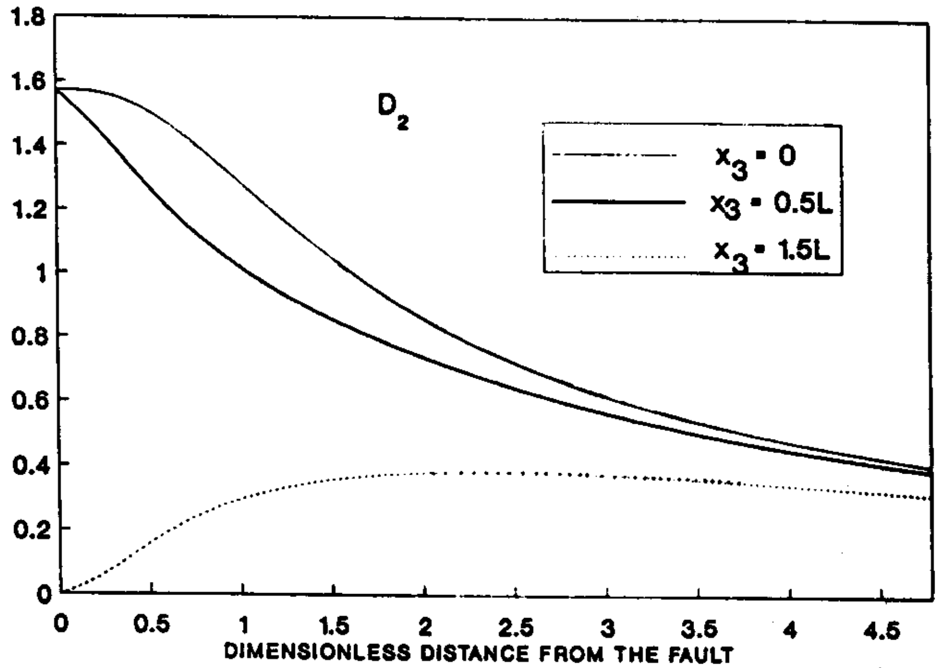


3(a)

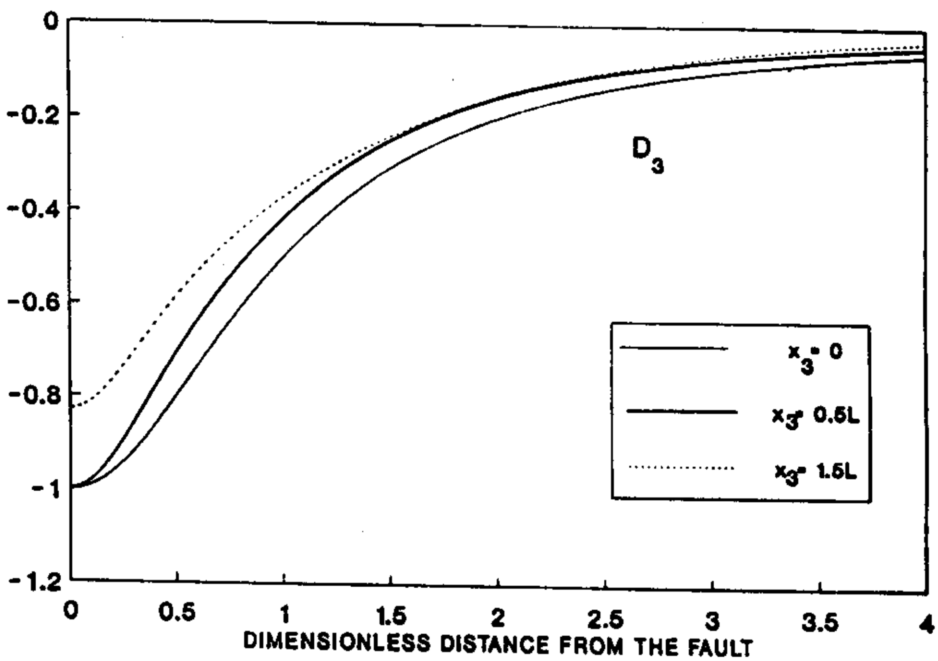


3(b)

Fig. 3 Variation of the dimensionless (a) normal stress P_{22}, (b) shearing stress P_{23} with depth

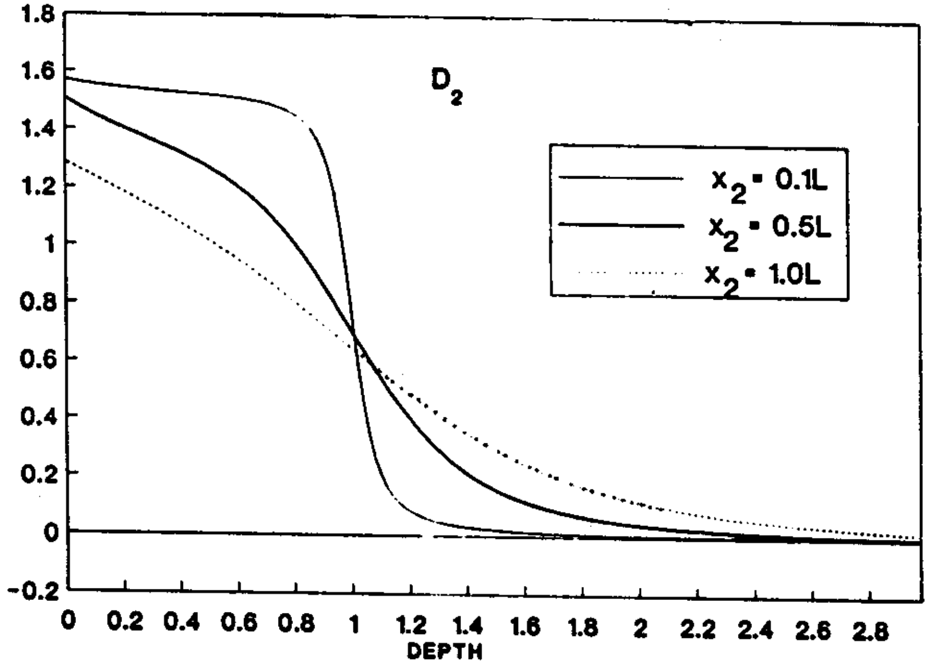


4(a)

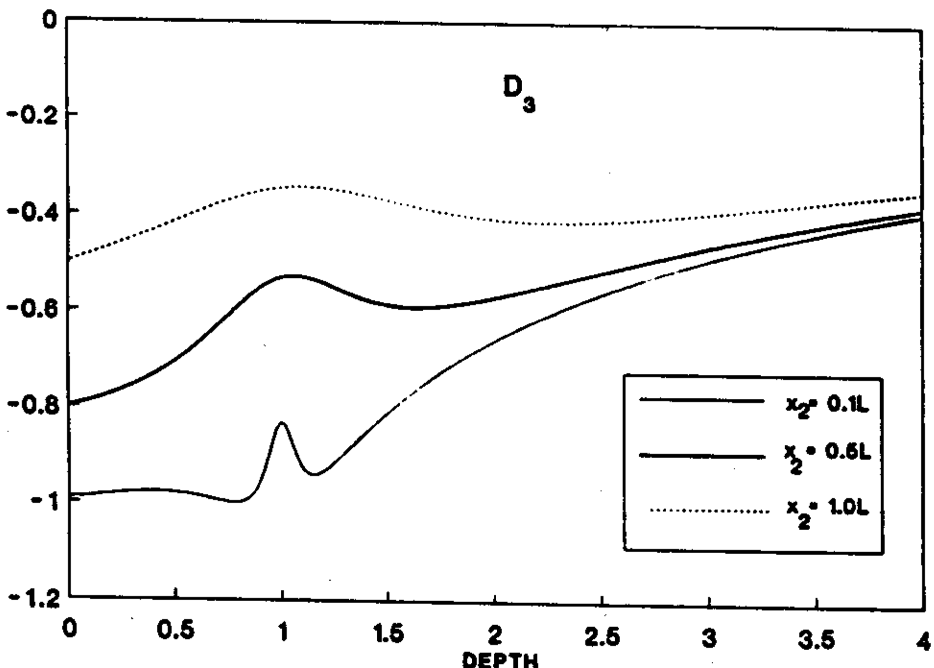


4(b)

Fig. 4 Variation of the dimensionless (a) horizontal displacement D_2, (b) vertical displacement D_3 with distance from the fault



5(a)



5(b)

Fig. 5 Variation of the dimensionless (a) horizontal displacement D_2, (b) vertical displacement D_3 with depth

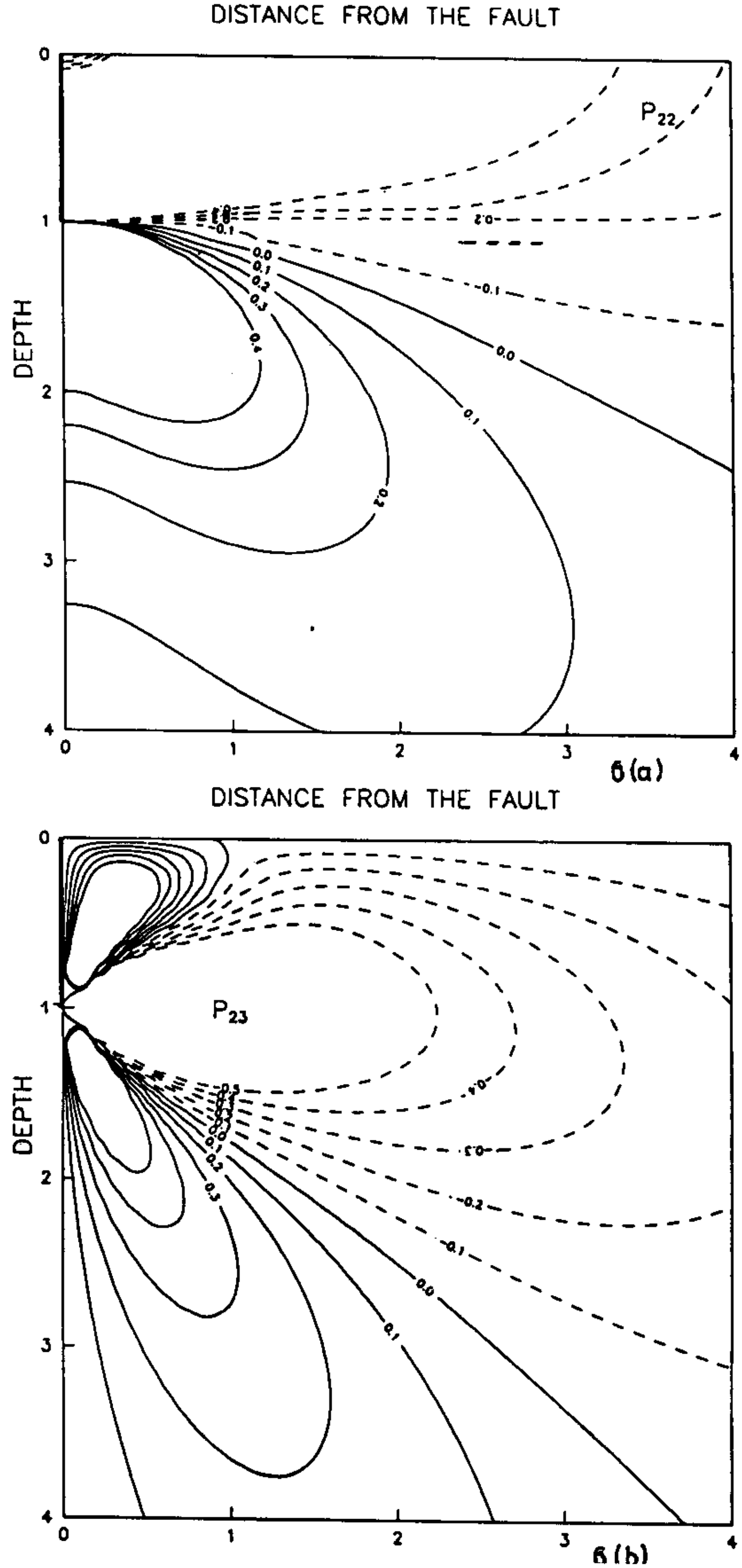


Fig. 6 Contour maps for dimensionless (a) normal stress P_{22}, (b) shearing stress P_{23}. (continuous lines indicate positive values; broken lines indicate negative values; the fault trace is shown by thick line)

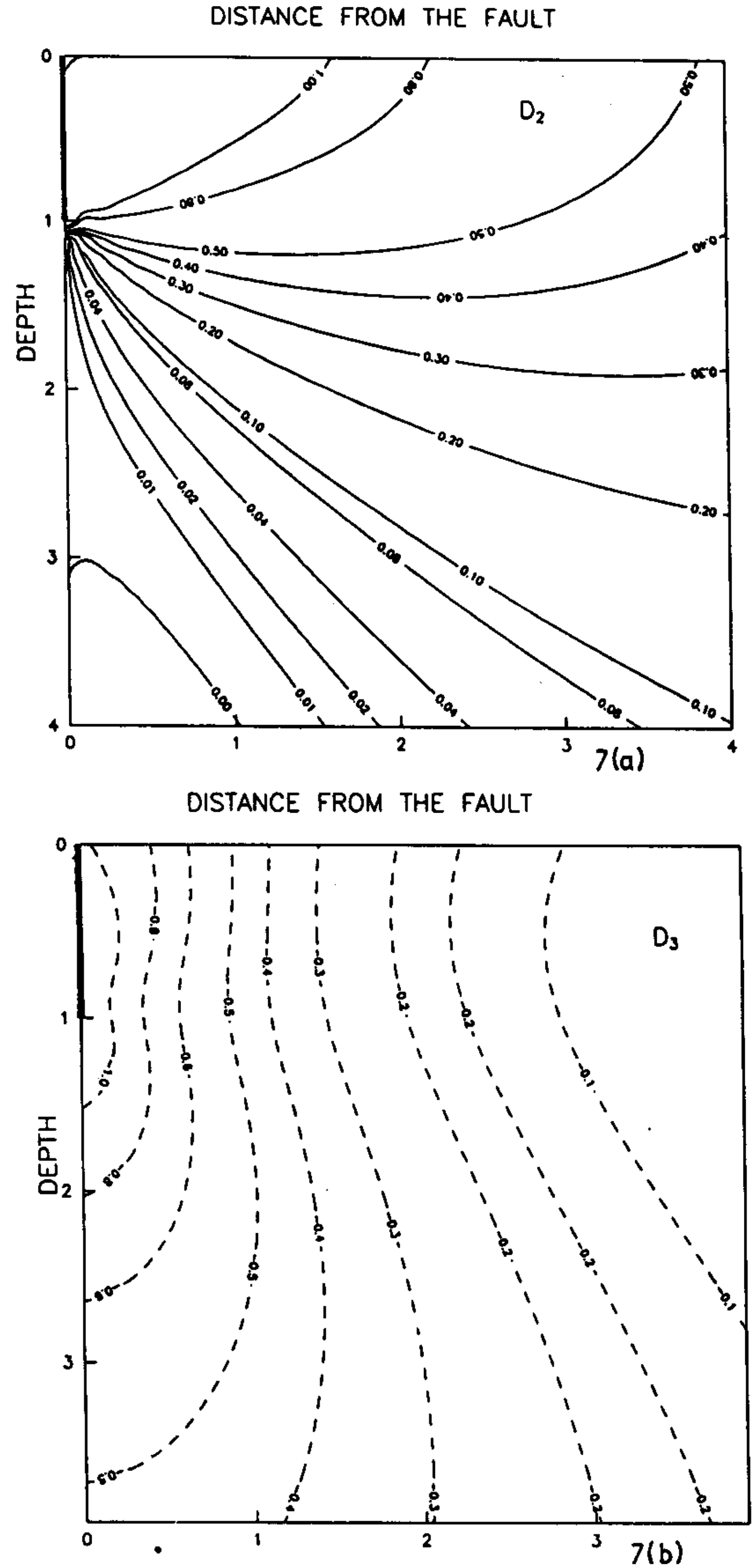


Fig. 7 Contour maps for dimensionless (a) horizontal displacement D_2, (b) vertical displacement D_3

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APPENDIX

1. Vertical dip-slip fault

$$L_0 = P_0 = Q_0 = 0, \qquad M_0 = \pm\frac{\mu b ds}{2\pi(1-\sigma)}$$

2. Vertical tensile fault

$$L_0 = M_0 = 0, \qquad P_0 = -Q_0 = \frac{\mu b ds}{2\pi(1-\sigma)}$$

3. Horizontal tensile fault

$$L_0 = M_0 = 0, \qquad P_0 = Q_0 = \frac{\mu b ds}{2\pi(1-\sigma)}$$

The upper sign is for $x_3 > h$, the lower sign is for $x_3 < h$, b is the magnitude of the displacement dislocation, and ds is the width of the line fault.