ISET Journal of Earthquake Technology, Paper No. 401, Vol. 37, No. 4, December 2000, pp. 47-63

LOW-CYCLE FATIGUE OF STRUCTURAL STEEL COMPONENTS: A METHOD FOR RE-ANALYSIS OF TEST DATA AND A DESIGN APPROACH BASED ON DUCTILITY

Claudio Bernuzzi *, Luis Calado** and Carlo A. Castiglioni*

* Department of Structural Engineering, Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133 Milan, Italy
** Department of Civil Engineering and Architecture, Instituto Superior Técnico
Avenida Rovisco Pais, 1049-001 Lisbon, Portugal

ABSTRACT

A method for the re-analysis of test data of structural steel components under cyclic actions is presented in this paper. The method is based on an $S-N$ line (i.e. stress range-fatigue endurance line) approach and, although proposed here for low-cycle fatigue, was derived and is valid also for high cycle fatigue. A statistical approach for the assessment of the $S-N$ lines, which is based on a selected probability of failure with reference to suitable levels of safety and reliability of the structural elements, is presented. Possible definitions of parameter S, as well as, failure criteria for the appraisal of N, are compared and their influence on the value of the slope of the $S-N$ line is discussed. Finally, a design approach is proposed, based on a direct assessment of the cumulative damage in steel components under variable amplitude loading histories. In order to validate and to analyse its accuracy, test data of beams, beam-columns and beam-to-column connections under cyclic and/or variable amplitude loading histories have been considered.

KEYWORDS: Connections, Experimental Behaviour, Fatigue, Damage, Failure. S-N Curves

INTRODUCTION

In the recent earthquake events of Northridge (1994) and Kobe (1995), a significant number of steel structures suffered extended damage. In buildings which did not collapse, local failures of steel elements and beam-to-column connections occurred without severe overall deformations, these failures remaining hidden behind undamaged architectural panels (Bertero et al., 1994). However, these disasters underlined the need of an efficient design approach for steel structures based on the selection of the energy dissipation mechanisms, which should permit to combine the stable hysteretic response of steel members and/or joints with the possibility of controlling simply, but reliably, the behavioural parameters of the relevant parts of the skeleton frame.

It has to be pointed out that the possibility to use such a seismic design approach implies an exhaustive knowledge of the low-cycle fatigue behaviour and strength of steel members and joints. At the present state of the art, such knowledge is still far from being satisfactory, despite several studies carried out in the past, or currently in progress, on the behaviour of subassemblies (Nader and Astaneh, 1991; Elnashai and Elghazouli, 1994), as well as of steel components (Plumier, 1994) under cyclic reversal loading.

This paper presents the main results of a joint research project, between the Universities of Lisbon (Portugal) and Milan (Italy) aimed to develop a method for the re-analysis of test data of structural steel elements under cyclic actions.

Furthermore, a design approach for the appraisal of the ductility of steel components under seismic loading is presented and discussed. Such an approach based on Miner's linear damage accumulation rule (Miner, 1945), and on S-N lines, is derived with reference to classical theory of elastic fatigue extrapolating its validity into the low-cycle (plastic) fatigue range. Failure criteria for the assessment of the number of cycles to failure (N) and definitions (proposed by various authors) of the stress (strain) range (S) are summarised and compared, with reference to the influence on the value of the slope of the $S-N$ line.

A statistical method for the assessment of the "design" $S-N$ lines, based on a given probability of failure with reference to suitable levels of safety and reliability of the structural elements, is also presented and analysed. The proposed methodology was applied to test data of beams, beam-columns and beam-to-column connections under cyclic loading to validate and to assess its accuracy.

FATIGUE BEHAVIOUR MODELS

An accurate prediction of fatigue failures requires the use of efficient techniques, which generally imply the definition of a function that relates a parameter (S), representative of the imposed cyclic actions, to the fatigue endurance (N) of the structural detail.

Most common approaches for the fatigue behaviour modelling can be classified into three categories, depending on the fatigue failure prediction function adopted:

- the $S-N$ line approach, which assumes S to be the nominal stress range $\Delta\sigma_0$;
- the local strain approach, which considers the local non-linear strain range $\Delta\varepsilon$;
- the fracture mechanics approach, which adopts the stress intensity factor range.

Reference is herein made to the $S-N$ line approach, because of its immediate applicability for structural design purposes.

The fatigue failure prediction function, used by the $S-N$ line approach, can be expressed by the following equation:

$$NS^m = K \tag{1}$$

where N is the number of cycles to failure at the constant stress (strain) range S. The non-dimensional constant m and the dimensional parameter K depend on both the typology and the mechanical properties of the considered steel component. In the Log-Log domain Equation (1) can be re-written as:

$$Log(N) = Log(K) - mLog(S) \tag{2}$$

Equation (2) represents a straight line with a slope equal to $-1/m$ called fatigue resistance line, which identify the safe and unsafe regions (Figure 1).

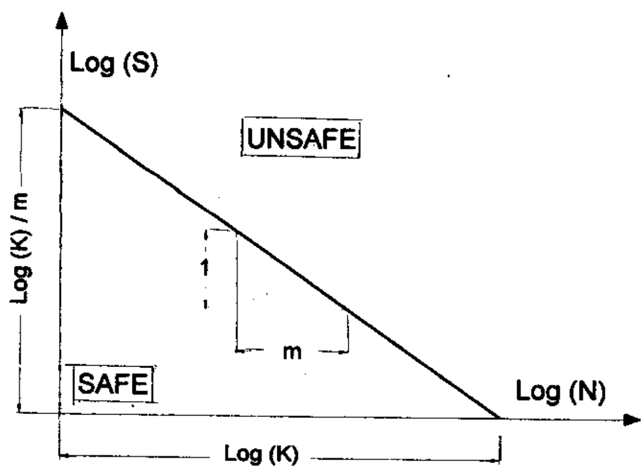

Fig. 1 Fatigue resistance line in the $Log(S) - Log(N)$ scale

Because of its simplicity, the $S-N$ line approach has been introduced into many fatigue design codes. The fatigue resistance lines adopted in design standards are built by means of a statistical analysis of constant amplitude fatigue tests data.

In case of variable amplitude loads, a direct assessment of the fatigue resistance is not possible and reference should be made to cycle-counting methods and to a suitable damage accumulation rule (Bannantine et al., 1989; and CEN, 1994). Usually, the linear damage accumulation rule proposed by Miner (1945) is adopted for calculation of an effective value, S_{eq}, to be used instead of S as argument in the fatigue failure prediction function.

The main advantage of the $S-N$ line approach is due to its simplicity. As this approach is able to interpret correctly the phase of stable crack propagation, it is commonly adopted in civil engineering design for the assessment of the fatigue strength of welded details where, due to the presence of fabrication imperfections, the crack initiation phase is practically absent.

METHODS OF RE-ANALYSIS OF LOW-CYCLE FATIGUE TESTS DATA

With reference to civil and industrial steel structures in seismic areas, the main components that should be considered and covered by a suitable design approach are members (under bending or compression and bending) and their connections (bolted or welded). As, usually, "slender" profiles are adopted for steel beams and columns, local buckling is likely to occur already during the first excursions in the plastic range. This effect should hence be considered as a stress (strain) concentrator factor. In the case of joints, the discontinuity in the geometry and, in general, in the stiffness of the connected elements generates the presence of a natural stress concentrator.

It is the author's opinion that a modern methodology to assess the low-cycle fatigue endurance of civil engineering structures should adopt parameters related to the global structural behaviour, such as displacements, rotations, bending moments, etc. As a consequence, the $S-N$ line approach may be adopted considering, as S, parameters related with the global structural ductility (e.g., inter-storey drifts or joint rotations).

Both parameters S and the number of cycles to failure (N) should be clearly defined, in order to apply Equation (1) in a consistent re-analysis of test data, in accordance with the basic assumptions of the selected damage model. The number of cycles to failure N can be identified on the basis of the adopted failure criterion, while S can be defined with reference to the proposals presented in the literature by various authors and summarised herein.

The slope $(-1/m)$ of the $S-N$ line (Figure 1), as well as the value of $Log\,K$, representing the intersection of the line with the horizontal $(Log\,N)$ axis can be defined by fitting the test data plotted in a $Log\,S-Log\,N$ scale.

1. Definition of the Strain Range (S)

As far as low-cycle fatigue is concerned, some of the most relevant proposals available in the literature for the definition of the strain range S are, in the following, presented and shortly discussed. It is interesting to notice that, although the various authors consider different definitions for the parameter relevant for the fatigue assessment, the same $S-N$ line type of approach can interpret their proposals. Of course, due to these differences in the original formulations for parameter S, it is to be expected that the exponent m and the constant K of Equation (1) are different from one formulation to the other. For this reason, these parameters will be indicated with a subscript referring to the authors of the formulation for S, with reference to the considered proposals available in literature.

1.1 Krawinkler and Zohrei Proposal

The concept to connect the parameter (S) of the fatigue failure prediction function with the global structural ductility was originally proposed by Krawinkler and Zohrei (1983). They proposed a relationship between the fatigue endurance and the plastic portion of the generalised displacement component (δ_{pl}) that can be expressed as:

$$N\left(\Delta\delta_{pl}\right)^{m_{KZ}} = K_{KZ} \tag{3}$$

where $\Delta\delta_{pl}$ represents the plastic portion of the deformation range.

1.2 Ballio and Castiglioni Proposal

Ballio and Castiglioni (1995), based on global displacement parameters instead of local deformation parameters, proposed a unified approach for the design of steel structures under low-and/or high-cycle fatigue. The fundamental hypothesis is the validity of the following equation:

$$\frac{\Delta\varepsilon}{\varepsilon_y} = \frac{\Delta\delta}{\delta_y} \tag{4}$$

where ε represents the strain, δ a generalised displacement component (e.g., a displacement v, or a rotation θ), Δ is associated with the range of variation in a cycle and subscript y identifies yielding of the material $\left(\varepsilon_y = f_y/E\right)$ as well as conventional yielding with reference to the generalised displacement component $\left(\delta_y\right)$.

Finally, the following parameter was identified:

$$\Delta\sigma^* = \frac{\Delta\delta}{\delta_y} f_y \tag{5}$$

representing an effective stress range associated with the real strain range in an ideal component made of an indefinitely linear elastic material, which can be assumed as S.

Hence, Equation (1) can be re-written as:

$$N(\Delta\sigma^*)^{m_{BC}} = K_{BC} \tag{6}$$

1.3 Feldmann et al. Proposal

On the basis of an extensive finite element analysis, Sedlacek et al. (1995) investigated the behaviour of beam-to-column joints under constant amplitude cyclic loading. They adopted a linear relationship between the plastic strain at the hot spot, ε_{pl} (strain at the relevant place where first crack occurs), and the plastic rotation $\left(\theta_{pl}\right)$:

$$\frac{\varepsilon_{2,pl}}{\varepsilon_{1,pl}} = \frac{\theta_{2,pl}}{\theta_{1,pl}} \tag{7}$$

where subscripts 1 and 2 refer to two different loading steps.

This linear relationship, that is somehow similar to Equation (4), can be rewritten in terms of the plastic portion δ_{pl} of the generalised displacement component δ and, when plotted in a Log-Log scale, plots as a "Wöhler" line.

Hence, it is possible to define an effective plastic stress range $\Delta\sigma_{pl}{}^*$:

$$\Delta\sigma_{pl}{}^* = \frac{\Delta\delta_{pl}}{\delta_y}\sigma\left(F_y\right) \tag{8}$$

that can be assumed as parameter S and Equation (1) can be re-written as:

$$N(\Delta\sigma_{pl}{}^*)^{m_{FSWK}} = K_{FSWK} \tag{9}$$

1.4 Bernuzzi et al. Proposal

Based on an extensive re-analysis of experimental data of beam-to-column joints as well as of beams and beam-columns tested under cyclic loading, Bernuzzi et al. (1997) proposed to use the total displacement range $\Delta\delta$ (e.g., the total range of interstorey drift $\Delta\nu$ for subassemblages or the total rotation range $\Delta\theta$ for joints) as parameter S.

As a consequence, Equation (1) can be re-written in the form:

$$N(\Delta\delta)^{m_{BCC}} = K_{BCC} \tag{10}$$

2. Definition of the Fatigue Endurance (N)

For the prediction of the low-cycle fatigue endurance of structural steel components, it seems convenient to adopt a failure criterion based on suitable parameters (i.e. stiffness, strength or absorbed energy) associated with the response of the component. Two failure criteria of this type are presented in the following.

2.1 Energy Reduction Failure Criterion

The "Energy Reduction Failure Criterion", originally proposed by Calado et al. (1989, 1995) and modified by Castiglioni (1999) is based on parameters (i.e. stiffness, strength or dissipated energy) associated with the response of the component. This criterion is characterised by a general validity for structural steel components under both constant and variable amplitude loading histories, and can be written as:

$$\frac{\eta_f}{\eta_0} = \alpha_f \tag{11}$$

Term η_f represents the ratio of the energy absorbed by the considered component at the last cycle before collapse (W_f) and the energy that might be absorbed by the same component, in the same cycle, if the material has an elastic-perfectly plastic behaviour (W_{eppf}). η_0 represents the same ratio but related to the first cycle in the plastic range $(\eta_0 = W_0/W_{epp0})$. In case of variable amplitude loading histories the same criterion remains valid but should be applied with reference to the half cycles, which can be defined in plastic range as the part of the hysteresis loop under positive or negative loads or as two subsequent load reversal points. In the case of constant amplitude loading, the ratio $\alpha_f = \eta_f/\eta_0 = W_f/W_0$, because $W_{eppf} = W_{epp0}$. The value of α_f which, in general should be determined by fitting the experimental results, depends on several factors (as the type of joint and the steel grade of the component). As it is particularly interesting to identify α_f a priori, in order to define a unified failure criterion for all types of steel components (members and joints), a value of $\alpha_f = 0.5$ is recommended by Calado and Castiglioni (1995) for a satisfactory and conservative appraisal of the fatigue life.

Later, Ballio et al. (1997) showed that the ratio η_f/η_0 at collapse depends on the cycle amplitude, although the available results did not allow a clear identification of the relationship.

During an extensive re-analysis of the data of constant amplitude cyclic tests carried out on more than 150 specimens of both members and joints, three types of collapse were observed. Independently on the structural component (member or joint), such failure modes occurred for different values of the ductility range $\Delta\nu/\nu_y$, defined as the ratio between the imposed displacement range $(\Delta\nu)$ and the conventional yield displacement (ν_y). The observed collapse modes are:

- *brittle failure mode*, which was usually achieved under cycles corresponding to small values of $\Delta\nu/\nu_y$. Specimen response was not affected by remarkable deterioration of strength, stiffness and/or energy absorption capability. Collapse was sudden, without warning signs, owing to a crack either at weld toes or in the base material;

- *ductile failure mode*, generally associated with a large ductility range $\Delta v/v_y$. The component performance was significantly affected by a remarkable and progressive deterioration of the key behavioural parameters, with the formation of a plastic hinge associated with local buckling of the beam flanges. Collapse was caused by the gradual propagation of a crack initiating at surface striations forming, due to attainment of the maximum tensile strain of the material, at the buckles at the plastic hinge location;
- *mixed failure mode*, which appears as a combination of the two previous failure modes. A progressive deterioration of the key behavioural parameters, such as stiffness, strength and dissipated energy, was observed. It was usually associated with both plastic hinge formation and local buckling phenomena. However, collapse was generally due to a crack at the weld toes.

Re-analysis of constant amplitude test data showed that a suitable threshold displacement, Δv_{Th}, can be identified, separating the two different failure modes.

In absence of experimental data, an approach to estimate by means of a simple equation the value of Δv_{Th} has been developed and validated (Castiglioni et al., 1997). In particular, it has been assumed that Δv_{Th} depends on the following parameters: the conventional elastic displacement, v_y; the beam web slenderness ratio $\lambda_w = d/t_w$, defined as the ratio between the depth of the profile, d, and the web thickness, t_w; the beam flange slenderness ratio $\lambda_f = c/t_f$, where c represents half width of the flange and t_f is its thickness; the weld quality and/or the severity of the detail, globally accounted for by the introduction of the numerical coefficient ξ. This term ranges from $\xi = 1.0$ for good quality (or no) welds to $\xi = 0.5$ for poor quality welds.

The threshold value, Δv_{th}, has been hence defined as:

$$\Delta v_{th} = \frac{\gamma v_y}{\xi \lambda_f \lambda_w} \qquad (12)$$

As to the non-dimensional coefficient γ, on the basis of the available experimental data, a value of 2000 ± 15% (i.e., in the range 1700 - 2300) was suggested, independently on the considered component (Castiglioni et al., 1997).

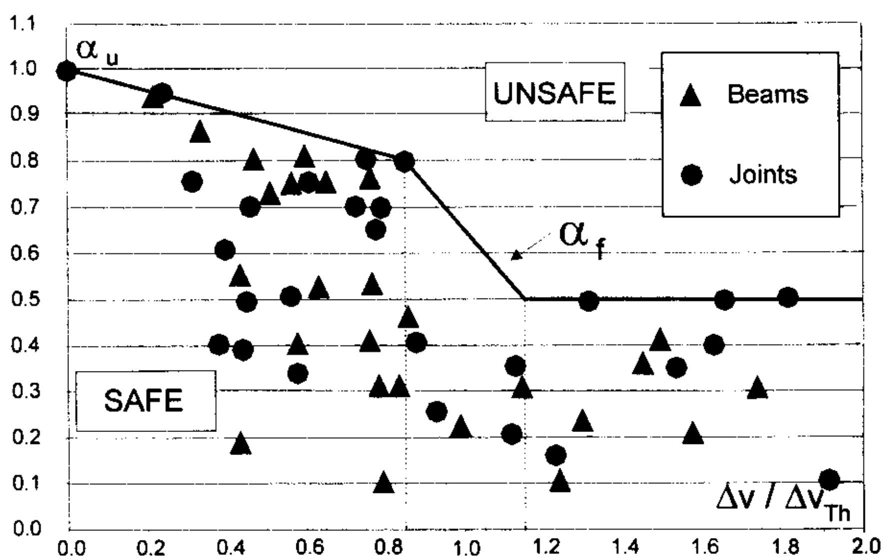

Fig. 2 Definition of parameter α_f

As previously said, Calado and Castiglioni (1995) recommended $\alpha_f = 0.5$ for a satisfactory appraisal of the fatigue life. This value leads to a very good appraisal of the fatigue endurance. In fact, it allows a definition of the number of cycles to failure N_f in good agreement with the experimental evidence for the steel components collapsed in a "ductile" or "mixed" mode. However, such criterion is not applicable in the case of "brittle" failures, which usually occurred for values of α_f ranging between 1.0 and 0.8, but always greater than 0.5.

Hence, a new a priori failure criterion was proposed, to extend the range of validity of the Calado and Castiglioni criterion also to "brittle" failure modes.

A conservative and satisfactory appraisal of the fatigue life can be obtained assuming (Castiglioni, 1999) a value of α_f in the failure criterion determined by fitting the experimental data (Figure 2) so that all the performed tests plot below the $\alpha_f(\Delta v/\Delta v_{th})$ line.

$$\begin{aligned} \alpha_f &= 1 - 0.235*(\Delta v/\Delta v_{th}) && \text{if} \quad \Delta v/\Delta v_{th} < 0.85 \\ \alpha_f &= 1.65 - (\Delta v/\Delta v_{th}) && \text{if} \quad 0.85 < \Delta v/\Delta v_{th} < 1.15 \\ \alpha_f &= 0.5 && \text{if} \quad \Delta v/\Delta v_{th} > 1.15 \end{aligned} \tag{13}$$

It can be seen that, in the range $0.85 < \Delta v/\Delta v_{th} < 1.15$, a linear variation of α_f in the range $0.5 < \alpha_f < 0.8$ is proposed; in this range of $\Delta v/\Delta v_{th}$, a mixed failure mode is to be expected.

For $\Delta v/\Delta v_{th} > 1.15$, a ductile failure mode is to be expected, and a value of $\alpha_f = 0.5$ is assumed, in agreement with the Calado and Castiglioni proposal.

For $\Delta v/\Delta v_{th} < 0.85$, a brittle failure mode is to be expected, and a linear variation of α_f in the range $0.8 < \alpha_f < 1$ is proposed.

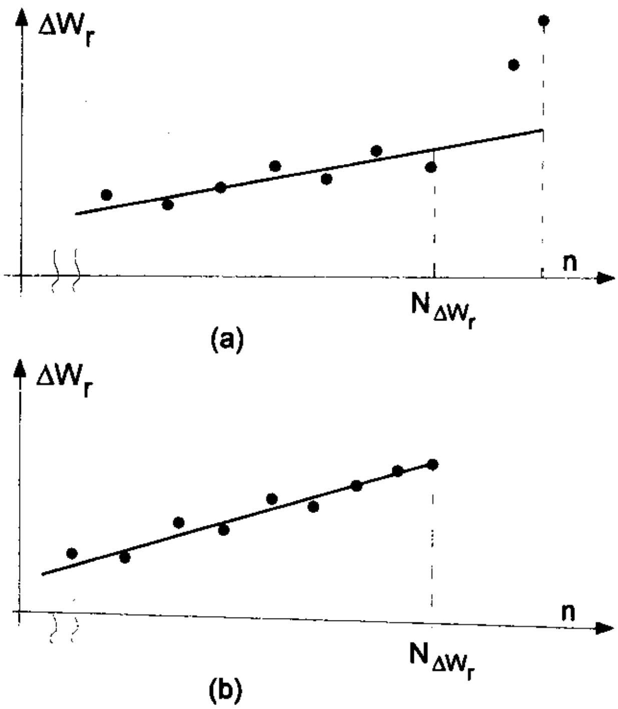

Fig. 3 Definition of the fatigue endurance N

2.2 Relative Energy Drop Failure Criterion $\left(N_{\Delta W_r}\right)$

This criterion was proposed for constant amplitude loading (Bernuzzi et al., 1997) and considers the drop of the hysteretic energy dissipation as the main parameter for the definition of the low-cycle fatigue endurance. In particular, focusing attention only on the cycles performed at the displacement range $\Delta\delta_i$, the relative energy drop, ΔW_r, can be defined as:

$$\Delta W_r = \left(\frac{W_{init} - W_i}{W_{init}}\right) \tag{14}$$

where W_{init} represents the absorbed energy in the first cycle at the assumed displacement range while W_i is the energy associated with the *i*th cycle at the same displacement range.

Failure is assumed to occur when the energy drop is evident, i.e., when the generic point of co-ordinates $n_k, \Delta W_{rk}$ (referring to cycle k), does not plot on the straight line interpolating the previous $k-1$ points (Figure (3a)) or in correspondence of the last cycle (Figure (3b)).

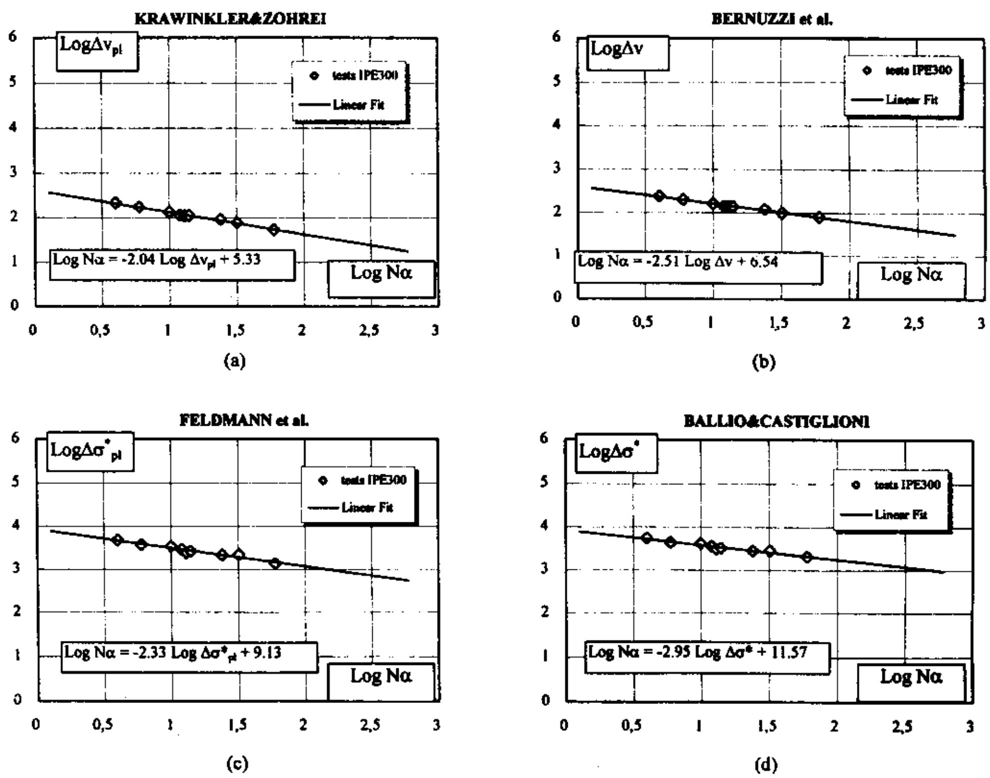

Fig. 4 Evaluation of the $S-N$ line for IPE300 beams on the basis of N_α and considering S as defined by Krawinkler, Ballio, Feldmann and Bernuzzi proposals

3. Definition of m

Parameter m identifies the slope $(-1/m)$ of the line interpreting, in a Log-Log scale, the relationship between number of cycles to failure (N) and the stress (strain) range (S).

Focusing the attention on m, some discrepancies can be found in the literature among the proposals of various authors. In particular, it can be noticed that Ballio and Castiglioni (1995), as well as Bernuzzi et al., (1997), suggested to adopt a value of $m=3$; on the contrary, Krawinkler and Zohrei (1983) and

Sedlacek et al. (1995) proposed an exponent $m = 2$. However, it should be noticed that both Feldmann and Krawinkler approaches adopt the plastic range of the assumed parameters, while both Ballio and Bernuzzi proposals are based on the total cyclic excursion (i.e. both the elastic and plastic contributions).

Hence, it appears that the value of the exponent m strongly depends on the definition of S. In order to clarify this influence, available data obtained in tests with cycles of constant amplitude (Ballio and Castiglioni, 1994; Ballio et al., 1997), have been plotted in a $Log(S)-Log(N)$ scale, according to the previously mentioned definitions of parameter (S). Best fitting such data by a straight line led to the definitions of (m), as well as of the intercept $(Log\,K)$ presented in Table 1, respectively related to beams, beam-columns and beam-to-column connections.

Table 1: Evaluation of $Log(K)$ and m Factors for Beams, Beam-Columns and Beam-to-Column Connections

BEAMS			HEA120	HEA140	HEA220	HEB220	IPE300
N_α	A	$Log(K)$	8.72	6.94	5.56	6.99	5.33
		m	3.05	2.50	1.93	2.52	2.04
	B	$Log(K)$	15.14	12.57	9.15	10.47	11.57
		m	3.90	3.26	2.23	2.57	2.95
	C	$Log(K)$	12.23	9.69	7.35	8.57	9.13
		m	3.15	2.51	1.77	2.09	2.33
	D	$Log(K)$	10.70	8.98	7.15	8.83	6.54
		m	3.74	3.23	2.53	3.23	2.51

BEAM-COLUMNS			HEA120 P/P_y=0.10	HEA220 P/P_y=0.05	HEA220 P/P_y=0.10	HEA220 P/P_y=0.125	IPE300 P/P_y=0.10
N_α	A	$Log(K)$	7.55	4.43	5.37	4.96	5.02
		m	2.68	1.46	2.05	1.92	2.12
	B	$Log(K)$	13.63	10.56	11.55	12.69	12.04
		m	3.55	2.70	3.07	3.40	3.22
	C	$Log(K)$	10.14	6.23	8.33	7.46	7.50
		m	2.63	1.48	2.20	1.95	2.00
	D	$Log(K)$	10.20	7.25	7.28	8.26	8.45
		m	3.64	2.66	2.80	3.34	3.61

CONNECTIONS			BCC1	BCC2	BCC3	BCC4	BCC5	BCC6
N_α	A	$Log(K)$	5.49	6.63	7.42	6.98	4.89	3.22
		m	1.99	2.49	2.80	2.88	1.96	1.08
	B	$Log(K)$	9.92	11.32	11.28	13.67	9.40	5.72
		m	2.61	2.81	2.68	3.49	2.33	1.25
	C	$Log(K)$	7.26	9.53	10.78	10.97	8.02	5.17
		m	2.00	2.53	2.58	2.82	1.99	1.11
	D	$Log(K)$	9.84	7.91	8.39	8.57	5.77	3.55
		m	3.80	2.89	3.16	3.49	2.32	1.21

A: Krawinkler and Zohrei B: Ballio and Castiglioni C: Feldmann et al. D: Bernuzzi el al.

All data presented in this table were obtained with reference to values of the fatigue endurance (N) determined in accordance with the *Energy Reduction Failure Criterion* (Equation (11)). However, very limited differences were noticed in the case of definition of (N) according to the relative energy drop criterion joints (Equation (14)) (Bernuzzi et al., 1997).

Figure 4 presents, in the case of IPE300 beams, the fatigue resistance lines in accordance with the four criteria considered, for the definition of (S).

The analysis of Table 1 allows to conclude that the slope of the $S-N$ lines is strongly affected by the considered definitions of S. In particular, with reference to parameter S, if it is considered only the excursion in the plastic range, a value $m=2$ is suggested, while if S is associated with the total range (i.e. considering both elastic and plastic contributions), a value of $m=3$ can be assumed. This seems to be a conclusion of general validity, for low-cycle fatigue of both members and joints.

ASSESSMENT OF K AND DEFINITION OF THE DESIGN S-N LINES

According to limit state design methodology, parameters governing the design should be defined on the basis of a statistical analysis, allowing to make reference to values associated with a given probability of failure P_f (or of survival, $1-P_f$). Such a probability depends on the suitable levels of the required safety and reliability of the whole structure as well as its components. Hence, the $S-N$ lines should be defined with reference to a given probability of failure P_f.

In this research, the procedure proposed in the International Welding Institute (1994) was used to define the design value $Log(K_d)$ and is briefly summarised herein.

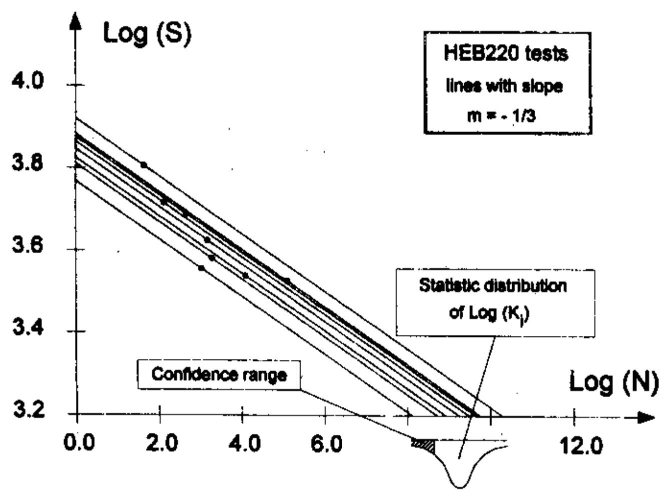

Fig. 5 Schematic representation of the procedure for the definition of the design $S-N$ curves

In the following, reference is made to the set of values $Log(K)$ representing the intersections with the N axis of the $S-N$ lines, each one having a constant slope $(-1/m)$ and passing for every single test data point (Figure 5).

The basic formula to determine the design value X_d of the random variable $X=Log(K)$ can be assumed as:

$$X_d = \mu - \phi^{-1}(\alpha)\cdot\sigma \tag{15}$$

where μ is the mean value of X, σ is the standard deviation of X, ϕ is the normal law of probability and α is the probability of survival.

Generally it is assumed that μ is distributed according to the t-Student model and the standard deviation σ^2 follows the chi-square law (χ^2). As σ and μ are unknown values, their estimated values from tests, $\bar{\mu}$ and $\bar{\sigma}$ respectively, should be associated with β, confidence level.

The design value of X can then be expressed as:

$$X_d = \bar{\mu} - \bar{\sigma} \cdot \left[\frac{t(\beta, n-1)}{\sqrt{n}} + \phi^{-1}(\alpha) \cdot \sqrt{\frac{n-1}{\chi^2\left(\frac{1-\beta}{2}, n-1\right)}} \right] \qquad (16)$$

According to Eurocode 3 (CEN, 1994), the confidence range β should be 75% of 95% probability of survival on the $Log(N)$ axis. Hence, by assuming α and β respectively equal to 0.95 and 0.75, Equation (14) can be re-written as:

$$X_d = \bar{\mu} - \bar{\sigma} \cdot \gamma \qquad (17)$$

where, in the case of limited amount of data, the value of γ can be obtained directly from Table 2 on the basis of the number of available data (n).

Table 2: Coefficients for Statistical Evaluation

n	2	3	4	5	6	7	8	9	10	11	12	13	14	15
γ	12.16	5.42	4.13	3.58	3.26	3.06	2.91	2.80	2.71	2.64	2.58	2.53	2.48	2.45

DESIGN APPROACH

Based on the results previously presented, a methodology for the design of steel components under cyclic (seismic) loading can be developed having a general validity, independently by the definitions of S as well as on the value of m and that can be applied in both high and low-cycle fatigue.

According to Miner's rule, which assumes the linear damage accumulation, damage index D can be expressed by means of the following relationship:

$$D = \sum_{i=1}^{L} \frac{n_i}{N_i} \qquad (18)$$

where n_i is the number of cycles carried out at the same range S_i, N_i is the number of cycles at which collapse would occur in case of constant amplitude S_i, and L is the number of set of constant amplitude cycles executed, being:

$$\sum_{i=1}^{L} n_i = n_{TOT} \qquad (19)$$

where n_{TOT} represents the total number of cycles in the loading history.

In the more general case of steel components under seismic loading, it can be assumed that the displacement history imposed on the component by the seismic event is made of n_{TOT} cycles, having amplitudes different from each other (i.e. $n_i = 1$ and $L = n_{TOT}$), and Equation (18) can hence be re-written as:

$$D = \sum_{i=1}^{n_{TOT}} \frac{1}{N_i} \qquad (20)$$

where term $(1/N_i)$ represents the damage associated with the cycle at the level S_i.

Damage index D assumes values ranging from 0 (no damage on the considered component) to 1 (conventional failure). By combining Equation (1) and Equation (20), it is possible to assess D in case of variable amplitude loading histories. In particular, the damage index can be expressed as:

$$D = \sum_{i=1}^{n_{TOT}} \frac{S_i^m}{K} = \frac{\sum_{i=1}^{n_{TOT}} S_i^m}{K} \tag{21}$$

For a given typology of steel component, under the assumption that constant K is known, Equation (21) allows a direct appraisal of the damage for a considered value of the parameter S as well as of the number of cycles associated with a given level of the cumulated damage (i.e., of the damage D).

In case of variable amplitude loading, it appears convenient to make reference to S_{eq}, effective value of parameter S that can be derived from Equation (21) as the value of S at which the steel components collapses for $N = n_{TOT}$ constant amplitude cycle at S_{eq} :

$$S_{eq} = \left(\frac{\sum_{i=1}^{L} n_i \cdot S_i^m}{n_{TOT}} \right)^{\frac{1}{m}} \tag{22}$$

For practical applications in seismic design, assuming a damage level D^*, it is possible to define in the $Log(S) - Log(N)$ domain an iso-damage line (Figure 6), by combining Equation (1) with Equations (21) and (22) obtaining:

$$Log(n_{TOT}) = Log(D^* K) - mLog(S_{eq}) \tag{23}$$

This relationship, which coincides with Equation (2) for the definition of the fatigue resistance (i.e., $D = 1$), allows to define a set of straight lines (iso-damage lines) with a slope of $(-1/m)$, each associated with a different level of damage D^*. It should be noted that a value of $D^* = 0.4$ can be assumed, as recommended by Chai et al. (1995) to identify a border line between repairable damage $(D^* < 0.4)$ and irreparable damage $(D^* > 0.4)$ situations (Figure 6).

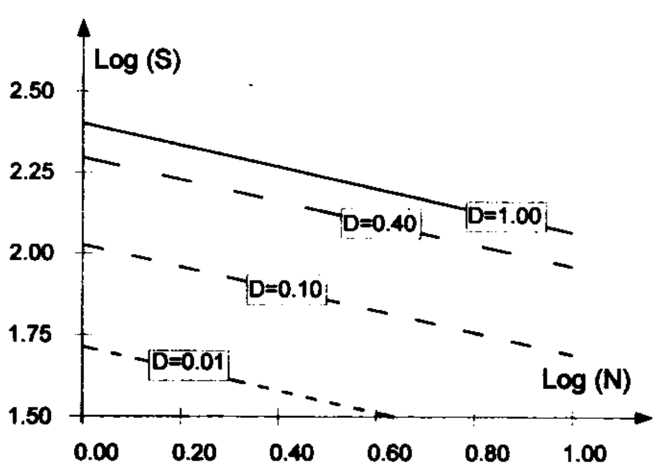

Fig. 6 Iso-damage lines

As an example, Figures 7 and 8 can be considered, which are the application of the previously mentioned approach to the test results of beam-to-column connections (BCC5) and beam specimens (IPE300), for different significant levels of damage ($D = 0.10, D = 0.25, D = 0.50, D = 0.75, D = 1.00$).

A direct use of the low-cycle fatigue approaches for practical design purposes, i.e., for the definition of the behaviour factor, was developed by Castiglioni et al. (1999). The procedure implies the use of linear elastic "time-history" analysis and of the Miner rule combined with the Ballio and Castiglioni low-cycle fatigue approach. On the basis of the values of the internal actions, the cumulated damage $D(a_{max})$ can be evaluated for each component/detail, based on a given value of the peak ground acceleration a_{max} at collapse. When $D(a_{max})$ reaches the damage design level, the behaviour factor is identified as the ratio (a_{max}/a_d), where a_d represents the design value of the peak ground acceleration.

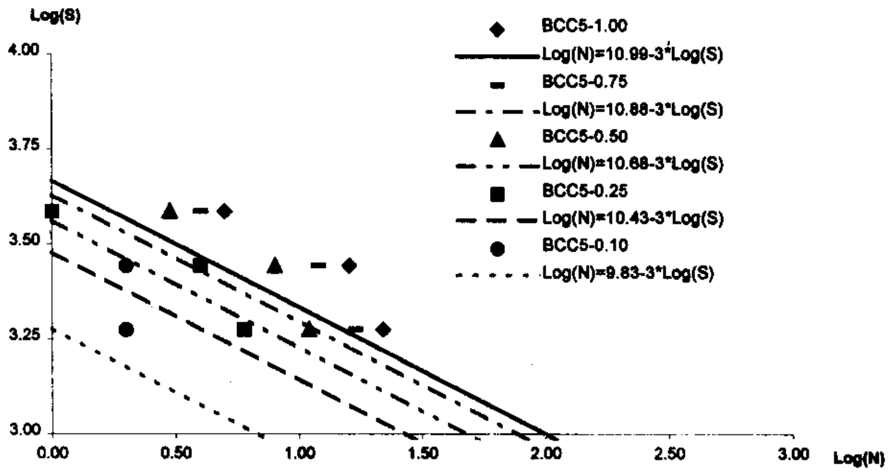

Fig. 7 Iso-damage lines for BCC5 specimens

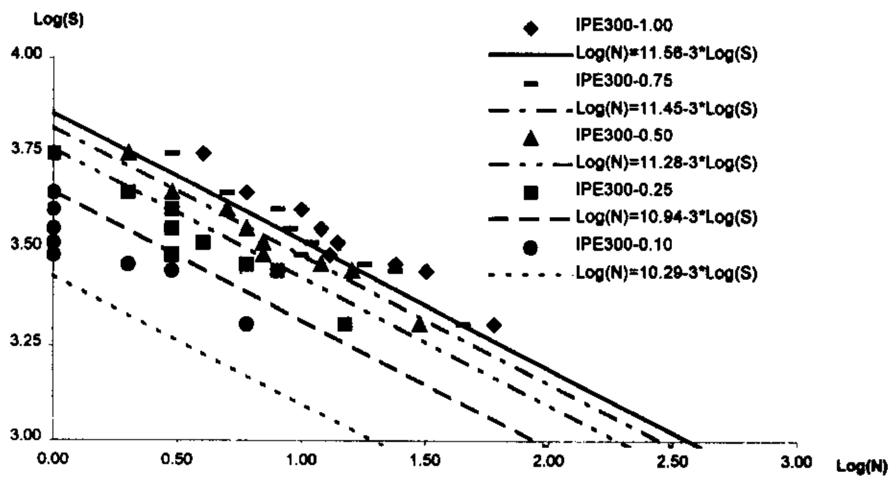

Fig. 8 Iso-damage lines for IPE300 specimens

VALIDATION OF THE METHOD UNDER VARIABLE AMPLITUDE LOADING HISTORIES

The approach previously presented can be directly and efficiently used for seismic design. In particular, under the assumption that, for the generic component, the fatigue resistance curve is known (i.e. the key parameters m and K have already been defined), an assessment of damage under random loading histories can be carried out with reference to the following phases:

- Re-analysis of the random loading history by means of a cycle-counting method;
- Evaluation of S_{eq}, i.e. the equivalent value of parameter S, according to Equation (22);
- Appraisals of the cumulated damage, based on the $S-N$ line of Equation (2) (associated to failure conditions $D=1$) or, given a level of acceptable damage D^*, appraisal of the fatigue endurance via the iso-damage line of Equation (23).

For the validation of the approach for seismic design, the multi-specimen testing programme carried out in Milano (for beams and beam-columns) as well as in Lisbon (for beam-to-column joints) was considered. The data related to the failure of specimen under constant amplitude loading histories (C.A.) were considered to define the fatigue resistance curve, the slope $(-1/m)$ as well as the constant K, while random loading tests (V.A.), which were carried out up to collapse, were used to validate the method and to appraise its degree of accuracy in the prediction of low-cycle fatigue failure.

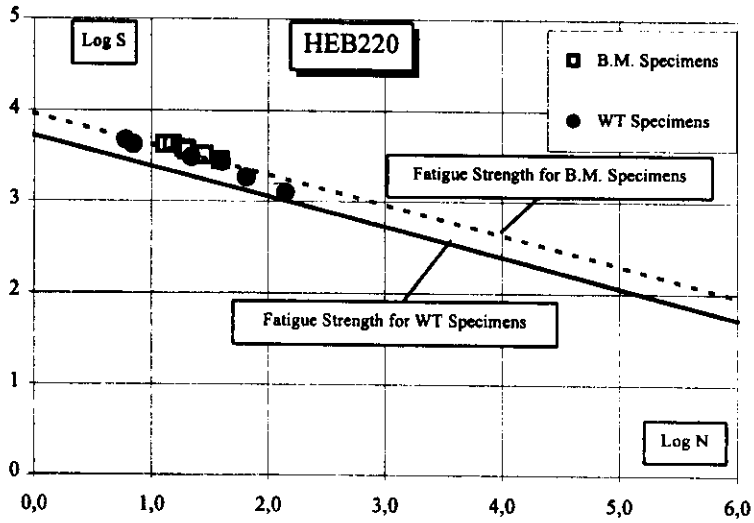

Fig. 9 $S-N$ lines for HEB220 type specimens (different lines fit data with different failure modes)

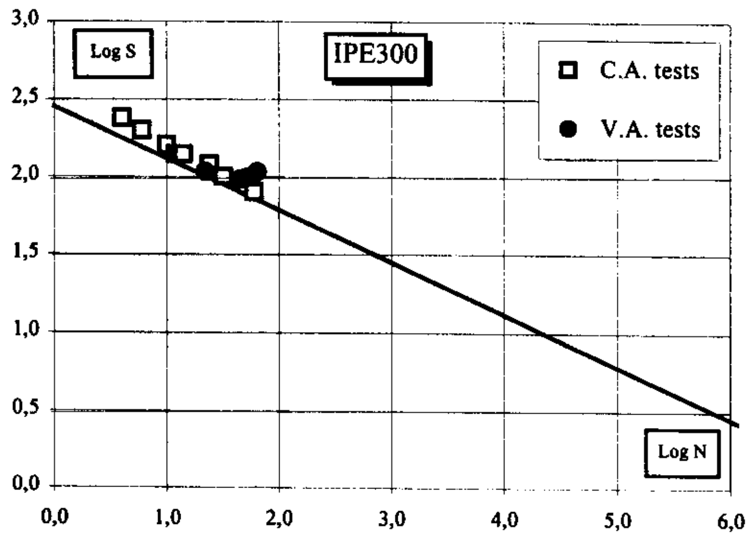

Fig. 10 Constant and variable amplitude test data on IPE300 type specimens fitted by the same $S-N$ line

As an example, reference can be made to the group of the data related to HEB220 beam specimens which failed for cracking in the base material (BM) at the plastic hinge location, or by cracking at the weld toes (WT) and to IPE300 beam specimens, which failed for cracking of the beam flanges. By considering only the Bernuzzi et al. and the Krawinkler and Zohrei criteria and making reference to the value of $m = 2$ and $m = 3$ for the fatigue resistance lines in the $Log(S) - Log(N)$ domain respectively, it is possible to observe (see Figures 9 and 10) a more than satisfactory degree of accuracy in the prediction of the fatigue endurance.

This remark is confirmed also by the data related to the other random tests on beams, beam-columns and beam-to-column joints.

Table 3 presents the value of the damage index D at collapse via the four approaches considered in this paper. It should be noted that the degree of approximation of D at failure is satisfactory, ranging between 0.83 and 1.59 when S is defined according to Bernuzzi et al.(1997), with a mean value of 1.12 and Coefficient of Variation (COV) of 16%, and ranging between 0.88 and 1.59 when S is defined according to Krawinkler and Zohrei, with a mean value of 1.16 and COV of 15%. The fact that these values are similar to those obtainable by usual application of Miner's rule to steel components under high cycle fatigue confirms the applicability of the proposed procedure in the case of steel components under low-cycle fatigue.

Table 3: Miner's Damage Sums for Variable Amplitude Tests

BEAMS		A	B	C	D
HEA220	*1*	1.00	1.06	1.06	1.00
	9	1.12	0.84	0.84	1.12
	10	1.19	1.25	1.25	1.20
	11	1.37	1.09	1.09	1.37
HEB220	*1*	1.59	1.28	1.28	1.59
	12	1.33	0.92	0.92	1.00
	13	1.12	1.28	1.28	1.19
IPE300	*9*	1.11	1.31	1.31	1.06
	10	1.00	1.05	1.05	1.00
	11	1.01	1.30	1.30	1.01
	15	1.18	0.91	0.91	1.18

CONNECTIONS		A	B	C	D
BCC1	*f*	1.00	1.12	1.12	0.83
	g	1.20	1.22	1.30	1.26
BCC2	*g*	1.18	1.67	1.45	1.12
BCC4	*b*	0.88	1.25	1.25	0.85
BCC5	*c*	1.27	1.52	1.52	1.16
BCC6	*c*	1.09	1.40	1.40	1.07

A: Krawinkler and Zohrei. B: Ballio and Cestiglioni C: Feldmann et al. D: Bernuzzi el al.

CONCLUSIONS

In this paper, a method for the re-analysis of test data of structural steel components under cyclic loading, which is based on an $S-N$ line approach, was proposed. Possible definitions of the fatigue endurance (N) and procedures for the assessment of the stress (strain) range (S) were presented and discussed, including their influence on the slope $(-1/m)$. It should be noticed that one of the scopes of this research was also to investigate the possible causes of the discrepancies that were found in the proposed values for the slope $(-1/m)$ of the $S-N$ lines presented by various authors in the literature.

The proposed methodology was applied to experimental data on beams, beam-columns and beam-to-column connections, in order to analyse its applicability and accuracy, with reference to a satisfactorily wide range of experimental results. Concerning the fatigue endurance, a good accuracy was observed between the two proposed criteria $(N_\alpha \; and \; N_{\Delta W_r})$

The re-analysis of the experimental data allows to conclude that the assessment of the slope $(-1/m)$ of the $S-N$ lines is strictly dependent on the definition of the parameter S. In fact, if the parameter S is based on the plastic range, like in Krawinkler and Zohrei and Feldmann et al. Proposals, a value of $m=2$ applies, while a value of $m=3$ is assessed if the total excursion (i.e. elastic and plastic) is used, as proposed by Ballio and Cestiglioni (1995), and Bernuzzi et al. (1997).

Furthermore, a design approach for the appraisal of the ductility of steel components under seismic loading, which is based on the Miner's linear damage accumulation rule, is presented, discussed and applied to data of steel components tested under variable amplitude loading histories.

ACKNOWLEDGEMENTS

This research was supported by grants from the Junta Nacional de Investigação Científica e Tecnológica (JNICT), from the Ministero dell'Università e della Ricerca Scientifica e Tecnologia (M.U.R.S.T) Co-financed Research Program 1997, and from the Consiglio Nazionale delle Ricerce (C.N.R.).

REFERENCES

1. Ballio, G., Calado, L. and Castiglioni, C.A. (1997). "Low Cycle Fatigue Behaviour of Structural Steel Members and Connections", Fatigue and Fracture of Engineering Materials and Structures, Vol. 20, No. 8, pp. 1129-1147.
2. Ballio, G. and Castiglioni, C.A. (1994). "Seismic Behaviour of Steel Sections", Journal of Constructional Steel Research, Vol. 29, pp. 21-54.
3. Ballio, G. and Castiglioni, C.A. (1995). "A Unified Approach for the Design of Steel Structures under Low and High Cycle Fatigue", Journal of Constructional Steel Research, Vol. 34, pp. 75-101.
4. Bannantine J., Comer, J. and Handrock, J. (1989). "Fundamentals of Metal Fatigue Analysis", Prentice Hall.
5. Bernuzzi, C., Calado, L. and Castiglioni, C.A. (1997). "Ductility and Load Carrying Capacity Prediction of Steel Beam-to-Column Connections under Cyclic Reversal Loading", Journal of Earthquake Engineering, Vol. 1, No. 2, pp. 401-432.
6. Bertero, V., Anderson, J. and Krawinkler, H. (1994). "Performance of Steel Buildings Structures during the Northridge Earthquake", Report UCB/EERC-94/09, University of California at Berkeley, U.S.A.
7. Calado, L. and Azevedo, J. (1989). "A Model for Predicting Failure of Structural Steel Elements", Journal of Constructional Steel Research, Vol. 14, pp. 41-64.
8. Calado, L. and Castiglioni, C.A. (1995). "Low Cycle Fatigue Testing of Semi-Rigid Beam-to-Column Connections", Proc. of 3rd International Workshop on Connections in Steel Structures, Trento, pp. 371-380.
9. Castiglioni, C.A., Bernuzzi, C., Calado, L. and Agatino, M.R. (1997). "Experimental Study on Steel Beam-to-Column Joints under Cyclic Reversal Loading", Proc. of the Northridge Earthquake Research Conference, Los Angeles, pp. 526-533.

10. Castiglioni, C.A. (1999). "Failure Criteria and Cumulative Damage Models for Steel Components under Low-Cycle Fatigue", Proc. of the XVII C.T.A., Naples, pp. 237-246.
11. Castiglioni, C.A., Mulas, M.G. and Zambrano, A. (1999). "Assessment of q-factors for Steel Frames, Based on Damage Accumulation Criteria", Proc. of the XVII C.T.A., Naples, pp. 257-268.
12. CEN (1994). "ENV-1993-1-1 Eurocode 3 - Design of Steel Structures - Part 1: General Rules and Rules for Buildings", European Committee for Standardization.
13. Chai, Y., Romstad, K. and Bird, S. (1995). "Energy-Based Linear Damage Model for High-Intensity Seismic Loading", Journal of Structural Engineering, Vol. 121, No. 5, pp. 857-864.
14. Elnashai, A. and Elghazouli, A. (1994). "Seismic Behaviour of Semi-Rigid Steel Frames", Journal of Constructional Steel Research, Vol. 29, pp. 149-174.
15 International Welding Institute (1994). "JWG XIII-XV Fatigue Recommendations", Doc. XIII-1539-94/XV-845-94.
16. Krawinkler, H. and Zohrei, M. (1983). "Cumulative Damage Model in Steel Structures Subjected to Earthquake Ground Motion", Computers and Structures, Vol. 16, No. 1-4, pp. 531-541.
17. Miner, M.A. (1945). "Cumulative Damage in Fatigue", Trans. ASME, Journal of Applied Mechanics, Vol. 67, pp. A159-A164.
18. Nader, M. and Astaneh, A. (1991). "Dynamic Behaviour of Flexible, Semi-Rigid and Rigid Steel Frames", Journal of Constructional Steel Research, Vol. 18, pp. 179-192.
19. Plumier, A. (1994). "Behaviour of Connections", Journal of Constructional Steel Research, Vol. 38, No. 2, pp. 95-123.
20. Sedlacek, G., Feldmann, M. and Weynand, K. (1995). "Safety Consideration of Annex J of Eurocode 3", Proc. of 3rd International Workshop on Connections in Steel Structures, Trento, pp. 453-462.