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# PERFORMANCE EVALUATION OF INSTRUMENTED BUILDINGS

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## ABSTRACT

Performance-based seismic design requires reliable methods to estimate earthquake demands to be met by structural capacities or to be limited through appropriate active or passive control systems. Several methods have been proposed to estimate seismic demand and capacity of existing structures through numerical models. The accuracy of these models in reproducing the real structural behavior depends on the correct calibration of the parameters used in the analyses. A satisfactory calibration can be achieved through identification and model updating methods using responses recorded on the structure during a moderate seismic event. Generally, due to economic reasons concerning the cost of instrumentation and of data analysis, responses are recorded in a limited number of locations; hence a reduced number of dynamic parameters may be estimated and used for the calibration of the numerical models. In order to obtain a more detailed description of the structure, the number of available responses can be increased by interpolating recorded ones. This paper describes a new technique to estimate the seismic demand on multistory buildings in terms of appropriate performance parameters such as story absolute accelerations, velocity and displacements, interstory drifts, story and base shear, from a limited number of recorded signals. A reduced number of signals is assumed to be recorded by sensors placed according to an optimal distribution defined by local minima of a function of the effective participation factors of the dominant modes of the structure. Unknown responses, in locations where sensors are not available, are calculated using a spline shape function to interpolate recorded responses along the height of the considered building. Recorded and interpolated signals are then used to estimate the seismic performance parameters. The method is applied to a real instrumented multistory building using records from recent earthquakes characterized by different intensities and frequency contents. Calculated performance parameters are compared to those obtained from recorded responses showing that the method leads to an excellent estimate of the seismic performance parameters even with a very limited number of recording sensors. The level of accuracy for a given number of recording sensors is linked to the order of the modes contributing to the response: the stronger the influence of the higher modes, the higher the number of sensors needed to attain a given level of accuracy in the estimate of seismic performance parameters.

**KEYWORDS:** Seismic Performance Parameters Estimation, Optimal Sensors Location, Multistory Buildings

## INTRODUCTION

Prescriptive building codes are increasingly being replaced around the world by performance-based standards improving structural performance, safety, design flexibility and cost efficiency.

Unlike prescriptive codes, which specify the exact requirements that must be met in order to make a structure safe, performance codes are based on design objectives. Performance-based design thus requires reliable methods to estimate the demand to be met by structural capacities and several methods have been proposed to estimate both seismic demand and capacity of structures through numerical models. A possible way to reliably predict the structural performance during strong earthquakes is to monitor the response to moderate seismic events that can be used to estimate the seismic performance of the building and/or to improve the dynamic models adopted to predict the response of the structure in the elastic range. For structures in high seismic areas the response to strong seismic events can give a quantitative measure of the demand placed upon them and provide information about different kinds of damage, helping in the choice of retrofitting or strengthening intervention and also suggesting possible improvement of seismic codes. Furthermore, knowledge of real structural demand during strong seismic events can lead to a better understanding of the structural behavior in nonlinear range.

Due to economic constraint only critical structures such as hospitals, fire stations, power stations, that is structures with an essential role in society, are usually instrumented. Besides, responses are usually recorded in a limited number of locations. Rational and economical methods of instrumentation are thus needed in order to extend performance monitoring to a greater number of structures. Criteria to optimally locate a limited number of recording sensors help reducing the cost of instrumentation and of data analyses through the reduction of the total number of sensors. The selection of the recording sensors locations depends on the kind of information required from the records. Generally optimal location is selected so that the recorded responses allow the best possible estimates of a model parameters using system identification techniques: the optimal location of a given number of sensors is defined as the one that minimizes the uncertainty on the parameters estimate. The differences between the approaches proposed in literature reside in the function chosen to mathematically define the uncertainty, that is the function to be minimized. In this respect Shah and Udwadia (1978) assume a suitable norm of the covariance matrix of the parameters estimates; Udwadia (1994) chooses the expected value of a Bayesian loss function expressed in terms of the Fisher information associated with the recorded responses. The same function is minimized in the criterion proposed by Heredia-Zavoni and Esteva (1998) and Heredia-Zavoni et al. (1999) that extends the one proposed by Udwadia (1994) explicitly taking into account the uncertainty about the structural parameters. The objective of minimizing the uncertainty in model parameters through an appropriate pattern of recording sensors is pursued minimizing the information entropy measure by Papadimitriou et al. (2000). This function has the advantage of allowing a direct comparison of configurations involving a different number of recording sensors which is useful for cost effective number and distribution of recording sensors. A different approach consists in minimizing the differences between responses: Datta et al. (2002) assume that the optimal location of a given number of recording sensors is the one minimizing the covariance kernel of the response modeled as a zero-mean, stationary, Gaussian process.

The principal aim of all these methods is to single out those locations where recorded responses lead (through the application of system identification techniques) to the most reliable identification of the system parameters. Frequently these parameters are used to calibrate a finite element model of the structure able to provide a representation of the actual structural behavior.

If modal frequency approaches are applied to the identification problem, optimal locations of a limited number of recording sensors are generally so chosen as to reliably identify the parameters of a great number of modes in a limited number of locations corresponding to the points of measurement.

In this paper a different approach is proposed: optimal locations of recording sensors are so chosen as to obtain the best approximation of responses reconstructed through interpolation of recorded ones via a spline shape function. This leads us to choose as recording locations the ones where the lowest number of modes gives their contribution to the response.

A criterion ($\gamma$-criterion) has been proposed by the author in a previous paper (Limongelli, 2003a) to place the available sensors in locations that are able to provide the best approximation of responses in a number of locations greater than the number of measurement points. The optimal distribution is defined by the local minimum points of a function $\gamma$ of the effective participation factors of a number of dominant modes that are equal to the number of available recording sensors. Locating the available sensors according to the $\gamma$-criterion and applying a spline interpolation technique to recorded signals, unknown structural responses (in locations where recording sensors are not available) can be reconstructed with remarkable accuracy in the case of structures subjected to seismic excitations.

In this paper the accuracy of the $\gamma$-method in estimating seismic response parameters has been checked using responses recorded on a real instrumented building during three seismic events of moderate to strong intensities and characterized by different frequency contents and epicentral distances from the building. Responses have been reconstructed at all the stories of the building and peak values of several seismic performance parameters (story acceleration, displacement, shear and interstory drift) have been calculated and compared to those derived from the "measured" responses.

Comparison has shown that a number of sensors allowing the dominant modes to be modeled through the spline shape function, leads to an excellent estimate of all the considered seismic response parameters. A high frequency content of the motion slightly reduces the accuracy of the reconstructed responses for a given number of recording sensors due to the increase of the contribution of the higher modes to the response. In this case an increase in the number of sensors is needed to improve the modeling of the higher modes and, thus, the match between real and reconstructed responses.

## RECONSTRUCTION OF UNKNOWN RESPONSES: THE $\gamma$ -METHOD

The $\gamma$ -method is a procedure to optimally reconstruct the responses at all the stories of a building through the responses recorded at a limited number of stories during a seismic event. This method is based on the assumption that 1) responses in terms of absolute acceleration are available in a limited number of locations along the height of the building, 2) the input ground acceleration is known, and 3) a number $N_s$ of recording sensors can be permanently installed on the structure according to an optimal pattern defined by the local minima of a function $\gamma$ of the effective participation factors of the structure.

The application of the $\gamma$ -method for a given number of available recording sensors $N_s$ consists of three consecutive steps:

- estimation of the modal parameters of the structure through a validated structural model of the structure, wherever available, or through application of system identification techniques to responses recorded during dynamic tests carried out on the structure. The availability of a number of sensors greater than $N_s$ during this phase could facilitate the estimation of the complete modal shapes.
- evaluation of the modal contribution factors and evaluation of a function $\gamma$ of the effective participation factors of the building; detection of the $N_m$ dominant modes of the structure as the ones characterized by the highest values of the modal contribution factors; detection of the optimal locations of the available $N_s$ recording sensors through the local minima of function $\gamma(N_s)$.
- installation of sensors according to the optimal distribution detected in the previous step and, on the occasion of a following seismic event, real-time instantaneous reconstruction of unknown responses through interpolation of the recorded ones.

Unknown responses, in the locations where recording sensors are not available, are reconstructed using a spline shape function. A spline function is a function composed by polynomials joined together with continuity conditions, thus allowing a smooth behavior of the resulting function with its derivatives (de Boor, 1978). The smoothness of the spline function depends on its degree: a spline of degree $m+1$ has a continuous *m*th derivative. Taking into account that the deformed shape of the building is described by the absolute displacement function and given the relationship between absolute acceleration and displacement, a cubic spline has been chosen to model the absolute acceleration function along the height of the building. In this way the slope and curvature continuity of the deformed shape of the building are ensured on the definition interval. The locations where responses are recorded are assumed as knots of the spline function whose values depend, in each subinterval between two knots, on 4 unknown coefficients. For each subinterval and for each time instant, the unknown coefficients are determined from continuity, interpolation, and boundary conditions.

At a given time $t$ the cubic spline interpolant to $\ddot{u}(z,t)$ is composed of $n+1$ cubic polynomials, each defined in one sub-interval $[z_i, z_{i+1}]$:

$$\ddot{u}(z,t) = \sum_{j=0}^{4} c_{j,i}(t)(z - z_i)^j; \qquad z \in [z_i, z_{i+1}] \tag{1}$$

where the abscissa $z_1, z_2, ........z_n$ define the locations where function $\ddot{u}(z,t)$ is recorded, that is the knots of the spline function. For each one of the $n+1$ polynomials, 4 unknown coefficients $(c_{0i}, c_{1i}, c_{2i}, c_{3i})$ must be estimated, hence a total number of $4(n+1)$ equations must be written to obtain a unique solution.

The continuity and interpolation conditions to be imposed in each one of the internal knots in order to obtain a continuous and smooth approximating function, give a total number of $4n$ equations (the spline function is assumed twice continuously differentiable so that it has also a continuous slope and a continuous curvature). The remaining 4 constraints are given by the boundary conditions to be imposed at the boundary of the interval of definition of the function (points $z = a$ and $z = b$). Once the coefficients of the spline have been calculated, unknown responses in the locations where recording sensors are not installed can be estimated through this function.

The reconstruction of unknown responses can be carried out for any distribution of recording sensors but there are distributions leading to a better match between the reconstructed and real responses. Given that responses reconstructed through the recorded ones will reflect the characteristics of the latter, and given that reconstruction is carried out through an approximation of the function $\ddot{u}(z,t)$, there are two factors that influence the characteristics and the accuracy of the reconstructed responses. The first one is the location of the recording sensors: for example if sensors are located at the nodes of a mode, recorded responses (hence reconstructed responses) will not contain the contribution of that mode. In this case the error between the real and reconstructed responses will depend in each location on the importance of the neglected mode in the response.

The second factor influencing the accuracy of the reconstructed responses depends on the ability of the spline function to interpolate the function $\ddot{u}(z,t)$ along the height of the building. It can be shown (Setola, 1998) that the free vibrations $u(x,t)$ of a beam-like structure can be reconstructed through a spline function $\hat{u}(x,t)$ with a bounded error that depends on the parameters of the $\nu$ modes excited by the initial conditions. Specifically, an upper bound of the error is given by:

$$\|u-\hat{u}\| \le \frac{5}{384}\frac{m}{EI}\left(\frac{\Delta}{L}\right)^4\left(\omega_1^2+\omega_2^2+....\omega_\nu^2\right)\phi_{\max} \quad \forall t \tag{2}$$

where $\|u\| = \max_{z\in[0,L]}\left|u\left(z,t\right)\right|$, $\Delta$ is the maximum distance between two consecutive sensors (considering the beam extremes as sensor locations), $\omega_i$ is the circular frequency of the *i*th mode, and $\phi_{\max} = \max_{i,z}\left|\phi_i\left(z\right)\right|$, $i=1,2,.........\nu,\ z\in\left[0,L\right]$ is the maximum amplitude of the excited $\nu$ modes.

It may be noted that the approximation error increases with the number of modes influencing the response and with the square of the circular frequency of each of those modes. The number $N_s$ of recording sensors influences the value of $\Delta$: if the sensors are uniformly distributed along the height of the building the maximum distance between two of them decreases with the increase of $N_s$. In other words, the accuracy of the spline function in reconstructing the response decreases with the order of the modes contributing to the response and increases with the number of recording sensors.

If a limited number of sensors is available two different choices can be made as to their locations. One possibility is to place the sensors in the locations where the greatest number of modes gives their contributions. The advantage is that the reconstructed responses will contain the contributions of a great number of modes. In this case the error of reconstructed responses depends mainly on the error linked to the low accuracy of the spline function in reconstructing the contribution of the higher modes. Another possibility is to place the sensors in locations where the lowest number of modes gives their contributions. In this case reconstructed responses will contain only (or mainly) the contributions of these modes but the spline function will be able to reconstruct with greater accuracy these contributions. In other words, given a certain number of sensors, the choice is between calculating a bad approximation of the contributions of a great number of modes and calculating a good approximation of the contributions of the lowest modes to the response.

In the case of a multistory building if a single recording sensor is available one can choose to place it on the roof of the building where three or four modes contribute to the response. Reconstructed responses in theory will contain the contributions of all these modes but the problem is that the spline function would not be able to interpolate with a great accuracy the function $\ddot{u}(z,t)$ along the height of the building. In this case the scarce efficiency of the spline in interpolating recorded responses is linked both to the high value of $\Delta$ in Equation (2) and to the high number of modes influencing the response at the point of measure. An alternative choice, as suggested in this paper, is to place the only recording sensor in a location where response is almost entirely due to the first mode contribution. In this way reconstructed responses will lack the contribution of the higher modes but the accuracy of the interpolation carried out with the spline function will be greater with respect to the preceding choice.

For a typical multistory building subjected to a seismic excitation, the response is influenced mainly by the lower modes of vibration with contributions decreasing with the order of the modes; hence the interpolation with the spline function of a limited number of responses gives very good approximation of the real responses.

The importance of each mode can be quantified through the values of the relevant modal contribution factors (Chopra, 1995) whose values have been used herein to select the $N_m$ dominant modes of the building. The *i*th modal contribution factor for the displacement of a multistory building is given by:

$$\bar{u}_i = \frac{\beta_i}{\omega_i^2} / \sum_{j=1}^{N} \frac{\beta_j}{\omega_j^2} \tag{3}$$

where $\beta_i$ is the vector of the effective participation factors of the *i*th mode and $\omega_i$ its circular frequency. Limongelli (2003a) showed that the locations where only $n$ dominant modes give a contribution, correspond to the $n$ local minimum points of a function $\gamma$ of the effective participation factors $\beta_i$ of $n$ 'dominant modes'. The function $\gamma$ of $n$ dominant modes is defined as:

$$\gamma(n,z) = \left|1 - \sum_{i=1}^{n} \beta_i(z)\right| = \sum_{i=n+1}^{N} \beta_i(z) \tag{4}$$

The local minima of this function correspond to locations along $z$ where the modes higher than the *n*th give the lowest contribution to the response and hence the location where response is mainly due to the lowest $n$ dominant modes.

Knowing the modal parameters of the structure (effective participation factors and circular frequencies), the local minima of function $\gamma$ calculated for a number $n$ of dominant modes give the optimal locations of $n$ recording sensors.

Given $N_s$ sensors (beyond the one recording the base acceleration) and given a multistory building with $N_m$ dominant modes, three different situations can occur:

- $N_s \le N_m$. If the $N_s$ sensors are placed in the local minima of function $\gamma(N_s)$, $N_s$ of the $N_m$ dominant modes can be interpolated with the spline function; hence the accuracy of the interpolation will depend on the importance of the response of the modes higher than the $N_s$th.
- $N_s > N_m$. If $N_m$ of the $N_s$ sensors are placed in the local minima of function $\gamma(N_m)$, all the dominant modes can be interpolated with the spline function. The remaining $N_s - N_m$ sensors can be placed in portions of the building where more information is required that is where more accuracy is required in the interpolation.

The proposed approach leads to a distribution of sensors different from the ones suggested by other works in literature. For example the optimal location of a single recording sensor is generally suggested as the top story while in this paper an intermediate story is found to be the best location. The difference derives from the aim pursued in this work that is to obtain the highest level of accuracy for responses reconstructed in the largest number of locations rather than to identify the parameters of the largest number of modes in a limited number of locations corresponding to the points of measurement (as is the case with several optimality criteria proposed in literature).

In the following the $\gamma$-method is applied to reconstruct responses of a real multistory building to different seismic events.

## ESTIMATES OF SEISMIC RESPONSE PARAMETERS: A REAL CASE

The $\gamma$-method has been applied to evaluate the performance of a nine-story reinforced concrete building during three moderate seismic events. The building is the Robert A. Millikan Library (see Figure 1), a reinforced concrete frame/shear wall structure located on the campus of the California Institute of Technology in Pasadena, California and instrumented with two Multi-Channel Central Recording Systems that include 36 accelerometers. The locations of the 36 recording sensors (three horizontals on each of the eleventh floors and three verticals in the basement) are reported in Figure 2.

The choice of the Millikan Library as a test structure for the $\gamma$-method originates from the availability of line of responses recorded at all the stories of the building[1]: assuming a limited number of responses to be known, the accuracy of the method in reconstructing the remaining "unknown" responses can be checked by comparison with the recorded ones.

In a previous paper (Limongelli, 2003b) the optimal locations of recording sensors for the Millikan Library according to the $\gamma$-method have been assessed for different numbers of the available sensors.

Fig. 1 The Millikan Library

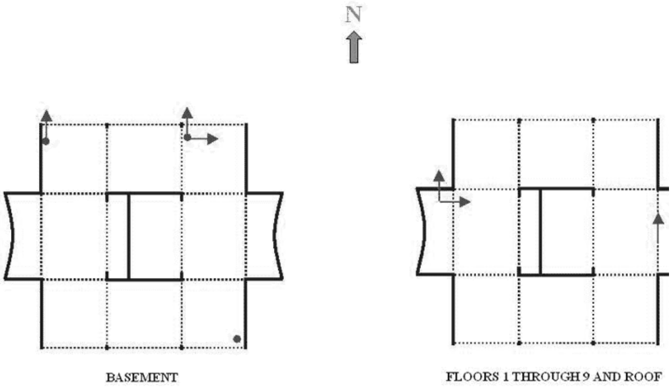


Fig. 2 Layout of the recording sensors[1]

In this paper two main objectives have been pursued:

- to verify the optimality criterion defined by the local minima of function $\gamma$ using responses recorded during seismic events with different characteristics; and
- to check the influence of the frequency content of the base motion on the accuracy of the $\gamma$-method in reconstructing unknown responses.

Three recent seismic events, characterized by different intensities and frequency contents, have been considered: the Alhambra earthquake (AH) of April 25, 1998, the Yorba Linda earthquake (YL) of September 3, 2002, and the Big Bear City earthquake (BB) of February 22, 2003. The values of the local magnitude $M_L$, epicentral distance $\Delta$, and the predominant frequency $f$, relevant to the three events, are reported in Table 1.

**Table 1: Characteristics of the Seismic Events**

| Event | $M_L$ | $\Delta$ [km] | $f$ [Hz] |
|---|---|---|---|
| Alhambra Earthquake | 3.8 | 7 | 4.7 |
| Yorba Linda Earthquake | 4.8 | 40.5 | 2.5 |
| Big Bear City Earthquake | 5.4 | 119.3 | 3.5 |

[1] Website of the National Strong Motion Program of USGS, http://nsmp.wr.usgs.gov/

The predominant period is calculated as the one corresponding to the maximum value of the Fourier amplitude spectrum of the base excitation. Since during the Big Bear City earthquake recording sensors placed at the base of the Millikan Library were not operational, the response recorded at the base of the USGS/NSMP office, located in Pasadena and not far from the Millikan Library, has been used to determine the fundamental period.

Figure 3 reports the comparison of the Fourier spectra of the three base excitations showing the differences in the frequency contents and specifically the higher frequency content of the Alhambra event with respect to the other two earthquakes.

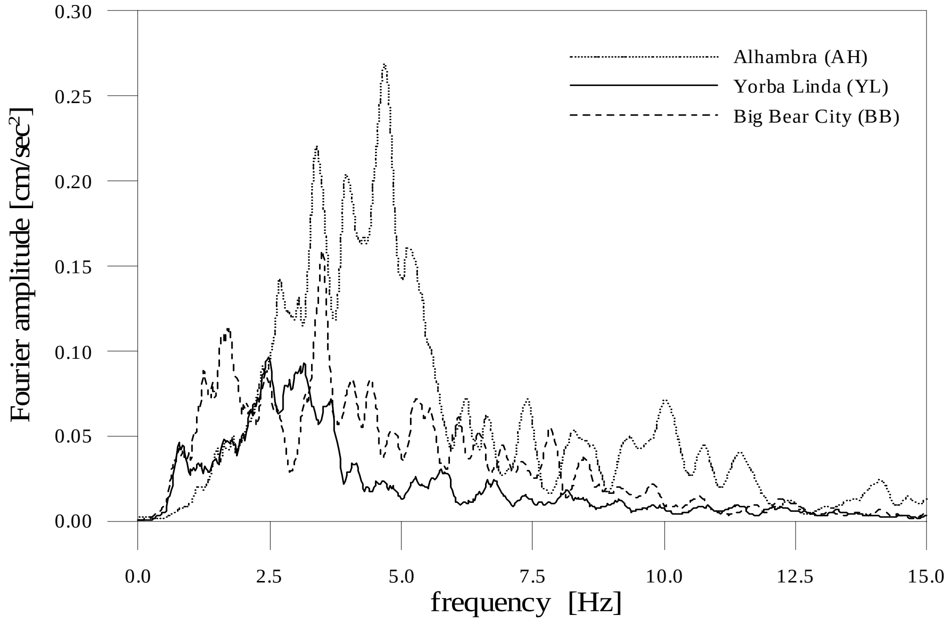


Fig. 3 Fourier spectra of the base excitations

For the three considered events, the spline interpolation technique has been applied to reconstruct responses at all the stories of the building. A number of recording sensors $N_s$ (beyond the one recording the base excitation), increasing from 1 to 4, have been assumed to be available. For all the possible layouts of the available $N_s$ sensors, responses in terms of absolute acceleration have been "reconstructed" at all the stories and compared with the recorded ones. The optimal pattern of the $N_s$ sensors has been singled out as the one corresponding to the minimum value of the mean value $\varepsilon_M$ of the errors between the recorded and reconstructed responses in terms of the absolute acceleration at story levels. As to the boundary conditions for the spline function, following the suggestion deriving from the inspection of the modal shapes and of the geometry of the building (having the first story embedded in the ground), a fixed-base at level 1 of the building has been assumed both in the NS direction and in the EW direction. During the Yorba Linda and the Big Bear City events the sensors recording responses in the North-South direction at the 2nd and 8th floors on the east side of the building, remained inoperative. For this reason only responses recorded on the west side at each floor of the building have been considered for the analyses relevant to the North-South direction of the building.

Figure 4 reports the optimal distributions of recording sensors, that is the ones corresponding to the minimum value of $\varepsilon_M$ for the three seismic events considered herein. These distributions are compared with the variation of function $\gamma$ along the height of the building determined respectively for $N_s = 1$, $N_s = 2$, and $N_s = 3$. The comparison shows that with a total number of three recording sensors (two located along the height of the building and the third recording the base acceleration) a very good approximation of the response is obtained both in time and in frequency domain. In almost all the cases the local minima of function $\gamma$ coincide with the optimal locations of available sensors in both EW and NS directions. The deviations from this behavior are characterized by very slight differences (with a maximum of one story) between a local minimum of $\gamma$ and the optimal location. This situation is probably due to the fact that function $\gamma$, which is a continuous function along the height of the building, is evaluated only at the locations corresponding to the story levels where effective participation factors have been identified using recorded responses, and hence local minima of $\gamma$ are approximated by the values of the function at the story levels. This comparison confirms the validity of the $\gamma$-criterion that is applied to detect the optimal location of a given number of recording sensors, and the local minima of function $\gamma$, calculated for a number of modes equal to the number $N_s$ of available sensors, indicate the optimal locations for these sensors.

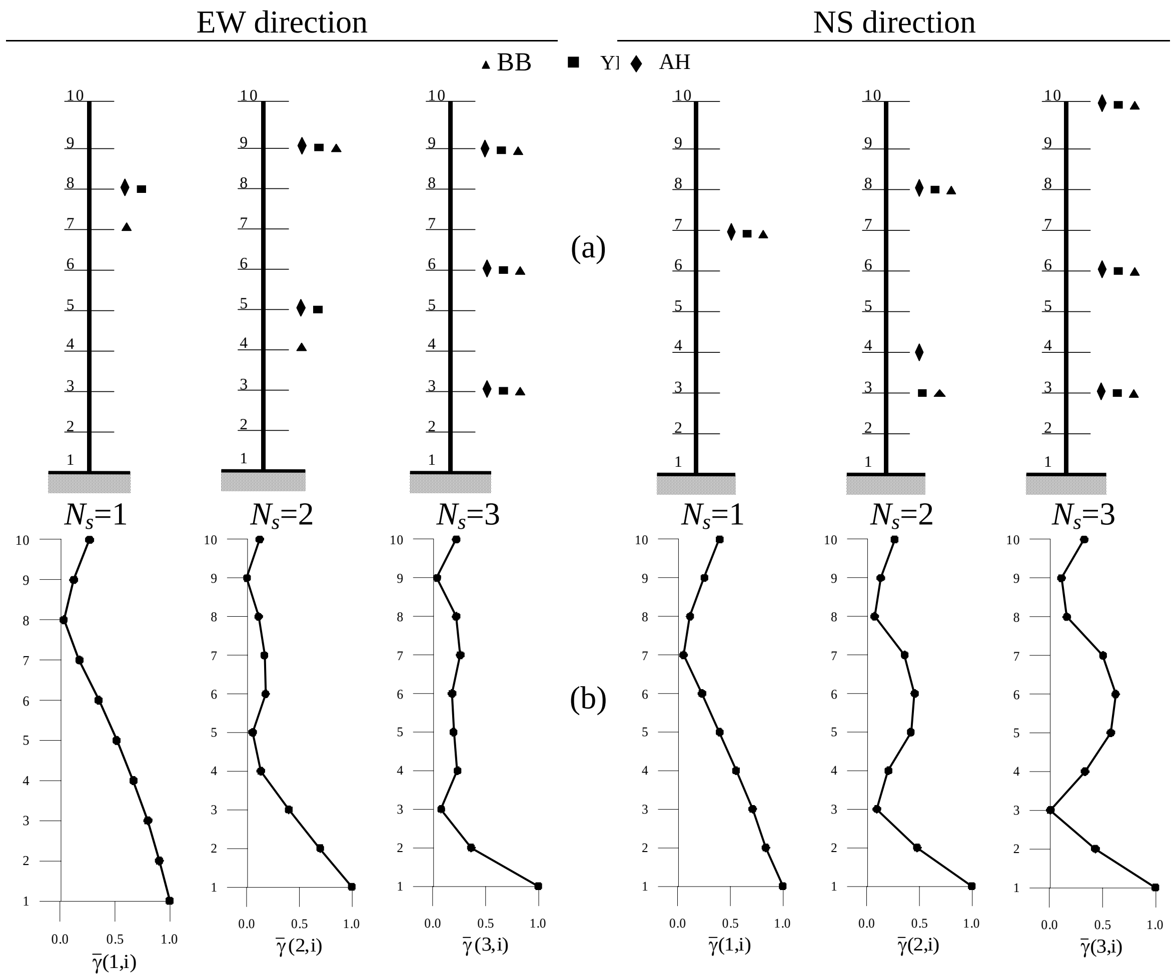


Fig. 4 Comparison between (a) optimal locations of sensors, and (b) local minima of function $\gamma$

In order to show the quality of the fit between the calculated and reconstructed responses (for $N_s = 2$), Figures 5-7 report the comparison in terms of absolute acceleration time history, magnitude of the transfer function, and phase of the transfer function for the top story of the building in the EW direction for the BB and AH events. In order to make the comparison clearer, only a segment of the time history and of the transfer function is reproduced. Responses have been reconstructed for the three events assuming sensors to be located according to the pattern indicated by the local minima of function $\gamma$. These would be the initial conditions if the method were to be applied to a generic building of known modal parameters and instrumented with a limited number of sensors. In order to study the influence of the higher mode contributions on the accuracy of the $\gamma$-method in reconstructing unknown responses, several response parameters (peak story acceleration, displacement, shear and interstory drift) have been calculated from the reconstructed signals and compared to the ones derived from the recorded responses. In the following, in order to distinguish between quantities derived from the recorded responses and quantities derived from the calculated responses, the former will be addressed as ‘recorded’ even if they are not directly measured and are instead calculated from the recorded responses.

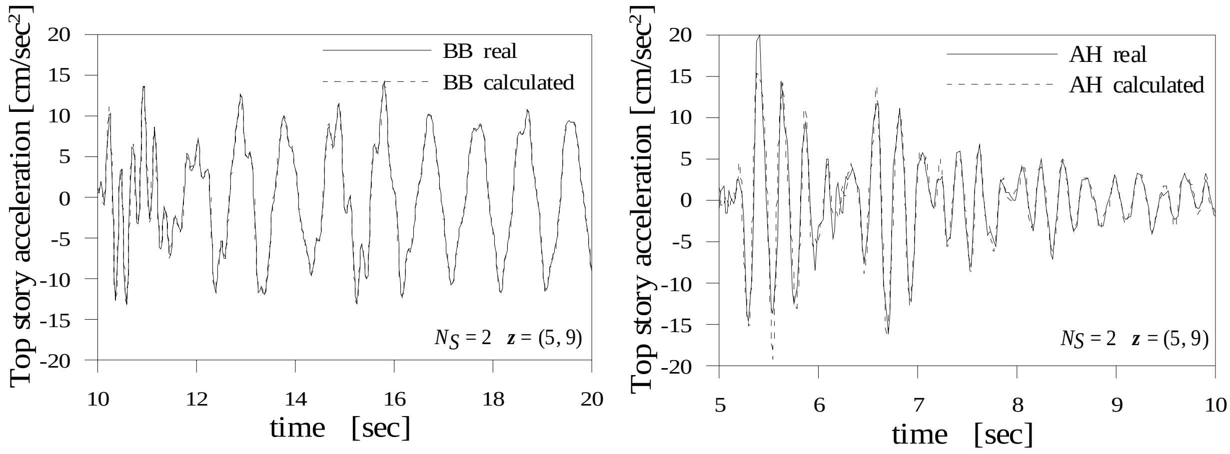


Fig. 5 Comparison between the calculated and recorded absolute accelerations: EW direction

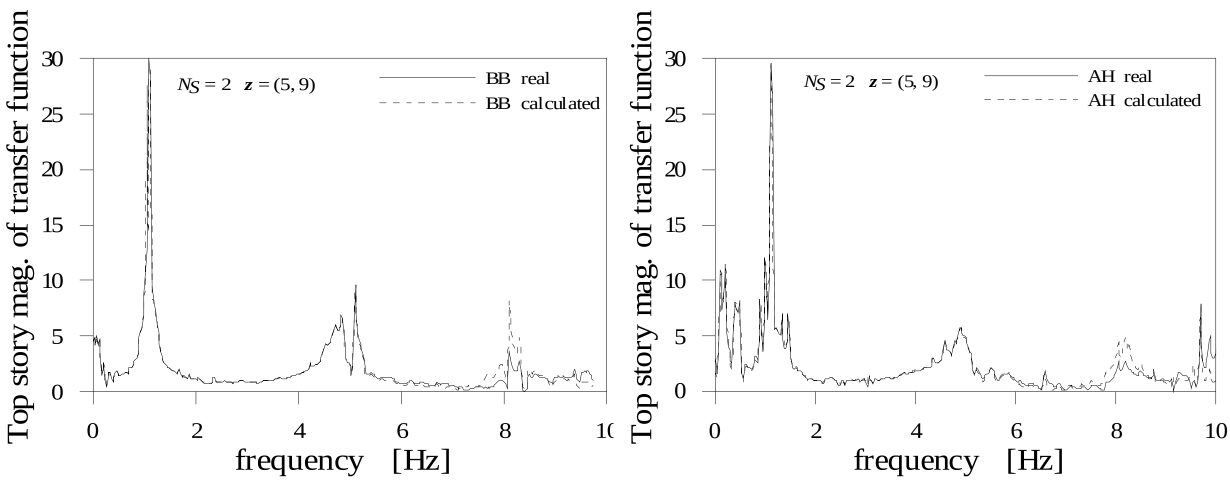


Fig. 6 Comparison between the calculated and recorded magnitudes of transfer function: EW direction

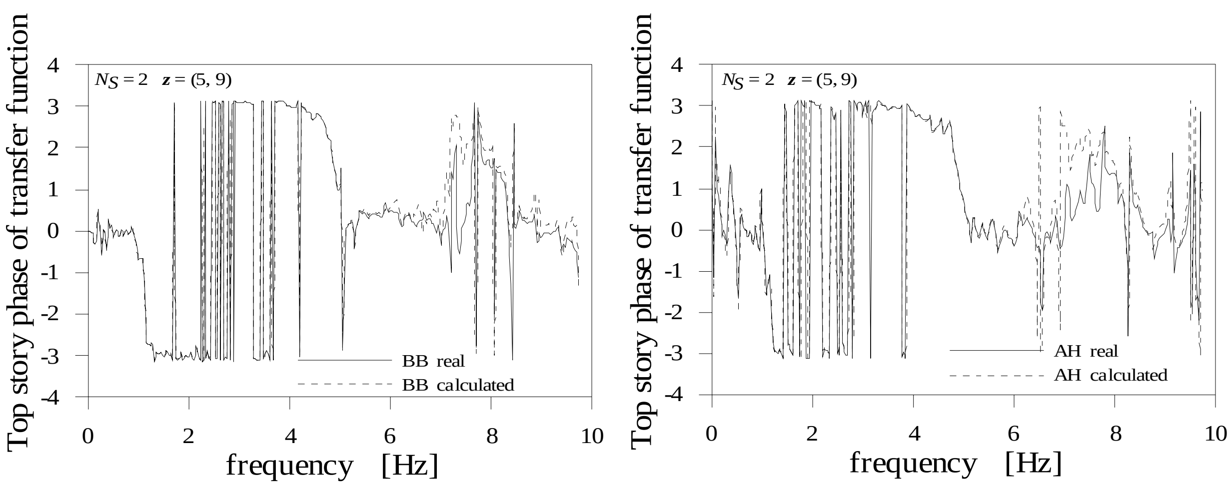


Fig. 7 Comparison between the calculated and recorded absolute phases of transfer function: EW direction

Different levels of accuracy of calculated responses are obtained for the three events with the same number of recording sensors. Figures 8 and 9 report comparisons between the peak values of story acceleration in EW and NS directions, respectively, for the number of recording sensors $N_s$ between 1 and 4.

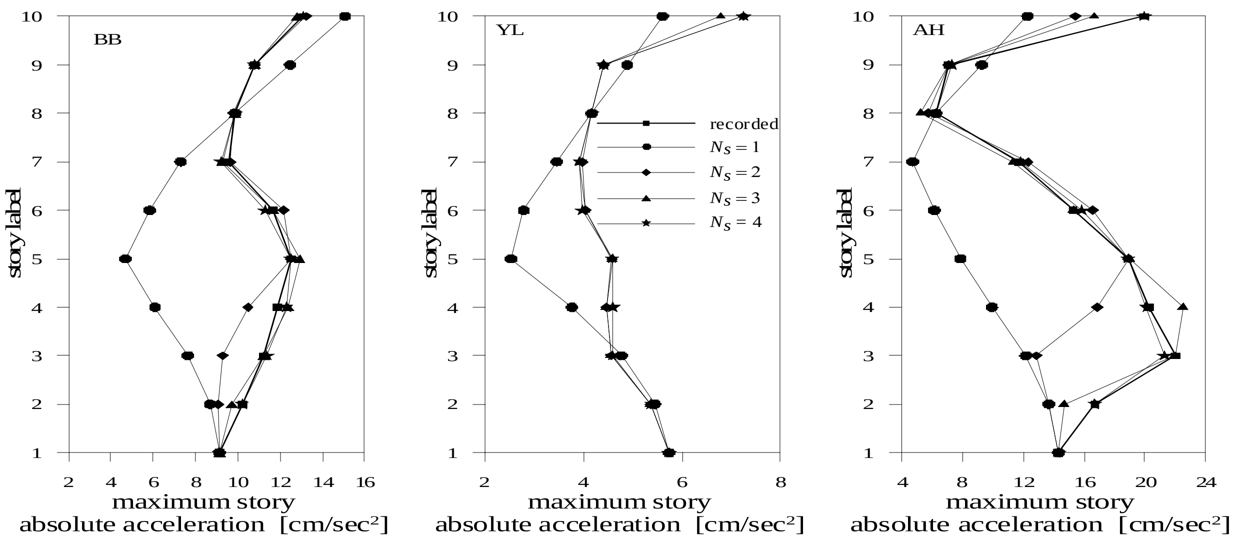


Fig. 8 Comparison between the calculated and recorded absolute acceleration maxima: EW direction

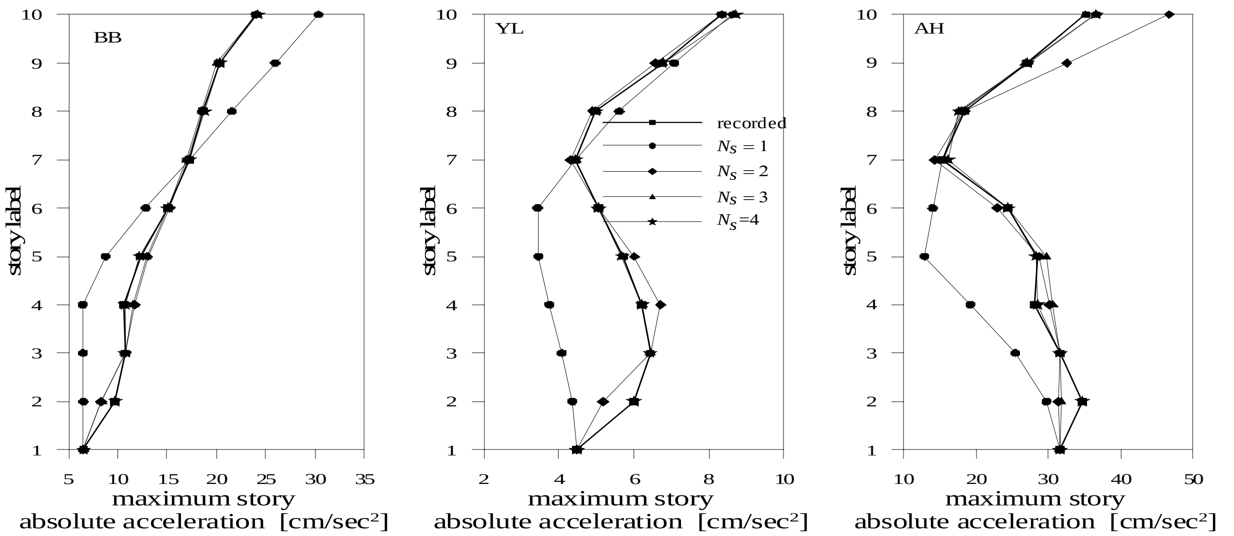


Fig. 9 Comparison between the calculated and recorded absolute acceleration maxima: NS direction

A total number of two recording sensors ( $N_s = 1$; hence only one sensor beyond the one at the base) appears in all the cases insufficient to give a good estimate of the absolute acceleration.

For the three events the maximum error ranges between 45% and 60% of the recorded value in the EW direction and between 40% and 55% in the NS direction. With the increase of $N_s$, for Big Bear and Yorba Linda events that are characterized by a lower frequency content with respect to the Alhambra earthquake (see Figure 3), two recording sensors (beyond the one recording the base input) give a fairly good estimate of story peak acceleration in both directions: the maximum error in EW direction for the BB and YL events is respectively equal to 15% and 8% of the recorded values and drops to about 5% of the recorded value for $N_s$ = 3. Further increases of $N_s$ in EW direction produce only minor reductions in the error between the recorded and calculated values of peak story absolute acceleration. On the contrary in the case of the AH event, the maximum error in the EW direction for $N_s$ = 2 is still high, reaching 41%

of the recorded value; only for $N_s$ = 3 it drops to about 15%. An analogous pattern is observed in the NS direction.

The other seismic performance parameters exhibit the same behavior. Figures 10 to 12 report the comparison between the recorded and calculated values of maximum story displacement, shear and interstory drift in the EW direction. In all cases for $N_s$ = 2 a very good estimate of the recorded peak values is obtained for the BB and YL events while, in order to obtain the same level of accuracy for the responses recorded during the Alhambra event, a greater number of sensors is needed. The different accuracy of the method for the Alhambra earthquake depends on the higher frequency content of this event that involves a greater influence of the higher modes on the response: a greater number of recording sensors (allowing to reproduce the higher modal shapes) is needed to obtain a comparable level of accuracy.

The above observation is clearly shown by the comparison between the recorded and calculated Fourier spectra of the response at the 3rd story for the three events reported in Figure 13: while for the BB and YL events both first and second modes contribute to the response at this story, for the Alhambra event the response is almost entirely due to the second mode (the frequencies of the first and second modes in the EW direction are equal to 1.12 and 4.77 Hz respectively).

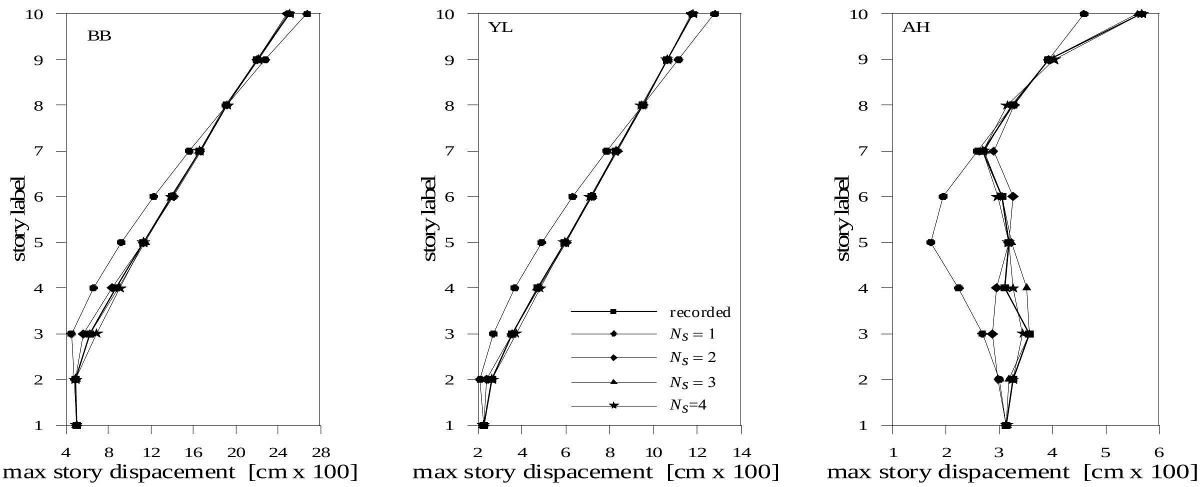


Fig. 10 Comparison between the calculated and recorded maximum absolute displacements: EW direction

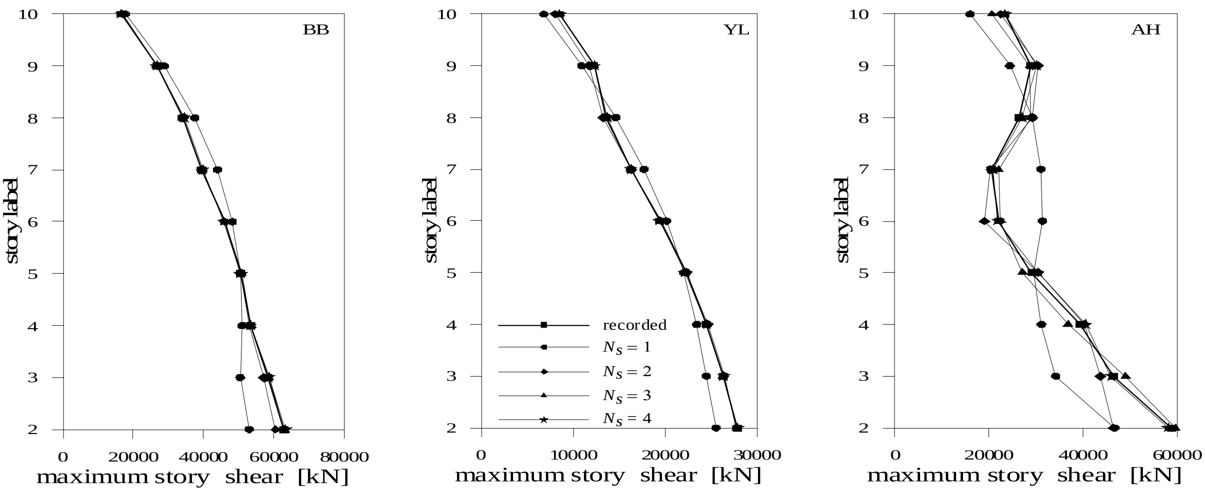


Fig. 11 Comparison between the calculated and recorded maximum story shears: EW direction

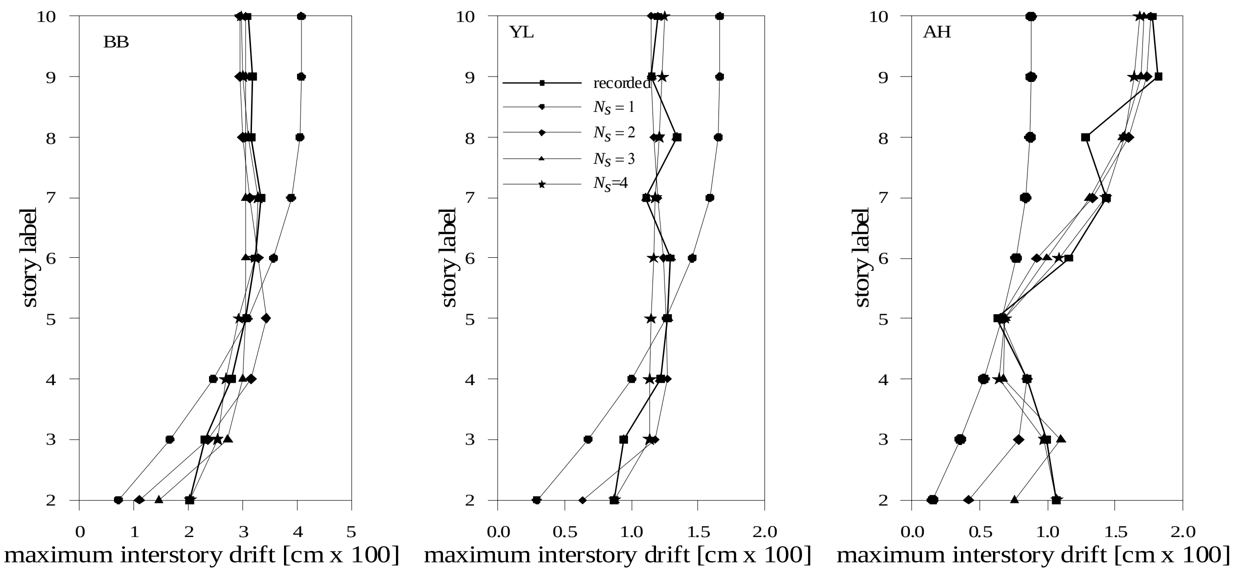


Fig. 12 Comparison between the calculated and recorded maximum interstory drifts: EW direction

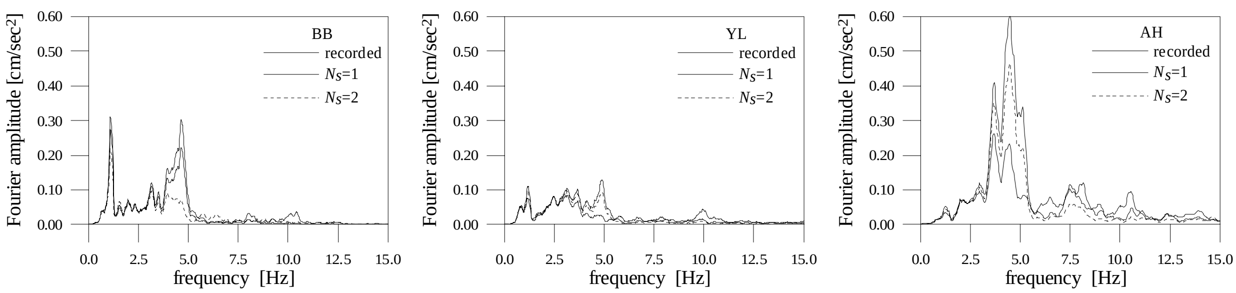


Fig. 13 Comparison between the calculated and recorded Fourier spectra of the response in the EW direction (3rd story)

Thus, for the Alhambra event a number of sensors able to accurately reproduce the second mode of the structure is needed to obtain an accurate reconstruction of the response at this story.

The variation of the maximum error for some response parameters is reported in Figures 14-16 for the EW and NS directions. For the Alhambra event the error for story acceleration, story shear, interstory drift and also story velocity and displacement (not reported here due to space limitation) in the two directions, is higher with respect to the ones relevant to the BB and YL events. For the BB and YL events, the increase in $N_s$ from 2 to 3 does not lead to a sensible reduction in the maximum error because two recording sensors are already able to give a very good estimate of the response. On the contrary for the AH event, in order to obtain the same level of accuracy, three or four recording sensors must be located along the height of the building.

The above results show that an optimum approximation of response parameters can be obtained if the distribution of sensors allows us to accurately model the modal shapes relevant to the modes dominating the response: higher is the frequency content of the excitation, higher will be the number of recording sensors required to give a certain level of accuracy of responses reconstructed through the $\gamma$-method. For the cases considered herein, a total number of 4 sensors ($N_s = 3$ plus the one recording the base acceleration) leads to the maximum errors between the recorded and calculated maximum values of the response in terms of acceleration, velocity displacement and story shear remaining within 10% of the relevant recorded values. A higher number of sensors are needed to obtain the same accuracy for the estimate of interstory drift.

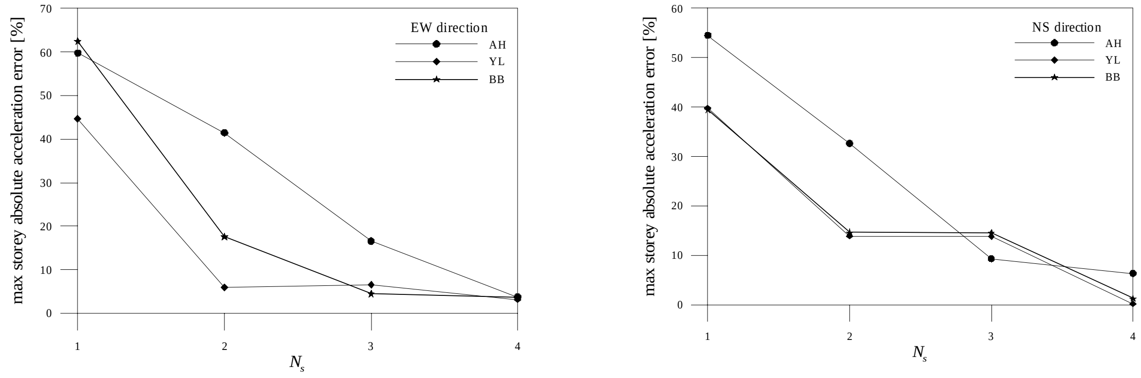


Fig. 14 Variation of the maximum error in story absolute acceleration with the number of recording sensors

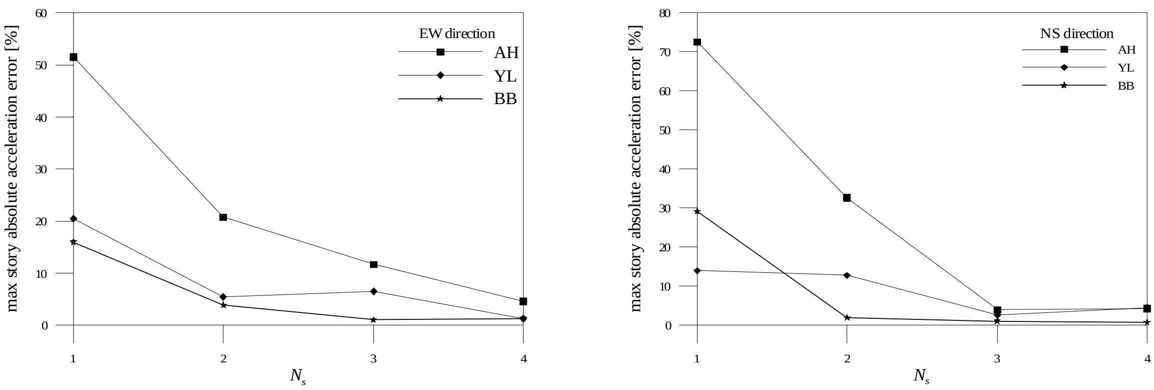


Fig. 15 Variation of the maximum error in story shear with the number of recording sensors

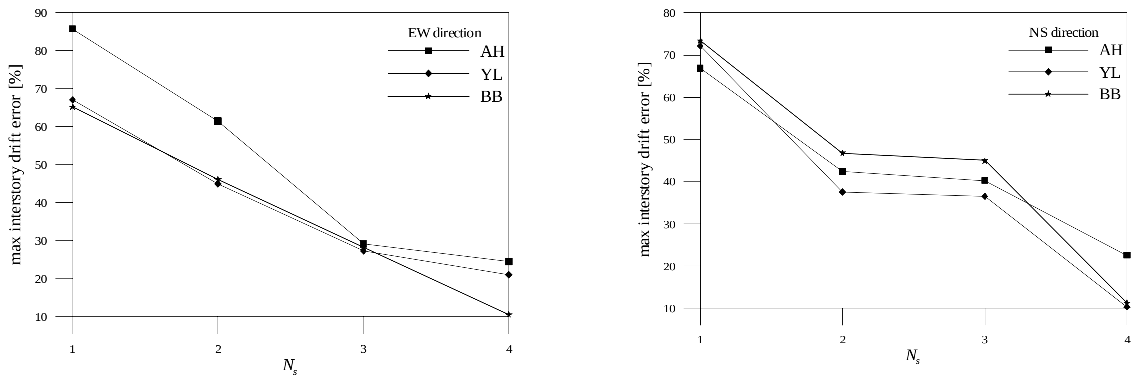


Fig. 16 Variation of the maximum error in interstory drift with the number of recording sensors

## CONCLUSIONS

The accuracy of a method to estimate seismic performance parameters for buildings instrumented with a limited number of recording sensors has been checked using data recorded on the Robert A. Millikan library during recent earthquakes. Three events characterized by different intensities and frequency contents have been considered and seismic performance parameters have been estimated for each of them using responses reconstructed through the $\gamma$-method proposed by the author in an earlier paper. The method is based on definition of the optimal location of a given number of recording sensors through a function of the effective participation factors of the structure. Assuming sensors to be located according to the optimality criterion, unknown responses are reconstructed by interpolating the recorded ones through a spline shape function.

The spline interpolation of responses recorded at the optimal locations gives an alternative method to the reconstruction of unknown responses carried out through previous identification of the modal parameters of the structure. In this regard the advantage of the $\gamma$-method is that it does not require at all previous identification of modal frequencies and damping, and the values of the effective modal participation factors influence only those sensor locations that are anyway approximated due to recording sensors assumed to be located only at the story levels. On the other hand the $\gamma$-method gives a poor approximation of responses if the number of available sensors is so limited as to prevent the interpolation of modes strongly contributing to such responses.

It is not possible to give a general rule to choose the method allowing greater accuracy in the reconstruction of unrecorded responses. The accuracy depends on the number of modal contributions influencing the response, on the number of available sensors, and on the behavior of the structure during the event. If a great number of modes influence the response, a very limited number of sensors (only one for example) are available, and the model parameters of the structure do not exhibit variations during the event, then probably the reconstruction of unknown response based on accurately identified modal parameters would supply more accurate results. If, on the contrary, a limited number of modes (for example two or three) give a significant contribution to the structural response and a number of sensors equal to the number of this fundamental modes are available, then the $\gamma$-method would probably allow to calculate more easily (since it does not require previous identification of modal frequencies and damping) and more accurately (since it reduces the influence of approximations associated with the evaluation of effective participation factors) the unknown responses. Furthermore, if the reconstruction of responses is carried out for structural control aims (for example to generate a signal able to reduce responses at the stories of the building) the $\gamma$-method that does not need the identification of modal parameters has the great advantage of reducing the computational time for unknown responses.

In this paper the accuracy of response parameters estimated from responses reconstructed applying the $\gamma$-method is assessed through comparison with the corresponding quantities derived from recorded signals. The comparison shows that the optimal distribution of a limited number of recording sensors leads to an estimate of performance parameters with maximum errors of about 10% of the real values provided the number of available sensors allows us to model the dominant modes through the spline shape function. Higher the order of the modes contributing to the structural response, higher will be the number of sensors required to obtain a given level of accuracy.

Results reported in this paper are based on the assumption of a linear behavior of the system. Studies are currently underway to check the applicability of the $\gamma$-criterion and of the response reconstruction method to more general systems characterized by different geometry and mechanical behavior.

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