

## **A PERFORMANCE-RELIABILITY-BASED CRITERION FOR THE OPTIMUM DESIGN OF BRIDGE ISOLATORS**

Giuseppe C. Marano and Rita Greco

Department of Civil Engineering and Architecture Science  
Polytechnic University of Bari, via Orabona  
4-70126-Bari, Italy

### **ABSTRACT**

This paper concerns a performance-reliability based criterion for the optimum design of bridge isolators. The meaning of "optimum design" concerns a structure designed in order to satisfy several performance requirements regarding the safety and the serviceability. In order to carry out this procedure a stochastic approach is developed and a Gaussian, zero-mean, filtered, nonstationary stochastic process, is adopted in order to model the earthquake motion. The hysteretic Bouc-Wen model is employed in order to reproduce the nonlinear constitutive behaviour of isolators, whereas a linear law is assumed in order to represent the piers. Covariance response is attained by means of the approximate stochastic linearization method and the system reliability, required in order to make explicit the safety and the serviceability performance objectives, is evaluated with the hypothesis of independent crossings. Finally, the optimum design, performed in two different phases, is carried out in a parametric form.

**KEYWORDS:** Bridge Isolation, Stochastic Process, Hysteretic Model, Equivalent Linearization, Performance-Based Seismic Design

### **INTRODUCTION**

Seismic isolation is a modern design approach intended for reducing destructive effects on structures caused by strong earthquakes. In bridges this method has the most important objective to protect low mass elements, such as the piers and the foundations, from the high inertia forces transmitted from the heavy mass of the deck. The technique is realized in a simple way by replacing conventional devices adopted to accommodate thermal movements with the isolation systems. In this way, seismic isolation acts by reducing the seismic forces and as a result, the bridge piers survive even under strong earthquakes.

Nowadays for the seismic protection of bridges various devices are available: Rubber Bearings (RB), Lead Rubber Bearings (LRB), High Damping Rubber Bearings (HDRB) and others, such as the Friction Pendulum (FP). The most important feature of these isolators is to supply, in a single combined element, vertical support, lateral flexibility, restoring force and dissipation energy capacity. An extensive review of the developments and recent applications has been provided by Symans et al. (1999).

The main objective of using the isolation technique is to reduce the seismic forces to (or near) the elastic limit capacity of structural elements so as to avoid (or limit) inelastic deformations and related damage phenomenon. In bridges, by using seismic isolation, shear forces transmitted from the superstructure to the piers are reduced by shifting the natural period of the bridge away from the frequency range where the energy content of earthquakes is high (this objective is achieved when RB, HDRB or LRB are used). As a result of employing the isolation strategy, the superstructure motion is decoupled from the piers motion during the earthquake, thus producing an effect of the reduction of inertia forces. At the same time, the seismic energy demand of the bridge is also reduced as a consequence of dissipation energy concentrated in isolators that are suitably designed for this purpose.

The RB isolator is realized by alternating rubber layers and steel plates vulcanised; the main characteristic of this device, whose constitutive law is approximately of a linear type, consists of increasing the natural period of the protected system. However, the damping, mainly of a viscous kind, is small. Higher dissipative capacities can be attained by using the HDRB. At present, this isolator is considered as one of the most attractive tool in passive seismic protection. The main characteristic of this device is to supply high dissipative capacity. Another significant attribute of HDRB is represented by a shear stiffness related to the shear deformation level. The stiffness, in fact, is high for low deformations (10%-20%), protecting the structure from undesirable displacements caused by low-intensity earthquakes

and wind loads. For higher deformations, the shear stiffness decreases and, therefore, the device acts by disconnecting the bridge deck motion from the pier motion. Finally, for deformations larger than 120%-150%, shear stiffness increases, but this deformation level is undesirable and design criteria could prevent it.

By placing a lead core in a RB isolator one can obtain the LRB, also named 'New Zealand' isolator (NZ): this is characterized by horizontal flexibility, energy dissipation and vertical load capacity. In fact, rubber provides the lateral flexibility to elongate the natural period of the structure whereas steel plates provide a high vertical support and confine the lead core. The latter produces a high dissipative capacity and, at the same time, it is able to support against wind loads and small or moderate earthquakes. The force-displacement relationship of HDRB and NZ can be well represented by using the differential Bouc-Wen model which is described by means of five analytical parameters governing the force-deformation cycle shape.

In seismic design of isolated structures it is important to decide with regard to the most appropriate mechanical characteristics of devices. To achieve this, the design variables for the problem must first be defined and these are: the isolator initial elastic stiffness, the isolator damping, the isolator elastic limit displacement and the post-elastic stiffness. The main goal of this study is the evaluation of optimum mechanical parameters of isolators utilized in seismic protection of bridges. Their mechanical characteristics will be designed in order to guarantee safety and serviceability objectives, and to maximize their performances in protecting bridges against destructive effects caused by strong earthquakes.

The study presented here refers, in particular, to isolators whose mechanical behaviour could be well described by a smoothed bilinear law, as in case of HDRB and LRB (Figure 1) that can be modelled by using the hysteretic nonlinear Bouc-Wen model (Bouc, 1967; Wen, 1976).

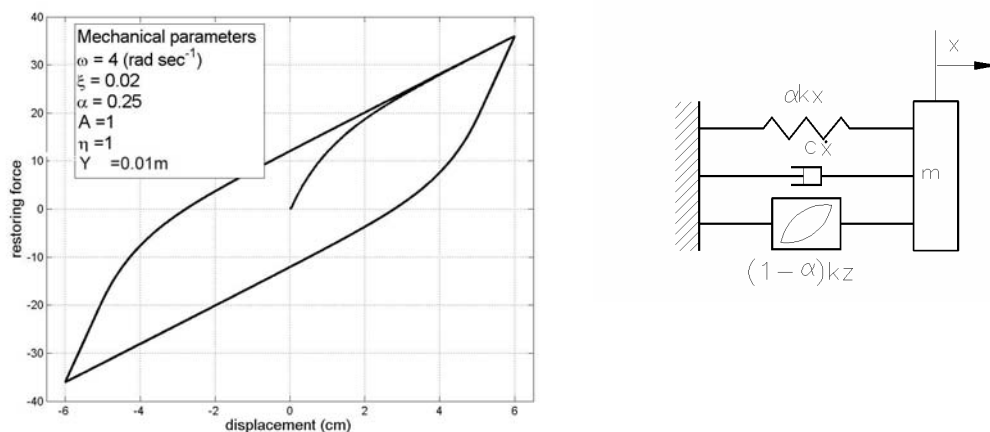


Fig. 1 Constitutive behaviour for a HDRB and its model

The proposed method, in agreement with current technical codes in the field, is based on the new seismic design philosophy called 'performance-based seismic design'. For this reason, two different excitation levels will be considered. The first one is a minor earthquake which can occur several times during the life-time of the structure. For this load condition, the performance objective requires no damage and the bridge and the isolators only suffer small stresses, smaller than their elastic limit capacity. In this situation, both the bridge and the isolators will be modelled by means of a visco-elastic constitutive law.

The second excitation level is a severe earthquake which has a low occurrence probability during the life-time of the structure. For bridges, which represent 'essential facilities', it is necessary that these remain operational and therefore even for a severe earthquake the damage should be limited or prevented (in relation to the bridge importance). Consequently, inelastic deformations of the piers should be avoided, whereas the hysteretic response of the isolators guarantees ductile behaviour and energy dissipation. For a severe earthquake, a nonlinear behaviour is adopted in order to reproduce the constitutive law of the isolators, whereas the pier is assumed linear as a result of the reduction of seismic forces produced by using the seismic isolation technique.

In order to carry out this procedure, a stochastic approach is implemented and a Gaussian, zero-mean, filtered, nonstationary stochastic process, is adopted in order to model the earthquake acting at the bridge

foundation. The nonlinear hysteretic Bouc-Wen model is assumed to represent the constitutive behaviour of the isolation devices under the severe loading, whereas a linear law is always supposed to model the pier-deck structure. The nonlinear stochastic problem is solved by means of the stochastic linearization method in order to achieve the system response covariance. In order to make explicit the safety and the serviceability objectives, which are defined in terms of limit state crossing probability, it is necessary to define the system reliability. Finally, the optimum design, performed in two different phases, is carried out in a parametric form.

**THE DYNAMIC MODEL OF THE ISOLATED BRIDGE**

In this section the dynamic model of the isolated bridge, adopted in order to perform the main objective of the study, is discussed (Figure 2).

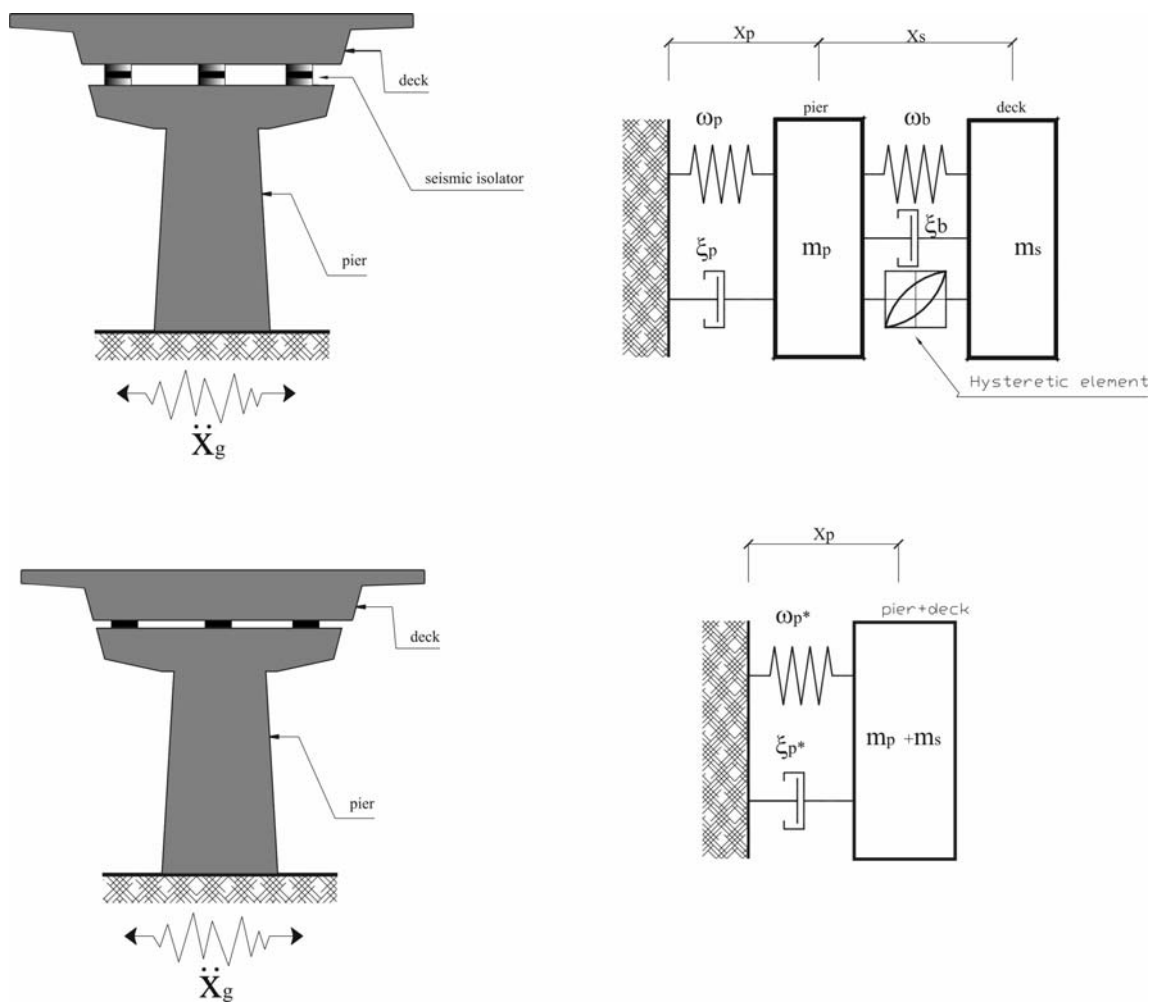


Fig. 2 Dynamic model of an isolated and a conventional bridge

Analysis of the seismic behaviour of bridges is a very complicated problem involving not only structural modelling of pier-deck elements, but also several other aspects — like soil-structure interaction (which considers the actual behaviour of soil foundation), multi-support seismic excitation (related to the difference of seismic input observed at different piers supports, especially in long bridges), and vertical oscillations (which are secondary occurrences observed in bridges under seismic excitations). All these phenomena, which have been extensively analysed (Monti and Pinto, 1998; Thakkar and Maheshwari, 1995), can significantly influence the bridge response for both conventional as well as isolated ones, but with stochastic seismic analysis becoming considerably complicated. Therefore, it is necessary to establish some assumptions in order to achieve, in a simple and reasonably accurate way, the main objective of this study — a preliminary design of isolator mechanical characteristics on the basis of some performance requirements based on earthquake severity. Therefore, the analytical model to be employed

should be simple, and at the same time, be able to capture the essential features of isolated bridge response. With this scope, a simplified 2 degrees-of-freedom system, as analysed below in detail, is adopted in order to develop the stochastic analysis of the isolated bridges under the following assumptions:

- the deck superstructure is assumed to move as a rigid body;
- the pier is assumed to have a linear behaviour. This is a reasonable assumption, since the isolation technique attempts to reduce the earthquake response in such a way that the pier remains within the elastic range. As will be shown later, this is more than a simplified assumption because one of the performance objectives involved in the design proposed here will require that the pier should remain within its elastic limit;
- the pier is assumed to vibrate in its first mode (this assumption is quite rational for regular bridges);
- the effects of the incoherence of support motion may be ignored;
- the effects of soil-structure interaction are ignored and the vertical motion is not considered herein.

Since in the isolation of bridges, the devices are located between the piers and the deck, it is possible, under the previous assumptions, to represent the structural system by means of a two degrees-of-freedom system, having masses  $m_p$  and  $m_s$ , which are respectively the mass of the pier and the mass of the deck. The pier, as established in the hypothesis, is assumed linear as a consequence of adopting the isolation technique, and can be well represented in its first vibration mode by means of the first natural frequency  $\omega_p = \sqrt{k_p / m_p}$ , and the damping coefficient  $\xi_p = c_p / 2\omega_p m_p$ , where  $k_p$  and  $c_p$  are the pier stiffness and viscous damping respectively. The deck, whose behaviour is assumed rigid, can be well-modelled through a concentrated mass  $m_s$ , positioned on the devices. The isolator elastic frequency  $\omega_b$  and the damping coefficient  $\xi_b$  are defined as  $\omega_b = \sqrt{k_b / m_s}$  and  $\xi_b = c_b / 2\omega_b m_s$ , where  $k_b$  and  $c_b$  are the elastic stiffness and damping of isolators, governing the elastic phase behaviour.

The conventional bridge, whose response is necessary in order to evaluate the isolation performance, is represented by means of a concentrated mass  $m_p + m_s$ , a frequency  $\omega_p^*$  and a damping coefficient  $\xi_p^*$ , related to the previous cited quantities through the relations,  $\omega_p^* = \omega_p / \sqrt{1 + \mu}$  and  $\xi_p^* = \xi_p / \sqrt{1 + \mu}$ , where  $\mu = m_s / m_p$  is the mass ratio.

The nonlinear dissipative Bouc-Wen model (BWM) is adopted in order to represent the dynamic behaviour of isolators under a severe earthquake. It is a single-degree-of-freedom nonlinear system having a mass  $m$ , whose nonlinear restoring force is expressed as

$$Q(x, \dot{x}, z) = c\dot{x} + \alpha kx + (1 - \alpha)kz \quad (1)$$

where  $x$  is the nonlinear oscillator displacement,  $c$  and  $k$  are the system damping and the elastic-initial stiffness respectively,  $z$  is an internal variable governing the hysteretic behavior and satisfying the differential equation:

$$\dot{z} = -\gamma |\dot{x}| |z|^{\eta-1} z - \beta \dot{x} |z|^\eta + A \dot{x} \quad (2)$$

The five parameters  $\beta$ ,  $\gamma$ ,  $\eta$ ,  $\alpha$  and  $A$  which appear in Equations (1) and (2) control the shape of the hysteretic force-deformation cycle and are well described by Cunha (1984). In detail,  $\eta$  is a natural number that controls the transition from the elastic to the post-elastic phase. When this parameter approaches infinity, the constitutive law becomes bilinear. The parameter  $\alpha$  is the ratio between the plastic phase stiffness  $k_f$  and the elastic initial stiffness  $k_i$  (this particular result is true for  $A = 1$ ), whereas  $\beta$  controls the nature of the constitutive law (hardening or softening). Wong et al. (1994) showed that  $\beta$  and  $\gamma$  determine the elastic limit displacement  $Y$  through the relation  $Y = 1 / (\beta + \gamma)$  (for  $\eta = 1$  and  $A = 1$ ).

## RESPONSE EVALUATION

In this section, the response of the isolated bridge subject to a seismic motion represented through a stochastic process will be assessed. In detail, the time-modulated Kanai-Tajimi process (Tajimi, 1960) is adopted in order to represent the ground motion at the bridge foundation. The nonstationary time-

modulated Kanai-Tajimi process is obtained by multiplying the stationary Kanai-Tajimi process with a time modulating function  $V(t)$ :  $\ddot{x}_g = \bar{\ddot{x}}_g V(t)$  where  $\bar{\ddot{x}}_g$  is the stationary Kanai-Tajimi process. In this study the exponential modulation function  $V(t) = \alpha_v t e^{-\beta_v t}$  (Figure 3) is adopted.

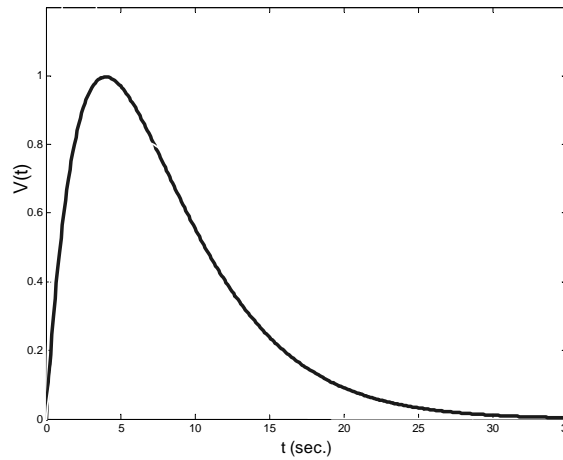


Fig. 3 Exponential modulation function  $V(t)$

The equations of motion of the isolated bridge, subjected to a seismic motion, represented by the time-modulated Kanai-Tajimi process are:

$$\begin{cases} m_p \ddot{x}_p + c_p \dot{x}_p - c_b (\dot{x}_s - \dot{x}_p) + k_p x_p - \alpha_b k_b (x_s - x_p) - (1 - \alpha_b) k_b z_b = -m_p \ddot{x}_g \\ m_s \ddot{x}_s + c_b (\dot{x}_s - \dot{x}_p) + \alpha_b k_b (x_s - x_p) + (1 - \alpha_b) k_b z_b = -m_s \ddot{x}_g \\ \dot{z}_b = -\gamma_b |(\dot{x}_s - \dot{x}_p)| |z_b|^{\eta_b - 1} z_b - \beta_b (\dot{x}_s - \dot{x}_p) |z_b|^{\eta_b} + A_b (\dot{x}_s - \dot{x}_p) \\ \ddot{x}_g = \ddot{x}_f + V(t)w = -2\xi_g \omega_g \dot{x}_f - \omega_g^2 x_f \end{cases} \quad (3)$$

where  $x_p$  and  $x_s$  are, respectively, the displacements of the top of the pier and of the deck relative to the ground.

The equations of motion have been written with the consideration that the isolator has a nonlinear behaviour. This is a more general hypothesis that can be particularized when the isolator exhibits a linear behaviour under low-intensity earthquakes. The parameters  $\beta_b$ ,  $\gamma_b$ ,  $\eta_b$ ,  $\alpha_b$  and  $A_b$ , which rule the shape of the isolator hysteretic cycle, have been introduced. Furthermore, in Equation (3) the equation of motion of the filter appears, where  $x_f$  is the response of the filter representing the ground, characterised by a frequency  $\omega_g$  and a damping coefficient  $\xi_g$ . Finally,  $w$  is the white noise excitation process at the bed rock and  $V(t)$  is the modulation function.

The dynamic problem formulated above is nonlinear because the seismic isolators have a nonlinear behaviour. In this study, the ‘stochastic linearization method’ is adopted in order to perform the analysis. The fundamental idea of this approximate technique is that the equation describing the nonlinear system can be substituted by a linear one that is equivalent in stochastic terms. Moreover, the hypothesis of a Gaussian response process should be assumed. The approximate linearized form of the original nonlinear equation is achieved by minimising, in a stochastic way, the difference between the nonlinear equation and the linearized one (Roberts and Spanos, 1990). Thus, the nonlinear equation governing the internal variable  $z_b$  is replaced by the linear one that is equivalent in stochastic meaning:

$$\dot{z}_b = -c_b^e (\dot{x}_s - \dot{x}_p) - k_b^e z_b \quad (4)$$

For  $A_b = 1$  and  $\eta_b = 1$ , with the hypothesis of variables  $z_b$  and  $\dot{u}_s = \dot{x}_s - \dot{x}_p$  being jointly Gaussian, Atalik and Utku (1976) provided the equivalent coefficients  $c_b^e$  and  $k_b^e$ :

$$\begin{cases} c_b^e = \sqrt{\frac{2}{\pi}} \left[ \beta_b \sigma_{z_b} + \gamma_b \frac{E[\dot{u}_s z_b]}{\sigma_{\dot{u}_s}} \right] - A_b \\ k_b^e = \sqrt{\frac{2}{\pi}} \left[ \gamma_b \sigma_{\dot{u}_s} + \beta_b \frac{E[\dot{u}_s z_b]}{\sigma_{z_b}} \right] \end{cases} \quad (5)$$

where  $\sigma_{z_b}$  and  $\sigma_{\dot{u}_s}$  are, respectively, the standard deviations of variables  $z_b$  and  $\dot{u}_s$  and  $E[\dot{u}_s z_b]$  is their covariance.

In matrix form the linearized equations of motion are converted into:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{q}(1 - \alpha_b)k_b z_b = -\mathbf{M}\mathbf{r}\ddot{x}_g \\ \dot{z}_b = -c_b^e \dot{u}_s - k_b^e z_b \\ \ddot{x}_g = \ddot{x}_f + wV(t) = -2\xi_g \omega_g \dot{x}_f - \omega_g^2 x_f \end{cases} \quad (6)$$

where the linearization coefficients  $k_b^e$  and  $c_b^e$  appear, and where  $\mathbf{M} = \begin{bmatrix} m_p & 0 \\ 0 & m_s \end{bmatrix}$ ;

$\mathbf{C} = \begin{bmatrix} c_p + c_b & -c_b \\ -c_b & c_b \end{bmatrix}$ ;  $\mathbf{K} = \begin{bmatrix} k_p + \alpha k_b & -\alpha k_b \\ -\alpha k_b & \alpha k_b \end{bmatrix}$ ;  $\mathbf{x} = \begin{Bmatrix} x_p \\ x_s \end{Bmatrix}$ ;  $\mathbf{q} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$ ;  $\mathbf{r} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ . By introducing the

coordinate change,  $\mathbf{x} = \mathbf{T}\mathbf{u}$  where  $\mathbf{u} = \{u_p \ u_s\}^T$  is the vector containing the pier displacement relative to the ground and the superstructure displacement relative to the pier, which corresponds with the isolator

displacement, and  $\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$  is the coordinate change matrix, Equation (6) can be written as:

$$\begin{cases} \mathbf{M}^* \ddot{\mathbf{u}} + \mathbf{C}^* \dot{\mathbf{u}} + \mathbf{K}^* \mathbf{u} + \mathbf{q}(1 - \alpha_b)k_b z_b = -\mathbf{M}^* \mathbf{r}^* \ddot{x}_g \\ \dot{z}_b = -c_b^e \dot{u}_s - k_b^e z_b \\ \ddot{x}_g = \ddot{x}_f + wV(t) = -2\xi_g \omega_g \dot{x}_f - \omega_g^2 x_f \end{cases} \quad (7)$$

where new matrices and vectors in the new coordinates appear as  $\mathbf{M}^* = \mathbf{M}\mathbf{T} = \begin{bmatrix} m_p & 0 \\ m_s & m_s \end{bmatrix}$ ;

$\mathbf{C}^* = \mathbf{C}\mathbf{T} = \begin{bmatrix} c_p & -c_b \\ 0 & c_b \end{bmatrix}$ ;  $\mathbf{K}^* = \mathbf{K}\mathbf{T} = \begin{bmatrix} k_p & -\alpha k_b \\ 0 & \alpha k_b \end{bmatrix}$ ;  $\mathbf{r}^* = \mathbf{r}\mathbf{T}^{-1} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ .

After the 'state vector'  $\mathbf{Y} = \{\mathbf{u} \ x_f \ z_b \ \dot{\mathbf{u}} \ \dot{x}_f\}^T$  is introduced, it is possible to write the 'state equation' as:

$$\dot{\mathbf{Y}}(t) = \mathbf{A}^e \mathbf{Y}(t) + \mathbf{B}(t) \quad (8)$$

where  $\mathbf{B}(t)$  has all elements equal to zero except  $B_7(t) = -wV(t)$ , and  $\mathbf{A}^e$  is the 'equivalent linearized state matrix' of order 7:

$$\mathbf{A}^e = \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 1} & \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & 0 & 0 & \mathbf{0}_{1 \times 2} & 1 \\ \mathbf{0}_{1 \times 2} & 0 & -k_b^e & \mathbf{C}_{1 \times 2}^e & 0 \\ -\mathbf{M}^{*-1} \mathbf{K}^* & \mathbf{r}^* \omega_g^2 & -\mathbf{M}^{*-1} \mathbf{q}(1 - \alpha_b)k_b & -\mathbf{M}^{*-1} \mathbf{C}^* & \mathbf{r}^* 2\xi_g \omega_g \\ \mathbf{0}_{1 \times 2} & -\omega_g^2 & 0 & \mathbf{0}_{1 \times 2} & -2\xi_g \omega_g \end{bmatrix} \quad (9)$$

It may be seen that the zero vectors, the zero matrices and the identity matrices appear in the matrix in Equation (9) and that  $\mathbf{C}_{1 \times 2}^e = \{-c_b^e \ 0\}$ .

Now, the stochastic response can be obtained by solving the differential equation of the covariance matrix  $\mathbf{Q}_{\mathbf{Y}\mathbf{Y}}(t)$ , whose time-varying elements are the second order moments  $E[Y_i Y_j]$  relative to the state vector  $\mathbf{Y}$ :

$$\dot{\mathbf{Q}}_{\mathbf{Y}\mathbf{Y}}(t) = \mathbf{A}^e \mathbf{Q}_{\mathbf{Y}\mathbf{Y}}(t) + \mathbf{Q}_{\mathbf{Y}\mathbf{Y}}(t) \mathbf{A}^{eT} + \mathbf{G}(t) \tag{10}$$

Here  $\mathbf{G}(t)$  is a matrix of order 7 having all elements except  $G_{77}(t) = 2\pi S_0 V^2(t)$  equal to zero, where  $S_0$  is the white noise intensity.

Figures 4 and 5 show the variances of the pier and deck displacements, obtained by using both — the approximate linearization technique, and the Monte Carlo simulation based on 1500 synthetic acceleration time-histories. Following parameters are adopted for the stochastic ground shaking modelling and for the isolated bridge system:  $\omega_g = 20$  rad/sec,  $\xi_g = 0.6$ ,  $S_0 = 0.062$  m<sup>2</sup>/sec<sup>3</sup>,  $\omega_b = 6.28$  rad/sec,  $\omega_p = 12.56$  rad/sec,  $\xi_p = 5\%$ ,  $\xi_b = 10\%$ ,  $\mu = 4$ ,  $Y = 10$  cm,  $\beta = 0.05$ , and  $\gamma = 0.05$ , whereas  $\alpha$  assumes the values 0.3, 0.6 and 0.9. The results show a relatively good agreement between the stochastic response of the linearized system and that of the original nonlinear one as obtained by means of simulation. However, in some conditions the linearized response is underestimated and this occurs especially in the isolator displacement when low values of the parameter  $\alpha$  are adopted.

**PROBABILISTIC PERFORMANCE-RELIABILITY OPTIMUM DESIGN OF BRIDGE ISOLATORS**

The stochastic nature of natural hazards and accidental loads and the nondeterministic character of material properties make it desirable to use probabilistic methodologies in order to provide, in a rational way, the assessment of structural performance and safety.

In this paper a probabilistic performance-reliability-based criterion for the optimum design of isolators used for the seismic protection of bridges is proposed. The meaning of optimum design involves design of a structure in order to satisfy a set of requirements concerning performance objectives that are related to system ‘safety’ and ‘serviceability’. In fact, structural seismic design should be performed not only to guarantee the life safety and to prevent structural collapse, but also to control the level of damage and the behaviour of components and systems.

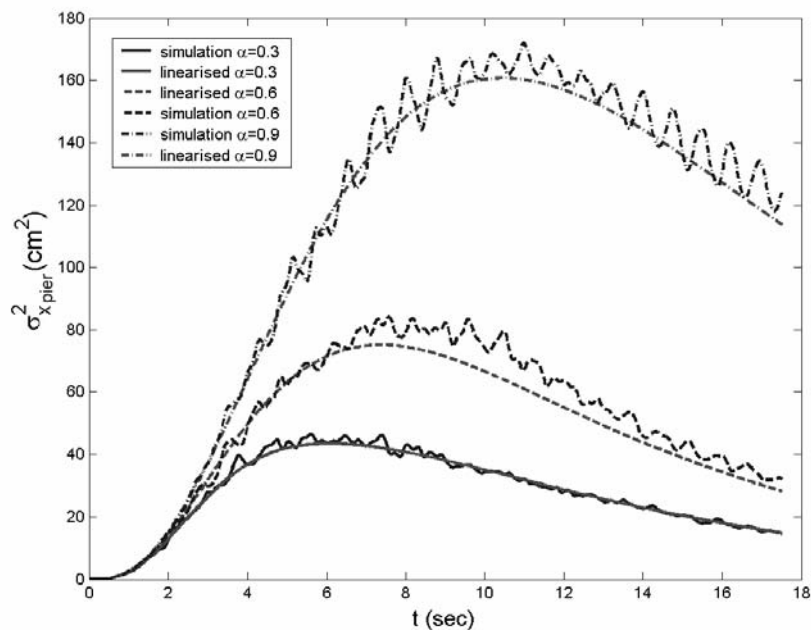


Fig. 4 Simulated and linearized pier response variances

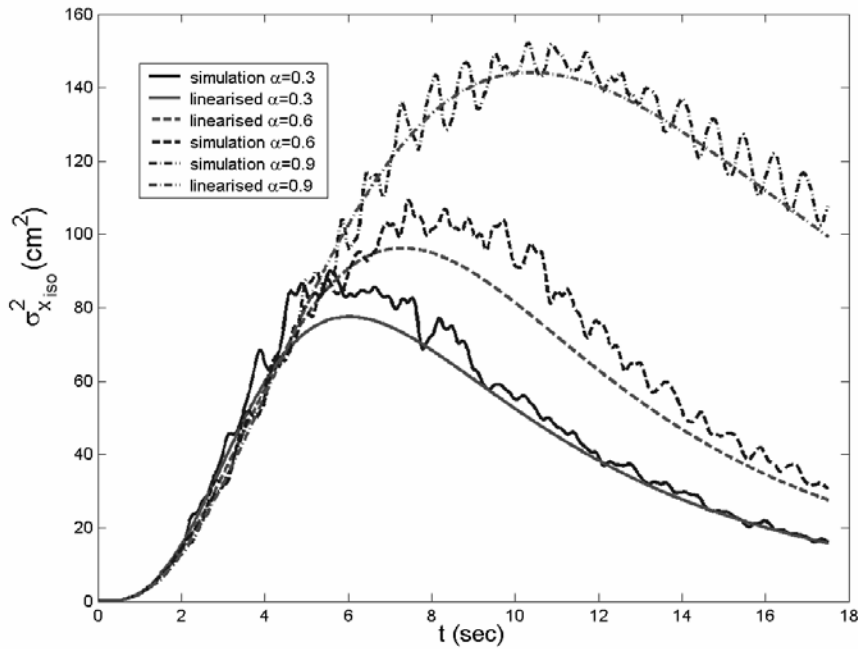


Fig. 5 Simulated and linearized isolator response variances

On the basis of these ideas a new design philosophy has been under development in recent years. This is known as the ‘performance-based design’ (SEAOC, 1995) which can be defined as a design to reliably achieve targeted performance objectives. Performance-based design is a general design philosophy in which the design criteria are expressed in terms of achieving stated performance objectives when the structure is subjected to stated levels of seismic hazard. It is possible to summarize that this seismic design philosophy accomplishes the following objectives:

1. To prevent nonstructural damage during minor earthquakes, which may occur frequently during the service life of the structure;
2. To prevent structural damage and minimize nonstructural damage during moderate earthquake ground shakings, which may occasionally occur;
3. To avoid collapse or serious damage during severe earthquake ground shakings, which may rarely occur.

Therefore, the meaning of this design philosophy is firstly, the definition of the earthquake probability and secondly, the performance objectives. These are summarised in the performance matrix, reproduced below (Table 1) which considers four levels of earthquake hazard (SEAOC, 1995).

Table 1: Earthquake Probability and Performance Objectives (SEAOC, 1995)

		Performance Objective			
		Fully operational	Operational	Life safe	Near collapse
Earthquake Probability	Frequent	■	Unacceptable Performance		
	Occasional	●	■		
	Rare	■	●	■	
	Very rare		■	●	■

Basic facilities

Essential or hazardous facilities

Safety critical facilities



Also, specific seismic codes for bridges developed around the world (TNZ, 1995; JRA, 2002; AASHTO, 1996; ATC, 1996; CEN, 1994) are based on ‘performance expectations’, which are related to different earthquake levels expressed by means of the occurrence probability.

Table 2 shows recommended ‘Seismic Performance Criteria’ (ATC, 1996) for a two-level design approach, which considers two service levels and three damage levels for bridge design, and these can be different for ordinary and important bridges.

**Table 2: Recommended Seismic Performance Criteria (ATC, 1996)**

Ground Motion at Site	Ordinary Bridges	Important Bridges
<b>Functional-Evaluation</b>	<b>Service Level-Immediate</b>	<b>Service Level-Immediate</b>
<b>Ground Motion</b>	<b>Damage Level- Repairable Damage</b>	<b>Damage Level- Minimal Damage</b>
<b>Safety-Evaluation</b>	<b>Service Level-Limited</b>	<b>Service Level-Immediate</b>
<b>Ground Motion</b>	<b>Damage Level- Significant Damage</b>	<b>Damage Level - Repairable Damage</b>

On the basis of this (current) philosophy used in seismic design practice, this section develops a method intended for carrying out the probabilistic optimum design of isolation devices for seismic protection of bridges. The design is for two earthquake levels — the minor earthquakes and the severe earthquakes (“very severe” ground motions are not included in this analysis).

After the earthquake hazard levels are stated, the next step is the description of the required performance objectives that, as explained before, should be fixed on the basis of the earthquake severity. For each hazard level a limit state will be established, defined as a state where the structure attains an undesirable structural behaviour.

Commonly, isolated structures are designed in accordance with the following requirements: to withstand minor and moderate earthquakes without damage to structural elements, nonstructural components, and contents; and to withstand severe earthquake ground motion without failure of the isolation system, without significant damage to structural elements (or without damage in relation to structure’s importance), without extensive damage to nonstructural components, without major disruption to the facility function, and subsequently without loss of life.

In conformity with the requirements previously explained, the optimum design method is developed in the following way.

**1. Minor Earthquake**

This is a frequent earthquake having a 50% probability of occurrence in 50 years. For this, the following objectives are established:

“The displacement of the top of the pier must be smaller than the elastic limit displacement. The isolator must remain in the elastic range in order to avoid large relative displacements of the superstructure with respect to the pier, as required for service loads, and to avoid plastic displacements that can reduce the isolator capacity under the severe earthquake.”

The design variables in this phase are the initial elastic stiffness  $k_b$ , the isolator damping  $c_b$ , and the isolator elastic limit displacement  $Y$ . More precisely, the first one is expressed by means of the frequency ratio  $I = \omega_b / \omega_p$ , that is the ratio between the initial elastic isolator frequency and pier frequency.

Concerning the damping coefficient  $\xi_b$ , previous studies (Greco et al., 2002) have verified that the optimum value  $\xi_{b\_opt}$ , for the usual range ( $5\% \leq \xi_b \leq 15\%$ ) which characterizes these isolators, always coincides with the highest value and, therefore, this parameter will not be designed. Instead, it will be recognised as a data problem.

Isolators’ performance is represented by means of two serviceability limit state probabilities: first, the probability that the displacement of the top of the pier crosses the pier elastic limit displacement  $X_{p\_e}$ , and secondly, the probability that the maximum isolator displacement crosses the elastic limit displacement  $Y$ .

The first requirement is expressed in terms of the pier reliability, i.e. the probability that the displacement  $X_p$  of the top of the pier does not cross the threshold elastic level  $X_{p\_e}$ :

$$P_S(t, X_{p\_e}) = P\left[|X_p(t)| \leq X_{p\_e} \quad \forall \quad 0 \leq t \leq \tau\right] \geq \bar{P} \quad (11)$$

where  $\tau$  is the earthquake duration and  $\bar{P}$  is the reliability target fixed for this limit state. In this study the reliability is determined with the hypothesis of independent threshold crossings having a Poisson distribution (Nigam, 1983):

$$P_S(t, \xi) = \exp\left\{-\int_0^t \alpha(t) dt\right\} = \exp\left\{-\int_0^t 2\nu_{\xi^+}(t) dt\right\} \quad (12)$$

where

$$\nu_{\xi^+}(t) = \nu_{0^+}(t) \exp\left[-\frac{\xi^2}{2\sigma_x^2(t)}\right] \quad \text{and} \quad \nu_{0^+}(t) = \frac{1}{2\pi} \frac{\sigma_{\dot{x}}(t)}{\sigma_x(t)} \quad (13)$$

and  $\xi$  is a generic threshold level.

The use of this approach for the reliability evaluation has some restrictions which are related both to the Poisson distribution hypothesis and to the Gaussian joint probability distribution assumption (for the displacement and the velocity processes) that is necessary in order to perform the stochastic linearization technique adopted here.

The assumption of independent barrier crossings of the random process  $X(t)$  is quite adequate in the analysis developed here. Indeed, it is well known that this hypothesis can be quite poor and excessively conservative when clumping effects occur in barrier crossings (for a more accurate analysis different and more complicated approaches can be used, as that proposed by Vanmarcke (1972) based on the use of envelope process). Moreover, it has been verified that the Poisson distribution hypothesis is a valid assumption for high threshold levels and so it could be used in the present analysis due to the fact that reliability levels investigated here are high (the minimum reliability target considered is  $1 \times 10^{-2}$ ) and therefore the barrier up-crossing events are effectively independent.

The second limitation in the reliability evaluation developed here is related to the actual nonlinear structural behaviour. In fact, the stochastic linearization technique, adopted here in order to estimate the system response covariance is able to provide, with a good agreement, the mean and the variance of the original nonlinear process. However, it is not able to give, with the same accuracy, information about the response probability distribution. Moreover, information on the tails of the joint probability distribution function, which is an essential element in the mean rate crossing evaluation of the exact Rice formulation (1944), can be affected by severe mistakes that could induce incorrect reliability results.

In spite of this problem the comparison between the approximate linearized analysis and the Monte Carlo simulation shows that for rare crossing events (when the reliability is high) there is a good agreement between the two methodologies (Figure 6).

In order to perform the optimum isolator design, Equation (11) is replaced by its inverse formulation by representing in explicit form the maximum pier displacement  $X_{p\_max}$  that has  $\bar{P}$  probability of no exceedance during the earthquake duration. Hence, the relation in Equation (11) is replaced by:

$$X_{p\_max}(t, \bar{P}) \leq X_{p\_e} \quad (14)$$

where  $X_{p\_max}(t, \bar{P})$  is the maximum pier displacement having a probability  $\bar{P}$  of no exceedance.

Similarly, for the isolator displacement it is required that  $X_{b\_max}(t, \bar{P}) \leq Y$  where  $X_{b\_max}(t, \bar{P})$  is the maximum isolator displacement having a probability  $\bar{P}$  of no exceedance.

## 2. Severe Earthquake

This is a rare earthquake having a 10% probability of occurrence in 50 years. For this the following objectives are established:

- The displacement of the top of the pier must be smaller than the elastic limit displacement. This is expressed in terms of the pier reliability formula (Equation (14)).
- The performance of the seismic isolation, i.e. the reduction of the seismic pier response, must be the best possible, and also one needs to reduce the isolator displacements.
- Moreover, it is necessary that the isolator displacement is smaller than the ultimate limit.

In an analogous manner, the last requirement is expressed as:

$$X_{b\_max}(t, \bar{P}) \leq X_{b\_u} \tag{15}$$

where  $X_{b\_max}(t, \bar{P})$  is the maximum isolator displacement having the probability  $\bar{P}$  of no exceedance and  $X_{b\_u}$  is the ultimate isolator displacement.

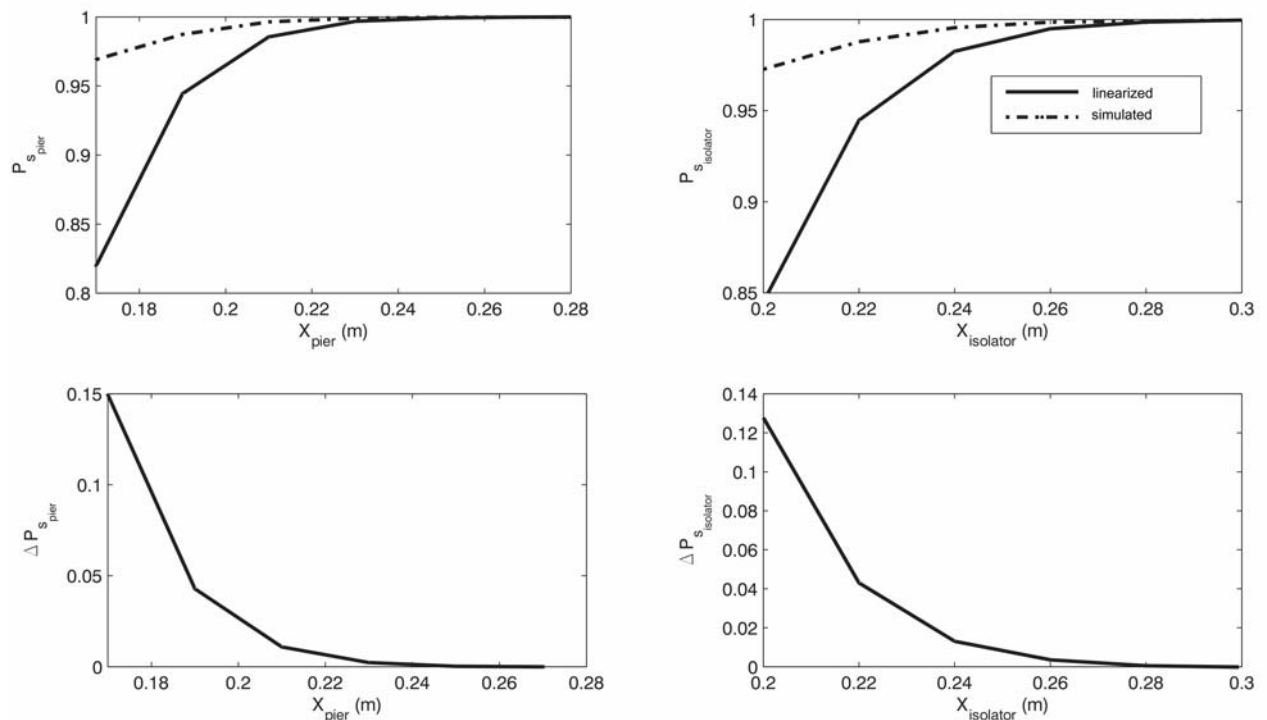


Fig. 6 Simulated and linearized pier and superstructure reliability, and corresponding error  $\Delta P_s$

For a severe earthquake, a suitable measure of the isolator performance is defined in probabilistic terms as the ratio  $\max \sigma_{x_p} / \max \sigma_{x_{p_0}}$ , of the maximum variance of the displacement of the top of the pier,  $\max \sigma_{x_p}$ , of the isolated bridge to the equivalent response,  $\max \sigma_{x_{p_0}}$ , for the conventional bridge.

The parameter characterizing the isolator behaviour under the severe earthquake loading, i.e. the post-elastic stiffness, is designed in this phase. The requirement that the pier should remain in the elastic range during a severe earthquake, without showing damage consequent to inelastic deformations, is more essential for the important bridges (as those are required to remain functional) than for the ordinary ones (for which a limited amount of damage can be accepted). It may be mentioned that current technical codes (see for example CEN(1994)) for bridges located in seismic areas authorize two different kinds of seismic behaviour and on the basis of this the performance requirements should be satisfied. More specifically, the bridges should be designed in order to show, under the design earthquake, a ductile behaviour or an “elastic behaviour with a limited ductility”. In sites where the seismic intensity is low, the latter behaviour is acceptable and the bridges can be designed in order to have an elastic behaviour without particular ductility requirements. In areas with moderate or high seismic intensity, bridges with a ductile behaviour, having a capacity to dissipate an amount of seismic energy, are preferable. This is the main objective that can be achieved both through the realization of plastic hinges, in proper locations, and by using isolation devices. Such devices are, in this situation, the sacrificial elements (ATC, 1996) and hence, it is possible to concentrate energy dissipation and, thus, the damage in those.

Of course, different performance objectives may be required for the severe ground motions according to specifications of the structure being analysed. If the performance objectives allow certain level of damage for the pier, a more suitable model needs to be adopted for the pier in order to reproduce its real nonlinear behaviour under severe earthquakes.

**DESIGN CRITERIA**

In this section, the optimum parameters ( $I_{opt}$ ,  $Y_{opt}$ ,  $\alpha_{opt}$ ) of isolators are attained by using the proposed method. The procedure is developed in two phases. In the first step, a range of optimum values for the isolator parameters governing the elastic response (i.e.,  $I_{opt}$ ,  $Y_{opt}$ ) is achieved by using the performance requirements concerning minor earthquakes. The performance objective regarding the pier response, that it should remain elastic, is utilized together with a limit  $Y_{max}$ , which is chosen as a higher value in design relative to the elastic limit displacement of isolators.

In the second step, firstly the requirements concerning the pier response for the severe earthquake are imposed for each pair of the optimum values ( $I_{opt}$ ,  $Y_{opt}$ ) as obtained in the first phase. In this way the optimum value  $\alpha_{opt}$  relative to this pair is reached. Then, by means of the safety condition for the isolator, the final optimum design parameters are obtained. This procedure is described below in detail and results obtained are represented in several graphs. The proposed procedure is also briefly shown in a schematic diagram in Figure 7.

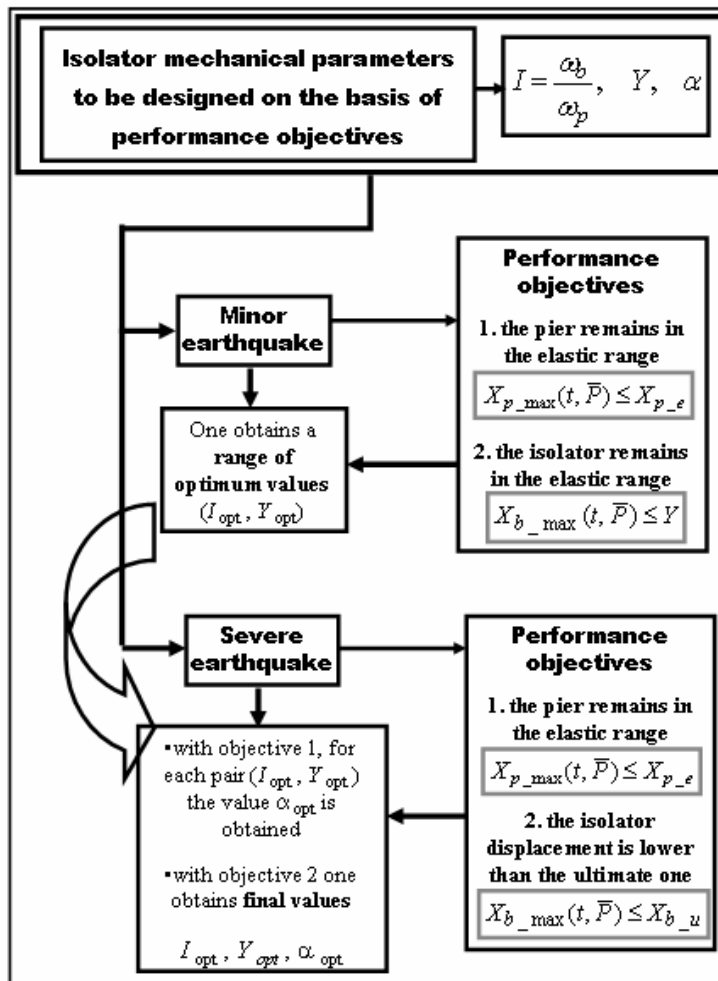


Fig. 7 Scheme of the procedure implemented in order to evaluate optimum isolator mechanical parameters

### 1. Design Criteria for the Minor Earthquake

In the first step of the proposed method, the seismic response and performance requirements for the minor earthquakes are considered (this is the elastic phase of the design). In Figure 8 the displacements  $X_{b\_max}(t, \bar{P})$  and  $X_{p\_max}(t, \bar{P})$  having a probability  $\bar{P}$  of no exceedance, are plotted against the frequency ratio  $I$ . The following parameters are adopted in the first phase of the optimum design:  $\omega_g = 22$  rad/sec,  $\xi_g = 0.42$ ,  $\omega_p = 15$  rad/sec,  $\xi_p = 0.05$ , and  $S_0 = 0.0034$  m<sup>2</sup>/sec<sup>3</sup>, corresponding to the peak ground acceleration  $\ddot{x}_{g\_max} = 0.15g$  where  $g$  is the acceleration due to gravity, as obtained by using the Kanai and Tajimi formula (Tajimi, 1960):  $S_0 = 0.141\xi_g\ddot{x}_{g\_max}^2 / \omega_g(1 + 4\xi_g^2)$ . The following values are also adopted:  $\alpha_v = 0.33$  and  $\beta_v = 0.125$ , which correspond to  $V_{max} = 1$  and  $t_{max} = 8$  sec, where  $t_{max}$  is the time when  $V(t)$  reaches  $V_{max} = 1$ . The value  $\bar{P} = 1 \times 10^{-2}$  is adopted as the target probability for the service earthquake. The performed optimum design method is described below.

Firstly the elastic limit displacement  $X_{p-e}$  of the piers is established. For example, if the value of 10 cm is fixed (this can be assigned in relation to the specific pier characteristics), from Figure 8 one can determine the point A', for which  $X_{p-e}$  is attained with the fixed target probability  $\bar{P}$ . Therefore, the corresponding value of  $I$  (see point B') identifies the optimum frequency ratio  $I_{opt}$  and point C' the corresponding elastic limit  $Y_{opt}$  of the isolators.

If another value  $X_{p-e} = 15$  cm is fixed, one can notice from Figure 8 that this is always larger than the pier displacement. In this case one identifies the points A, B and C where  $I_{opt}$  is equal to unity. At this point the problem is not unequivocally completed, because all values smaller than  $I_{opt}$  (point B), now named  $maxI_{opt}$  (with the corresponding  $Y_{opt}$  (point C) now named  $minY_{opt}$ ), are admissible, and for each value smaller than  $maxI_{opt}$ , it can define a pair of values  $I_{opt}$  and  $Y_{opt}$ .

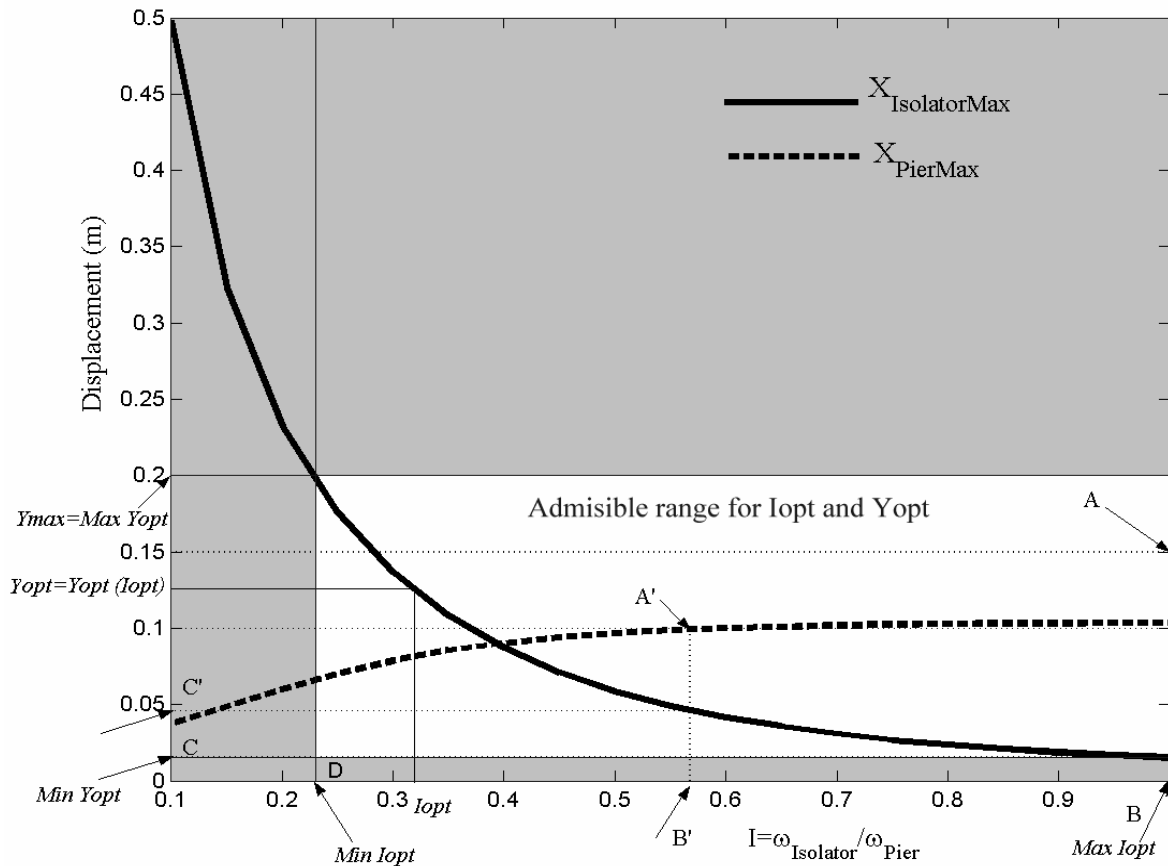


Fig. 8 Admissible range for the pair  $I_{opt}$ ,  $Y_{opt}$

Now, it is also needed to fix a limit  $Y_{max}$  for the isolator elastic displacement. In this way one can define the value  $minI_{opt}$  (point D) and, then a range of optimum values ( $I_{opt}$ ,  $Y_{opt}$ ), where  $Y_{opt} = Y_{opt}(I_{opt})$ , in

case of the service earthquake for the specified performance requirements. These pairs of optimum values will be utilised in the next step of the optimum design as shown below.

**2. Design Criteria for the Severe Earthquake**

In this sub-section, optimum design in the post-elastic phase is carried out by starting from the range of optimum pairs  $(I_{opt}, Y_{opt})$  as obtained earlier. The input for the system is the expected severe earthquake, and the following parameters are adopted:  $\omega_g = 22$  rad/sec,  $\xi_g = 0.42$  and  $S_0 = 0.0379$  m<sup>2</sup>/sec<sup>3</sup>, corresponding to the peak ground acceleration  $\ddot{x}_{g\_max} = 0.50g$ . For each pair  $(I_{opt}, Y_{opt})$  the stochastic response to this earthquake is evaluated. In Figure 9 the displacements  $X_{b\_max}(t, \bar{P})$  and  $X_{p\_max}(t, \bar{P})$  having a probability  $\bar{P} = 1 \times 10^{-3}$  of no exceedance, are plotted against the parameter  $\alpha$ .

The isolator elastic frequency is defined by  $I_{opt}$ , whereas in order to model the Bouc-Wen mechanical law, the parameters  $\gamma_{opt}$  and  $\beta_{opt}$  corresponding to  $Y_{opt}$ , where  $Y_{opt} = Y_{opt}(I_{opt})$ , are adopted. In the specific case represented in Figure 9 the pair of  $I_{opt} = 0.331$  and  $Y_{opt} = 8.5$  cm has been used. After that, since for the severe earthquake it is required that the pier displacement does not exceed the elastic limit  $X_{p-e}$ , the line representing this value (15 cm) is plotted in the graph. In this way it is possible to identify the point A and  $\alpha_{(A)}$ .

In order to arrive at an optimum design in a parametric form, which can be simple to use, the objective performance on the isolator displacement for the severe earthquake is not introduced here. For this reason, all values smaller than  $\alpha_{(A)}$  are admissible and the optimum one is that which minimizes the isolator displacement (point B). Finally one can identify  $\alpha_{opt(B)}$  and the related pier displacement  $X_{p\_max}(\alpha_{opt})$  (point C).

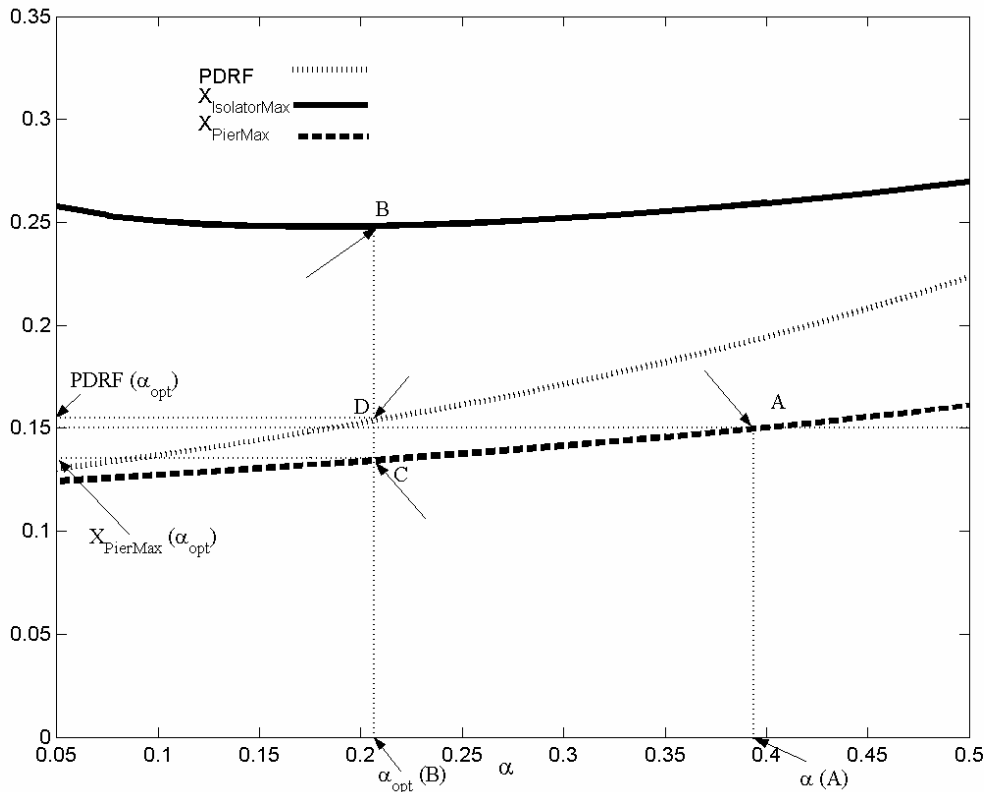


Fig. 9 Evaluation of  $\alpha_{opt}$  for each pair  $(I_{opt}, Y_{opt})$

In Figure 9, the measure of the performance of the optimally designed isolator, i.e. the ratio  $\max \sigma_{x_p} / \max \sigma_{x_{p_0}}$  (briefly named as PDRF — Pier Displacement Reduction Factor), is also plotted. Then point D gives the PDRF relative to the optimum isolator parameters.

This procedure, previously carried out for a pair of values  $(I_{opt}, Y_{opt})$ , is numerically implemented and extended to all pairs  $(I_{opt}, Y_{opt})$  obtained in the elastic phase of design. The results are plotted in Figure 10, where on the  $x$ -axis there is the optimum isolation ratio  $I_{opt}$  and on the  $y$ -axis there are  $\alpha_{opt}$  (i.e.,  $\alpha_{opt(B)}$  in Figure 9), the corresponding isolator displacement  $X_{b\_max}(\alpha_{opt})$  and PDRF, resulting from these optimum values.

Finally, when the ultimate isolator displacement  $X_{b\_u}$  is fixed (in Figure 10,  $X_{b\_u} = 25$  cm is assigned), one obtains the final  $I_{opt}$  (point A) and consequently  $Y_{opt}$ ,  $\alpha_{opt}$  (point B) and the corresponding PDRF (point C).

The results obtained through the proposed procedure can be generally adopted. This is because a different elastic pier displacement  $X_{p-e}$  and a different  $Y_{max}$  only produce a variation in the admissible range  $\min I_{opt} - \max I_{opt}$ , and then the same optimum plots can be used for the isolator design.

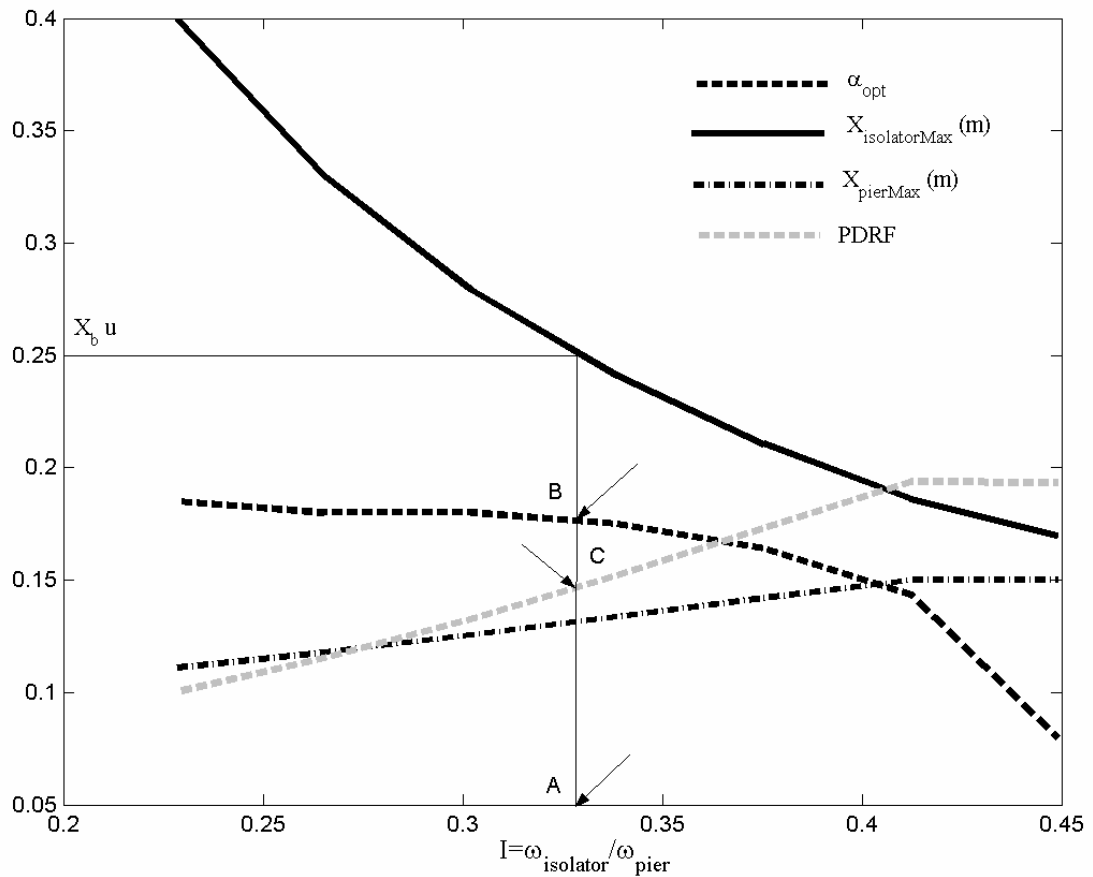


Fig. 10 Evaluation of  $\alpha_{opt}$

**CONCLUSIONS**

In this study a design procedure for isolator devices used for the seismic isolation of bridges is developed. In order to obtain a structural model that is able to provide, in a simple way, the main objective of isolator design, some simplified hypotheses are made. Then a system with two degrees of freedom, representing respectively the pier and the rigid deck positioned on isolators, is considered. Thus, the results obtained in this study can be referred only to those bridges whose configurations are consistent with the simplifying hypotheses made here.

The adopted design method is performance-reliability-based, where it is intended to design a structure satisfying the requirements related to the ‘safety’ and the ‘serviceability’ of the structure. This approach performed in a stochastic way provides, in a simple way, the optimum mechanical parameters of the isolators, starting from the serviceability and safety constraints required for the structural elements like

the elastic limit pier displacement, the elastic limit isolator displacement, and the ultimate isolator displacement.

## REFERENCES

1. AASHTO (1996). "Standard Specifications for Highway Bridges", American Association of State Highway and Transportation Officials, Washington, DC, U.S.A.
2. Atalik, T.S. and Utku, S. (1976). "Stochastic Linearization of Multi-Degree-of-Freedom Non-linear Systems", *Earthquake Engineering and Structural Dynamics*, Vol. 4, pp. 411-420.
3. ATC (1996). "Improved Seismic Design Criteria for California Bridges — Provisional Recommendations", Report ATC-32, Applied Technology Council, Redwood City, CA, U.S.A.
4. Bouc, R. (1967). "Forced Vibration of Mechanical Systems with Hysteresis", *Proceedings of the Fourth Conference on Nonlinear Oscillations*, Prague, Czechoslovakia.
5. CEN (1994). "Eurocode 8: Design Provisions for Earthquake Resistance of Structures", Comité Européen de Normalisation, Brussels, Belgium.
6. Cunha, A.M.F. (1984). "The Role of the Stochastic Equivalent Linearization Method in the Analysis of the Non-linear Seismic Response of Building Structures", *Earthquake Engineering and Structural Dynamics*, Vol. 23, pp. 837-857.
7. Greco, R., Marano, G.C. and Uva, G. (2002). "Stochastic Optimization of High Damping Base Isolators", *European Earthquake Engineering*, Vol. XVI, No. 1, pp. 65-77.
8. JRA (2002). "Design Specifications of Highway Bridges — Part V: Seismic Design", Japan Road Association, Tokyo, Japan.
9. Monti, G. and Pinto, P.E. (1998). "Effects of Multi-Support Excitation on Isolated Bridges", *Proceedings of U.S.-Italy Workshop on Seismic Protective Systems for Bridges*, New York, U.S.A.
10. Nigam, N.C. (1983). "Introduction to Random Vibrations", MIT Press, Cambridge, MA, U.S.A..
11. Rice, S.O. (1944). "Mathematical Analysis of Random Noise", *Bell System Technical Journal*, Vol. 23, pp. 282-332.
12. Roberts, J.B. and Spanos, P.D. (1990). "Random Vibration and Statistical Linearization", John Wiley & Sons, Chichester, U.K.
13. SEAOC (1995). "Performance-Based Seismic Engineering of Buildings", Vision 2000 Committee Report to California Office of Emergency Services, Structural Engineers Association of California, Sacramento, CA, U.S.A.
14. Symans, M., Kelly, J.M. and Steven, W. (1999). "Hybrid Seismic Isolation of Bridge Structures", *Proceedings of the Second World Conference on Structural Control*, John Wiley & Sons, Chichester, U.K., Vol. 2, pp. 923-932.
15. Tajimi, H. (1960). "A Statistical Method of Determining the Maximum Response of a Building during Earthquake", *Proceedings of the Second World Conference on Earthquake Engineering*, Tokyo, Japan.
16. Thakkar, S.K. and Maheshwari, R. (1995). "Study of Seismic Base Isolation of Bridges Considering Soil-Structure Interaction", *Proceedings of the Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, University of Missouri-Rolla, MO, U.S.A., pp. 397-400.
17. TNZ (1995). "Bridge Manual", Transit New Zealand, Wellington, New Zealand.
18. Vanmarcke, E.H. (1972). "Properties of Spectral Moments with Applications to Random Vibration", *Journal of the Engineering Mechanics Division, Proc. ASCE*, Vol. 98, No. EM2, pp. 425-442.
19. Wen, Y.K. (1976). "Method for Random Vibration of Hysteretic Systems", *Journal of the Engineering Mechanics Division, Proc. ASCE*, Vol. 102, No. EM1, pp. 150-154.
20. Wong, C.W., Ni, Q.Y. and Ko, J.M. (1994). "Steady-State Oscillation of Hysteretic Differential Model. I: Response Analysis", *Journal of Engineering Mechanics, ASCE*, Vol. 120, pp. 2299-2325.