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A MODAL COMBINATION RULE FOR PEAK FLOOR ACCELERATIONS IN MULTISTORIED BUILDINGS

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ABSTRACT

It is useful to estimate peak floor accelerations consistent with the specified seismic hazard for ensuring the safety of rigid nonstructural components in structural systems. A modal combination rule is formulated here to estimate peak floor accelerations in a multistoried building directly in terms of the dynamic properties of the building and pseudo spectral acceleration ordinates of the base excitation. The formulation is developed under the framework of stationary random vibration theory for a linear, lumped-mass, classically damped, multi-degree-of-freedom system with the help of some approximations. A numerical study shows that the proposed rule performs well with the maximum average absolute error in any combination of building and excitation being less than 20% in case of 5% damping. Two simpler SRSS-type variants of the proposed rule, one considering modal cross-correlation and another ignoring this, are also shown to perform reasonably well, particularly when the building is not flexible to the ground motion.

KEYWORDS: Rigid Nonstructural Components, Peak Floor Accelerations, Modal Combination Rule, Pseudo Spectral Acceleration Spectrum, SRSS Method

INTRODUCTION

Safety of a structural system against seismic hazard is ensured in practice by designing it as per the codal provisions in force at its location. This process usually ensures that the main skeleton of the system that consists of beams, columns, shear walls, floor diaphragms, structural connections, etc. remains intact without collapse during the extreme event expected during the life of the structure. Much of the attention paid in the past 30–40 years to the improvement of aseismic design procedures has been devoted to ensuring better and economical performance of such structural components as those provide stability and strength to the structure to survive during the earthquake ground motion. There are, however, nonstructural components also in a building that are attached to the main skeleton at different locations. Those may include masonry panels, parapets, chimneys, ceilings, water heaters, pressure vessels, generators, piping, storage tanks, escalators, equipments, and lighting fixtures, among various possibilities depending on the functional requirements expected of the building. Scant attention has been paid to the task of ensuring the safety of such components, except in critical installations like nuclear power plants, and as a result, there have been numerous cases of large damage, and thus heavy financial losses, in the last 10–15 years even when the damage to the main skeletons was not significant. In some cases, this has even led to undesirable consequences, like hospitals being closed down during the 1994 Northridge earthquake (Hall, 1994). Damage to nonstructural components also poses threat to the lives of the building occupants in the near vicinity.

Nonstructural components respond primarily to the accelerations of the floors on which those are supported. The peak values of these floor accelerations may often be greater than the peak ground acceleration (PGA), depending on the building characteristics and the location of the floor, and thus, the nonstructural components may be effectively subjected to amplified ground motions. There may be a further amplification if the fundamental periods of these components are close to the natural periods of the structural system, resulting in severe damage to the components and to their attachments to the structural system. It is also important that the nonstructural components are not usually as ductile as the supporting structure and, therefore, those may fail even during small-to-moderate magnitude ground motions. It may not always be sufficient to simply anchor these components to the supporting system and, therefore, one may have to properly design these components and their attachments.

Some efforts have been made in the past 10 years to improve the codal provisions to avoid damage to the nonstructural components (see Singh et al. (2006) for an up-to-date review), but much still remains to be done in this direction. The present codal provisions (see, for example, ASCE (2003)) are still oversimplified and do not adequately account for the role of all the governing parameters. As shown by Taghavi and Miranda (2005) and Singh et al. (2006), these provisions may in fact lead to too conservative estimates. Despite the significant research efforts, like those by Singh et al. (1998, 2006), Villaverde (1997), Soong et al. (1998), the present codal provisions for nonstructural components have yet to strike the right balance between simplicity and rigour. It is nevertheless clear that the future provisions in various codes will continue to depend on the use of pseudo-spectral acceleration (PSA) spectrum for the characterization of the input excitation. It is also clear that the future provisions will depend on the estimation of linear response of nonstructural components and the supporting structure, and that the nonlinear behaviour of these components and/or supporting structure will be accounted for via the use of some kind of response modification factor (Rodriguez et al., 2002).

The nonstructural components may be considered as rigid if those are sufficiently stiff to vibrate in phase with their attachment points. For such components it is desirable to properly estimate the absolute floor accelerations consistent with the specified seismic hazard. Restricting discussion just to the use of response-spectrum based techniques, there is no modal combination rule derived till date to predict the peak floor accelerations in a structural system by directly using the response spectrum ordinates. This is despite the fact that several researchers like Goodman et al. (1955), Rosenblueth and Elorduy (1969), Wilson et al. (1981), Singh and Mehta (1983) have proposed schemes to estimate the largest peak in the response of a base-excited linear system by combining the response maxima in different modes, after Biot (1934, 1942) outlined the basic superposition of modal responses in earthquake engineering. Nevertheless, there have been several efforts to estimate the PSA ordinates corresponding to the floor motions, known popularly as floor response spectrum, and since peak floor accelerations are zero-period ordinates of floor response spectra, those response spectrum-based formulations can be theoretically used to estimate the peak floor accelerations as well. For example, the papers by Singh and co-workers (Singh, 1980; Singh and Sharma, 1985; Suarez and Singh, 1987) and Der Kiureghian and co-workers (Der Kiureghian et al., 1983; Igusa and Der Kiureghian, 1985) give elegant formulations to estimate floor response spectra for classically-damped structural systems and directly in terms of ground spectrum input by making varying sets of assumptions. The simplest of these, e.g., that by Singh (1980), is based on ignoring the interaction between the support and supported systems and may thus be used by lumping the mass of the nonstructural component with the supporting mass and by estimating the zero-period ordinate of the floor response spectrum. The formulations including the interaction, e.g., those by Suarez and Singh (1987) and Der Kiureghian et al. (1983), can be used by lumping the mass of the nonstructural component and by estimating the zero-period ordinate of the floor response spectrum for zero value of the supported mass. It is also possible to use a generalized response spectrum formulation (Singh et al., 2006), but clearly there remains a need to develop a closed-form expression or a modal combination rule that can be used to estimate the peak floor accelerations directly in terms of the PSA ordinates and modal properties of a linear structural system.

This study considers the development of a modal combination rule from the power spectral density function (PSDF) of the floor acceleration response of a linear, lumped-mass, multistoried shear building. For this purpose, both excitation and response processes are assumed to be stationary, and the effects of nonstationarity are included in peak floor acceleration via the use of response spectrum ordinates and nonstationarity factors as in Gupta (2002). The modal combination rule and its two simpler variants on the lines of SRSS (square-root-of-sum-of-squares) rule (Goodman et al., 1955) are obtained by making suitable assumptions regarding nonstationarity factors and peak factors. Performance of the proposed rule and its variants is investigated through consideration of three example buildings and six example ground motions.

FORMULATION OF THE PROPOSED RULE

1. PSDF of the Absolute Acceleration Response

Let us consider a symmetric shear building as shown in Figure 1 where the lumped floor masses m_i, $i = 1, 2, ..., n$ are interconnected through massless column springs of stiffnesses k_i, $i = 1, 2, ..., n$, and the viscous dampers representing the interstory dampings of magnitudes c_i, $i = 1,2,...,n$. The building is subjected to the ground acceleration $\ddot{z}(t)$ at its base. The n-coupled equations of motion for this system can be written as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -\ddot{z}[m]\{\Gamma\} \tag{1}$$

where $[m]$, $[c]$ and $[k]$ respectively are the $n \times n$ mass, damping and stiffness matrices in terms of m_i, c_i and k_i, $i = 1,2,...,n$; $\{\Gamma\}$ is the $n \times 1$ ground displacement influence vector; $\{x\}$ is the $n \times 1$ vector comprising of the relative displacements $x_i(t)$, $i = 1,2,...,n$ of the floor masses; and $\{\dot{x}\}$ $(= \frac{d}{dt}\{x\})$, $\{\ddot{x}\}$ $(= \frac{d^2}{dt^2}\{x\})$ are the time derivatives of $\{x\}$. It is assumed that the building is classically damped and therefore the viscous damping matrix $[c]$ can be diagonalized by the transformation $[\Phi]^T[c][\Phi]$ where $[\Phi]$ $(= [\{\phi^{(1)}\}\{\phi^{(2)}\}\cdots\{\phi^{(n)}\}])$ is the $n \times n$ modal matrix of the eigenvectors $\{\phi^{(j)}\}$, $j = 1,2,...,n$ obtained by solving the eigenvalue problem, $\omega^2[m]\{\phi\} = [k]\{\phi\}$. The *j*th element of this diagonal form is denoted as $2\zeta_j\omega_j M_j$ where ω_j and ζ_j respectively are the natural frequency and damping ratio in the *j*th mode and $M_j = \{\phi^{(j)}\}^T[m]\{\phi^{(j)}\}$ is the *j*th modal mass.

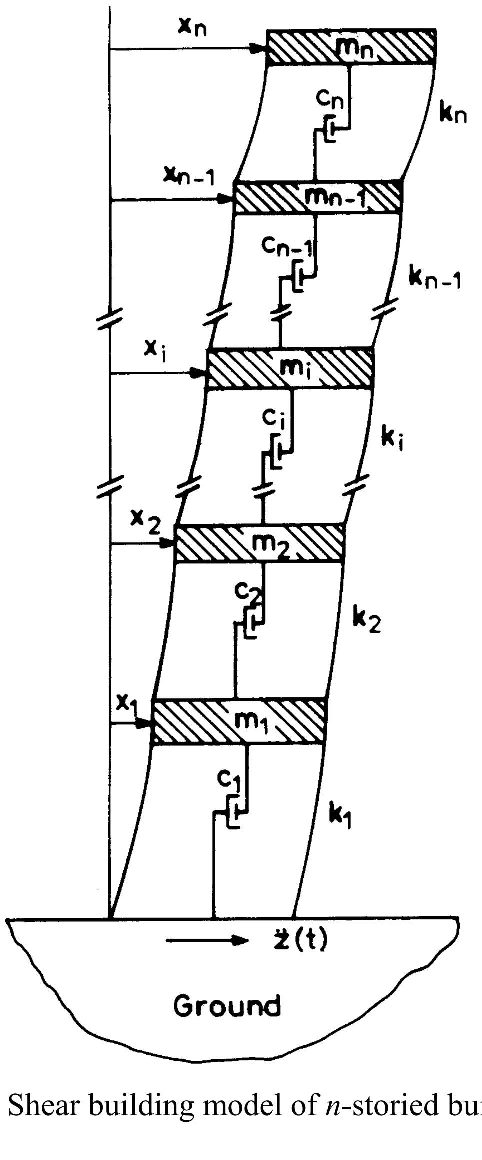

Fig. 1 Shear building model of *n*-storied building

On using normal mode decomposition of the relative displacement of the *i*th floor, the transfer function relating the absolute acceleration response of the *i*th floor to the input ground acceleration may be expressed as

$$H_{a_i}(\omega) = 1 - \omega^2 \sum_{j=1}^{n} \phi_i^{(j)} \alpha_j H_j(\omega) \tag{2}$$

where $\phi_i^{(j)}$ is the *i*th element of the *j*th mode shape vector, $\alpha_j = \{\phi^{(j)}\}^T[m]\{\Gamma\}/M_j$ is the modal participation factor in the *j*th mode, and

$$H_j(\omega) = \frac{-1}{\omega_j^2 - \omega^2 + 2\mathrm{i}\zeta_j\omega_j\omega} \tag{3}$$

(with i = $\sqrt{-1}$) is the transfer function relating the relative displacement of the equivalent SDOF oscillator in the *j*th mode to the input base excitation. On assuming stationarity in the excitation and the response, the PSDF of a response may be obtained by multiplying the PSDF of the excitation with the squared modulus of the corresponding transfer function. The PSDF of the absolute acceleration response of the *i*th floor may thus be expressed as

$$S_{a_i}(\omega) = S_{\ddot{z}}(\omega)\left[1 + \sum_{j=1}^{n}\left\{2\phi_i^{(j)}\alpha_j \mathrm{Re}\left(\bar{H}_j(\omega)\right) + \left(\phi_i^{(j)}\right)^2 \alpha_j^2 \left|\bar{H}_j(\omega)\right|^2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k \mathrm{Re}\left(\bar{H}_j(\omega)\bar{H}_k^*(\omega)\right)\right\}\right] \tag{4}$$

where $S_{\ddot{z}}(\omega)$ is the PSDF of the input base excitation $\ddot{z}(t)$, $\bar{H}_j(\omega)$ $(= -\omega^2 H_j(\omega))$ is the transfer function relating the relative acceleration of the equivalent SDOF oscillator in the *j*th mode to the input base excitation, and $\bar{H}_k^*(\omega)$ is the complex conjugate of $\bar{H}_k(\omega)$. On the right hand side of Equation (4), the second term represents the cross-correlation of the ground acceleration with the relative acceleration response of the *i*th floor, and the third and fourth terms together represent the PSDF of the relative acceleration response of the *i*th floor. In the latter, the term involving the summation over k represents the cross-correlation of the *j*th mode with the remaining $n-1$ modes. On expanding this term by using the partials for $\mathrm{Re}(\bar{H}_j(\omega)\bar{H}_k^*(\omega))$ (Vanmarcke, 1972) and on rearranging terms, Equation (4) becomes

$$S_{a_i}(\omega) = S_{\ddot{z}}(\omega)\left[1 + \sum_{j=1}^{n}\left\{\left(2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)}\alpha_k D_{jk}\right)\phi_i^{(j)}\alpha_j\omega_j^2\left|\omega H_j(\omega)\right|^2 + \left(\phi_i^{(j)}\alpha_j - 2 + \sum_{k=1,k\neq j}^{n}\phi_i^{(k)}\alpha_k\left(C_{jk} - D_{jk}\right)\right)\phi_i^{(j)}\alpha_j\left|\bar{H}_j(\omega)\right|^2\right\}\right] \tag{5}$$

where C_{jk} and D_{jk} are the coefficients given in terms of ζ_j, ζ_k and $\varrho = \omega_k/\omega_j$ as

$$C_{jk} = \frac{1}{B_{jk}}\left[8\varrho\zeta_j\left(\zeta_k + \zeta_j\varrho\right)\left\{\left(1-\varrho^2\right)^2 - 4\varrho\left(\zeta_j - \zeta_k\varrho\right)\left(\zeta_k - \zeta_j\varrho\right)\right\}\right] \tag{6}$$

$$D_{jk} = \frac{1}{B_{jk}}\left[2\left(1-\varrho^2\right)\left\{4\varrho\left(\zeta_j - \zeta_k\varrho\right)\left(\zeta_k - \zeta_j\varrho\right) - \left(1-\varrho^2\right)^2\right\}\right] \tag{7}$$

with

$$B_{jk} = 8\varrho^2\left[\left(\zeta_j^2 + \zeta_k^2\right)\left(1-\varrho^2\right)^2 - 2\left(\zeta_k^2 - \zeta_j^2\varrho^2\right)\left(\zeta_j^2 - \zeta_k^2\varrho^2\right)\right] + \left(1-\varrho^2\right)^4 \tag{8}$$

2. Largest Peak of the Absolute Acceleration Response

Ordered peaks (largest, second largest, third largest, …) of a response process are estimated in stationary random vibration theory by (i) computing moments of the PSDF of the process, (ii) computing root-mean-square (r.m.s.) value of the process and peak factors for the ordered peaks from the moments, and (iii) by multiplying the r.m.s. value with the peak factors (Gupta and Trifunac, 1988; Gupta, 2002). This procedure is followed in this section to formulate the expression for the largest peak of the absolute acceleration response at the *i*th floor.

Taking *p*th moment of $S_{a_i}(\omega)$ about the origin leads to

$$\lambda_p^{a_i} = \lambda_p^G + \sum_{j=1}^{n}\Bigg[\Bigg\{2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)}\alpha_k D_{jk}\Bigg\}\phi_i^{(j)}\alpha_j\omega_j^2\lambda_{p,j}^V + \Bigg\{\phi_i^{(j)}\alpha_j - 2 + \sum_{k=1,k\neq j}^{n}\phi_i^{(k)}\alpha_k\left(C_{jk} - D_{jk}\right)\Bigg\}\phi_i^{(j)}\alpha_j\lambda_{p,j}^A\Bigg]; \quad p = 0,1,2,... \tag{9}$$

In this equation,

$$\lambda_p^G = \int_0^\infty \omega^p S_{\ddot{z}}(\omega)\mathrm{d}\omega \tag{10}$$

is the *p*th moment of the PSDF of the ground acceleration process, $\ddot{z}(t)$;

$$\lambda_{p,j}^V = \int_0^\infty \omega^p S_{\ddot{z}}(\omega)\left|\omega H_j(\omega)\right|^2 \mathrm{d}\omega \tag{11}$$

is the *p*th moment of the PSDF of the relative velocity response of a SDOF oscillator with ω_j frequency and ζ_j damping ratio and subjected to the base acceleration $\ddot{z}(t)$; and

$$\lambda_{p,j}^A = \int_0^\infty \omega^p S_{\ddot{z}}(\omega)\left|\bar{H}_j(\omega)\right|^2 \mathrm{d}\omega \tag{12}$$

is the *p*th moment of the PSDF of the relative acceleration response of this oscillator.

By calculating the moments from Equation (9) for p = 0, 2 and 4 and then multiplying the r.m.s. value $(=\sqrt{\lambda_0^{a_i}})$ with a suitable peak factor, the largest peak amplitude of the desired absolute acceleration response can be determined at a given level of confidence. The peak factor depends on $\lambda_0^{a_i}$, $\lambda_2^{a_i}$, $\lambda_4^{a_i}$, strong motion duration of the excitation, and the level of confidence at which the response amplitude is to be obtained (see Gupta (2002) for details). The largest peak amplitude so obtained has to be multiplied with a suitable nonstationarity factor in order to account for the fact that the response process is not a stationary process. Such a factor may be close to unity if $S_{\ddot{z}}(\omega)$ is a spectrum-compatible PSDF (see, for example, Kaul (1978), Unruh and Kana (1981), Christian (1989)). Thus, the largest peak amplitude of the absolute acceleration of the *i*th floor becomes

$$a_{i,\max} = \eta^{a_i}\beta^{a_i}\Bigg[\lambda_0^G + \sum_{j=1}^{n}\Bigg\{\Bigg(2 + \sum_{k=1,k\neq j}^{n}\phi_i^{(k)}\alpha_k D_{jk}\Bigg)\phi_i^{(j)}\alpha_j\omega_j^2\lambda_{0,j}^V + \Bigg(\phi_i^{(j)}\alpha_j - 2 + \sum_{k=1,k\neq j}^{n}\phi_i^{(k)}\alpha_k\left(C_{jk} - D_{jk}\right)\Bigg)\phi_i^{(j)}\alpha_j\lambda_{0,j}^A\Bigg\}\Bigg]^{\frac{1}{2}} \tag{13}$$

where η^{a_i} is the corresponding peak factor and β^{a_i} is the nonstationarity factor, for this response process.

In continuation with the above logic, λ_0^G in Equation (13) may be expressed as $(PGA/\eta^G\beta^G)^2$, where PGA is the largest peak amplitude of the ground acceleration process $\ddot{z}(t)$ for the same level of confidence to which η^{a_i} corresponds, η^G is the corresponding peak factor, and β^G is the nonstationarity factor for the acceleration process. In the same way, $\lambda_{0,j}^V$ may be expressed as $(SV_j/\eta_j^V\beta_j^V)^2$, where SV_j is the largest peak amplitude of the relative velocity response of the SDOF oscillator with ω_j frequency and ζ_j damping ratio in response to the excitation process $\ddot{z}(t)$, η_j^V is the corresponding peak factor, and β_j^V is the nonstationarity factor associated with the relative velocity response process. Further, $\lambda_{0,j}^A$ may be expressed as $(RSA_j/\eta_j^A\beta_j^A)^2$, where RSA_j is the largest peak amplitude of the relative acceleration response of the SDOF oscillator (with ω_j frequency and ζ_j damping ratio) in response to the excitation process $\ddot{z}(t)$, η_j^A is the corresponding peak factor, and β_j^A is the nonstationarity factor associated with the relative acceleration response process. Thus, $a_{i,\max}$ may be expressed as

$$a_{i,\max} = \left[\left(\frac{\eta^{a_i}}{\eta^G}\right)^2 \left(\frac{\beta^{a_i}}{\beta^G}\right)^2 PGA^2 + \sum_{j=1}^{n}\left\{\left(2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)}\alpha_k D_{jk}\right)\phi_i^{(j)}\alpha_j\omega_j^2 \left(\frac{\eta^{a_i}}{\eta_j^V}\right)^2 \left(\frac{\beta^{a_i}}{\beta_j^V}\right)^2 SV_j^2 \right.\right.$$
$$\left.\left. + \left(\phi_i^{(j)}\alpha_j - 2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)}\alpha_k\left(C_{jk} - D_{jk}\right)\right)\phi_i^{(j)}\alpha_j \left(\frac{\eta^{a_i}}{\eta_j^A}\right)^2 \left(\frac{\beta^{a_i}}{\beta_j^A}\right)^2 RSA_j^2 \right\}\right]^{\frac{1}{2}} \quad (14)$$

In the next section, suitable approximations will be made to develop a modal combination rule from this equation.

3. Approximations for the Proposed Rule

One can use Equation (14) to estimate the largest peak amplitude of the absolute acceleration at the ith floor for the same level of confidence for which PGA, SV_j and RSA_j have been estimated. Hence, if the seismic design levels at a site are characterized by certain PGA, and spectral velocity (SV) and relative spectral acceleration (RSA) curves, this equation can be used to estimate the largest floor acceleration at the ith floor consistent with these design levels. For this, however, one needs to have reasonable estimates of η ratios (i.e., η^{a_i}/η^G, η^{a_i}/η_j^V and η^{a_i}/η_j^A) and β ratios (i.e., β^{a_i}/β^G, β^{a_i}/β_j^V and β^{a_i}/β_j^A). Further, it is unusual to have the SV and RSA curves available in a design situation. Suitable approximations, therefore, need to be made in order to obtain a useful expression for the peak floor accelerations from Equation (14).

It is proposed to first assume that various η and β ratios are unity. It will be shown in the next section through numerical examples that these ratios are usually not unity. While the η ratios are not very far from unity, the β ratios show considerable scatter around their mean values, depending on the characteristics of the structural system and excitation. However, since the mean beta ratios for the first few modes stay close to unity, this assumption is deemed to be appropriate.

Secondly, pseudo-spectral velocity (PSV) curves may be used in place of the SV curves as per the existing engineering practice. The RSA values may also be estimated approximately from the knowledge of PGA, pseudo-spectral acceleration (PSA), and energy distribution in the ground motion. As shown by Trifunac and Gupta (1991), this approximation of RSA, known as PRSA (pseudo-relative spectral acceleration), may be expressed as

$$\begin{aligned} PRSA(T) &= \sqrt{\{PSA(T)\}^2 - PGA^2}; \quad T \le T_c \\ PRSA(T) &= \sqrt{\{PSA(T)\}^2 + PGA^2}; \quad T > T_c \end{aligned} \quad (15)$$

where $PRSA(T)$ and $PSA(T)$, respectively, are the PRSA and PSA values for the SDOF oscillator of period T, and T_c is the period corresponding to the centre of gravity of the Fourier spectrum $|\ddot{Z}(\omega)|$ of ground motion. T_c will be referred to in this study as the mean period of ground motion.

Incorporating the above approximations, Equation (14) leads to the proposed modal combination rule as

$$a_{i,\max} = \left[PGA^2 + \sum_{j=1}^{n} \left\{ \left(2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k D_{jk} \right) \phi_i^{(j)} \alpha_j PSA_j^2 \right.\right.$$
$$\left.\left. + \left(\phi_i^{(j)} \alpha_j - 2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k \left(C_{jk} - D_{jk} \right) \right) \phi_i^{(j)} \alpha_j PRSA_j^2 \right\} \right]^{\frac{1}{2}} \quad (16)$$

Since the PSDF of the relative acceleration response at the ith floor cannot be negative, it is necessary to apply the following check in the above rule:

$$\sum_{j=1}^{n} \left[\sum_{k=1,k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k D_{jk} PSA_j^2 + \left(\phi_i^{(j)} \alpha_j + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k \left(C_{jk} - D_{jk} \right) \right) \phi_i^{(j)} \alpha_j PRSA_j^2 \right] \geq 0 \quad (17)$$

If this check fails (due to the various approximations that have been made), the summation of terms on the left-hand side of (17) should be taken as zero. In view of this, the proposed rule may be expressed as

$$a_{i,\max} = \left[PGA^2 + \sum_{j=1}^{n} 2\phi_i^{(j)} \alpha_j \left\{ PSA_j^2 - PRSA_j^2 \right\} + ra_{i,\max}^2 \right]^{\frac{1}{2}} \quad (18)$$

or, depending on how the mean period of ground motion compares with the natural periods of the system, T_j (= $2\pi/\omega_j$), $j = 1, 2, ..., n$,

$$a_{i,\max} = \left[PGA^2 \left(1 + \sum_{j=1}^{n} 2\phi_i^{(j)} \alpha_j \right) + ra_{i,\max}^2 \right]^{\frac{1}{2}}; \; T_1 < T_c$$
$$= \left[PGA^2 \left(1 - \sum_{j=1}^{n} 2\phi_i^{(j)} \alpha_j \right) + ra_{i,\max}^2 \right]^{\frac{1}{2}}; \; T_n > T_c \quad (19)$$
$$= \left[PGA^2 \left(1 - \sum_{j=1}^{\hat{n}} 2\phi_i^{(j)} \alpha_j + \sum_{j=\hat{n}+1}^{n} 2\phi_i^{(j)} \alpha_j \right) + ra_{i,\max}^2 \right]^{\frac{1}{2}}; \; T_{\hat{n}+1} < T_c < T_{\hat{n}}$$

where

$$ra_{i,\max}^2 = \sum_{j=1}^{n} \left[\sum_{k=1,k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k D_{jk} PSA_j^2 + \left(\phi_i^{(j)} \alpha_j + \sum_{k=1,k\neq j}^{n} \phi_i^{(k)} \alpha_k \left(C_{jk} - D_{jk} \right) \right) \phi_i^{(j)} \alpha_j PRSA_j^2 \right] \quad (20)$$

or 0, whichever is greater

A simpler variant of the proposed rule may be obtained by ignoring the cross-correlation between the ground acceleration and the relative floor acceleration. In that case, we obtain

$$a_{i,\max} = \left[PGA^2 + ra_{i,\max}^2 \right]^{\frac{1}{2}} \quad (21)$$

with restriction as in Equation (20) remaining applicable. Further, on ignoring the cross-correlation of the jth mode with the remaining $n-1$ modes (in the relative acceleration response), a further simpler variant of the proposed rule is obtained as

$$a_{i,\max} = \left[PGA^2 + \sum_{j=1}^{n} \left(\phi_i^{(j)} \right)^2 \alpha_j^2 PRSA_j^2 \right]^{\frac{1}{2}} \quad (22)$$

Since this form of the modal combination rule does not consider any cross-correlation, it will be referred to in this study as the SRSS rule for the peak floor acceleration response. Further, since Equation (21) ignores only the cross-correlation between the ground and relative floor accelerations, it will be referred to as the quasi-SRSS rule for the peak floor acceleration response.

It may be noted that both variants of the proposed rule lead to peak floor accelerations greater than or equal to PGA. Peak floor accelerations may however be less than PGA, when $\sum_{j=1}^{\hat{n}} \phi_i^{(j)} \alpha_j$ >

$\sum_{j=\hat{n}+1}^{n} \phi_i^{(j)} \alpha_j$ and the peak relative acceleration $ra_{i,\max}$ is not large enough in comparison with PGA. Such a situation may arise when the structural system is flexible with respect to the ground motion. If the system is only moderately flexible and just a few of its significant modes have periods greater than the mean period of the ground motion, lower floors may experience "less than PGA" peak accelerations. This may however be true for upper floors also when the system is very flexible and the periods of several significant modes exceed the mean period. It is shown in the next section that both variants usually lead to conservative to overconservative estimates of peak floor accelerations and are more suitable for use when the structural system is not flexible with respect to the ground motion.

NUMERICAL ILLUSTRATION OF THE PROPOSED RULE

1. Example Buildings and Excitations

In order to illustrate the proposed rule, six earthquake ground motions with the details as in Table 1 are considered. Five of these motions (Nos. 1–3 and 5–6) are recorded motions while one motion (No. 4) has been synthetically generated for a Mexico City site during the 1985 Michoacan earthquake (see Gupta and Trifunac (1990) for details). The Fourier spectra of these motions (as normalized to the unit maximum value) are shown in Figures 2(a)–2(f) (solid lines without dots). Also shown are the 5%-damping PSA spectra after normalization with respect to their respective maxima (solid lines with dots). It may be observed that all six motions cover a wide range of energy distributions. The dominant period in these motions varies from about 0.48 s in the Parkfield motion (see Figure 2(e)) to about 5.5 s in the Borrego Mountain and San Fernando motions (see Figures 2(a) and 2(f)). The Michoacan motion (see Figure 2(d)) is also a long-period motion with the dominant period of about 2.6 s. The Imperial Valley motion (see Figure 2(b)) and the Kern County motion (see Figure 2(c)) are medium-period motions with dominant periods as 0.85 and 0.65 s, respectively. In terms of the band of significant energy, the Michoacan motion is on one extreme with significant energy over a narrow band of 1.8–3 s. The Kern County motion is on another extreme with significant energy over a large band of 0.2–5 s. In the San Fernando motion also, the energy is concentrated in a narrow band of periods, while in the Imperial Valley motion, the band of energy is fairly wide. The remaining two motions (Borrego Mountain and Parkfield) fall in between with the band of energy not being narrow or wide.

Table 1: Details of the Example Ground Motions

Record No.	Earthquake	Site	Component
1	Borrego Mountain Earthquake, 1968	Engineering Building, Santa Ana, Orange County, California	S04E
2	Imperial Valley Earthquake, 1940	El Centro Site, Imperial Valley Irrigation District, California	S00E
3	Kern County Earthquake, 1952	Taft Lincoln School Tunnel, California	N21E
4	Michoacan Earthquake, 1985	Mexico City	Synthetic
5	Parkfield Earthquake, 1966	Array No. 5, Cholame, Shandon, California	N05W
6	San Fernando Earthquake, 1971	Utilities Building, 215 West Broadway, Long Beach, California	N90E

Three example buildings are considered such that the range of fundamental periods typically found in multistoried buildings is covered to a large extent. The first example building, henceforth denoted as Building-1 (or BD1), is a 24-story symmetric building with 2 s as its fundamental period. This building is same as that considered by Singh et al. (2003). The second example building (Building-2 or BD2) is a 15-story symmetric building with 1.2 s as its fundamental period. This building is similar to that considered by Ray Chaudhuri and Gupta (2003). Building-3 (or BD3) is the third example (symmetric) building with 5 stories and 0.514 s fundamental period. This example building is similar to that considered by Hu et al. (2007). The values of floor masses and story stiffnesses for the three example buildings are given in Table 2, and natural frequencies in various modes of vibration are given in Table 3. The fundamental periods of the example buildings are also indicated in Figures 2(a)–2(f) (dashed lines) in order to clearly show the relative stiffnesses of the example buildings with respect to the example ground motions. On assuming that the effects of soil-structure interaction are negligibly small, it may be observed that BD1 is very stiff to the San Fernando motion, stiff to the Michoacan and Borrego Mountain motions, flexible to the Imperial Valley and Kern County motions, and very flexible to the Parkfield motion. On the other hand, BD3 is very stiff to the Borrego Mountain, Michoacan and San Fernando motions, little stiff to Imperial Valley and Kern County motions, and is in near resonance with the Parkfield motion. The example buildings are assumed to be classically damped with damping ratio of 0.05 in all modes.

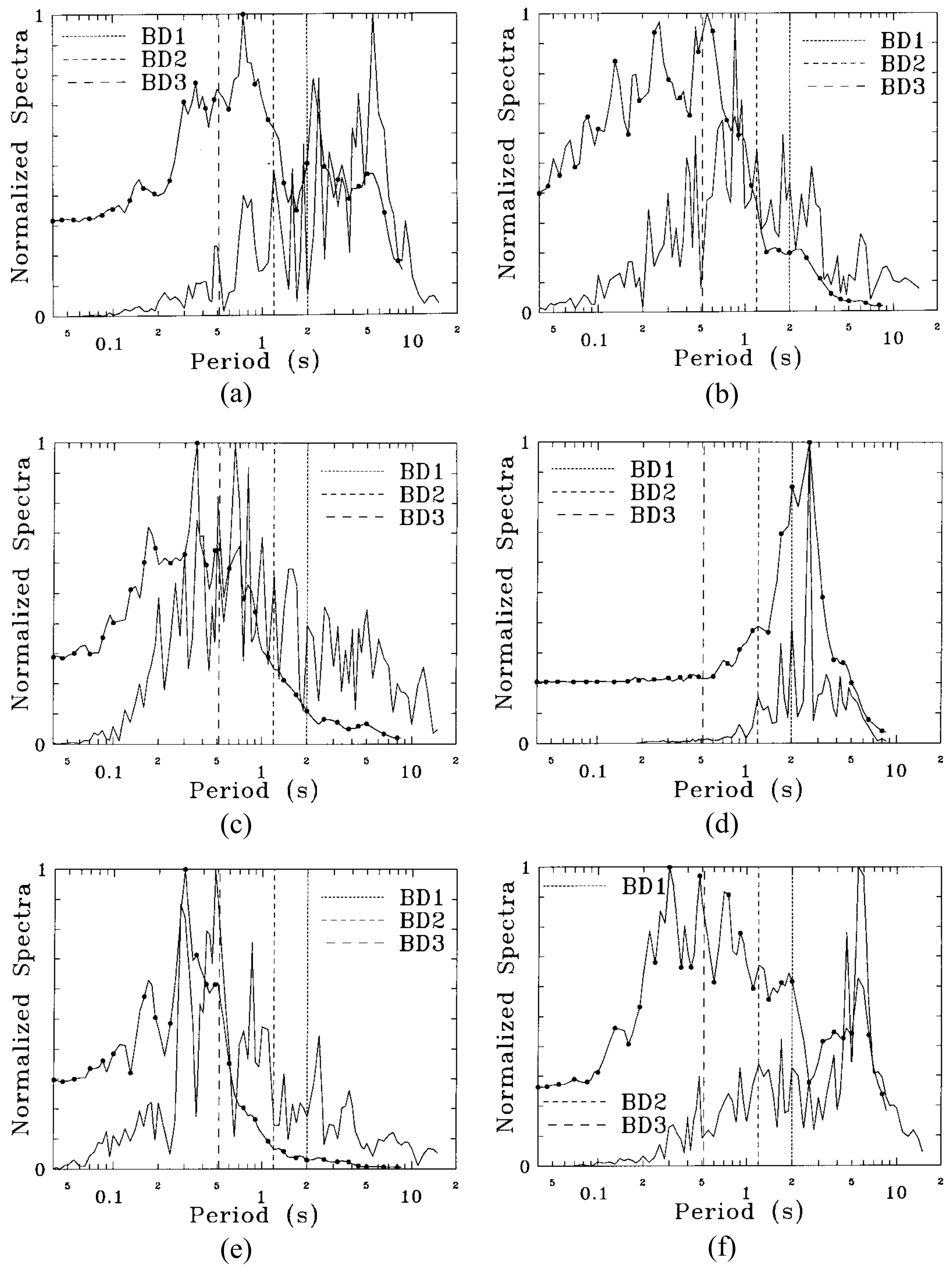

Fig. 2 Normalized Fourier amplitude spectrum (solid line without dots) and PSA spectrum (solid line with dots) for (a) Borrego Mountain, (b) Imperial Valley, (c) Kern County, (d) Michoacan, (e) Parkfield, and (f) San Fernando earthquake motions (dashed lines indicate the fundamental periods for BD1, BD2 and BD3)

Table 2: Mass and Stiffness Properties of the Example Buildings

i	Floor Mass m_i (t)			Story Stiffness k_i (kN/mm)		
	BD1	BD2	BD3	BD1	BD2	BD3
1	7,426	280	166	6650	525	290
2	7,426	200	166	6260	536	290
3	6,918	200	166	5880	536	290
4	6,970	200	166	5880	536	290
5	5,849	200	141	5510	536	290
6	5,587	200		5480	536	
7	5,569	200		5480	536	
8	4,063	200		5100	536	
9	3,678	200		5010	536	
10	3,678	200		5010	536	
11	3,678	200		4960	536	
12	3,415	200		4920	536	
13	3,415	200		4920	536	
14	2,855	200		4720	536	
15	2,469	200		4670	536	
16	2,469			4670		
17	2,329			4610		
18	1,769			4220		
19	1,769			4220		
20	1,524			4260		
21	1,278			4240		
22	1,261			4260		
23	928			4250		
24	771			4420		

Table 3: Natural Frequencies of the Example Buildings

Mode No.	Frequencies (Hz)		
	BD1	BD2	BD3
1	0.50	0.83	1.94
2	1.24	2.49	5.65
3	2.00	4.10	8.86
4	2.78	5.64	11.29
5	3.51	7.11	12.78
6	4.30	8.50	
7	4.97	9.83	

8	5.70	11.09	
9	6.34	12.26	
10	6.98	13.32	
11	7.58	14.25	
12	8.09	15.03	
13	8.61	15.65	
14	8.96	16.11	
15	9.37	16.38	
16	9.80		
17	10.51		
18	11.10		
19	11.59		
20	12.39		
21	13.24		
22	14.49		
23	16.42		
24	19.33		

2. Results and Discussion: η and β Ratios

The proposed rule is based on assuming η and β ratios uniformly as unity. It will, therefore, be first seen via a numerical study how good this assumption is. For this purpose, it is assumed that the response spectra associated with the example ground motions represent expected levels of the largest peak responses to the ground motion processes to which these example motions correspond. The peak floor acceleration responses of the example buildings, as computed from the time-history analyses for the example motions, also thus correspond to the expected levels of the responses to these processes. The PSDF of the base excitation process, $S_{\ddot{z}}(\omega)$, is obtained in case of each example motion by dividing the squared Fourier spectrum of the record by πT_s where T_s is the strong motion duration of the record given by Trifunac and Brady (1975). The values of T_s are obtained as 54.74, 24.44, 30.54, 47.28, 7.52, 44.34 s respectively for the Borrego Mountain, Imperial Valley, Kern County, Michoacan, Parkfield and San Fernando motions. Further, for the calculation of PRSA curves, the mean period T_c is obtained as 0.38, 0.17, 0.25, 0.96, 0.19, 0.39 s respectively for these motions.

The calculations of η^{a_i} and β^{a_i} are based on the expression of PSDF $S_{a_i}(\omega)$, as given in Equation (5), and on the maximum values of the absolute acceleration $a_i(t)$, as obtained via time-history analyses. The calculations of η^G and β^G are based on the $S_{\ddot{z}}(\omega)$ and PGA values. Further, η_j^V and β_j^V are computed from the use of PSDF, $|\omega H_j(\omega)|^2 S_{\ddot{z}}(\omega)$, and SV curve, and η_j^A and β_j^A from the use of PSDF, $|\overline{H}_j(\omega)|^2 S_{\ddot{z}}(\omega)$, and RSA curve.

The results for the η and β ratios are shown in Figures 3(a)–3(c) and 4(a)–4(d), respectively. Figures 3(a) and 4(a) show the results for η^{a_i}/η^G and β^{a_i}/β^G, respectively, in the case of the three example buildings (Building Nos. 1–3 referring to BD1, BD2 and BD3, respectively). Each of these figures shows the scatter of 6n values (for six ground motions and n floors) of the η or β ratio for each example building. The solid lines depict the respective average values while the dashed line shows the value assumed for developing the proposed rule. Figures 3(b) and 3(c) show the plots for η^{a_i}/η_j^V and η^{a_i}/η_j^A, respectively, in the case of BD1. Figures 4(b) and 4(c) show the plots for β^{a_i}/η_j^V in the case of BD1 and BD2, respectively. Further, Figure 4(d) shows the plot for β^{a_i}/η_j^A in the case of BD1. Each of these figures (i.e., 3(b)–3(c) and 4(b)–4(d)) shows the scatter of 6*n* values (of η or β ratio) for each of the n modes. The solid curve depicts the variation of average ratio with the mode number, while the dashed line shows the assumed value.

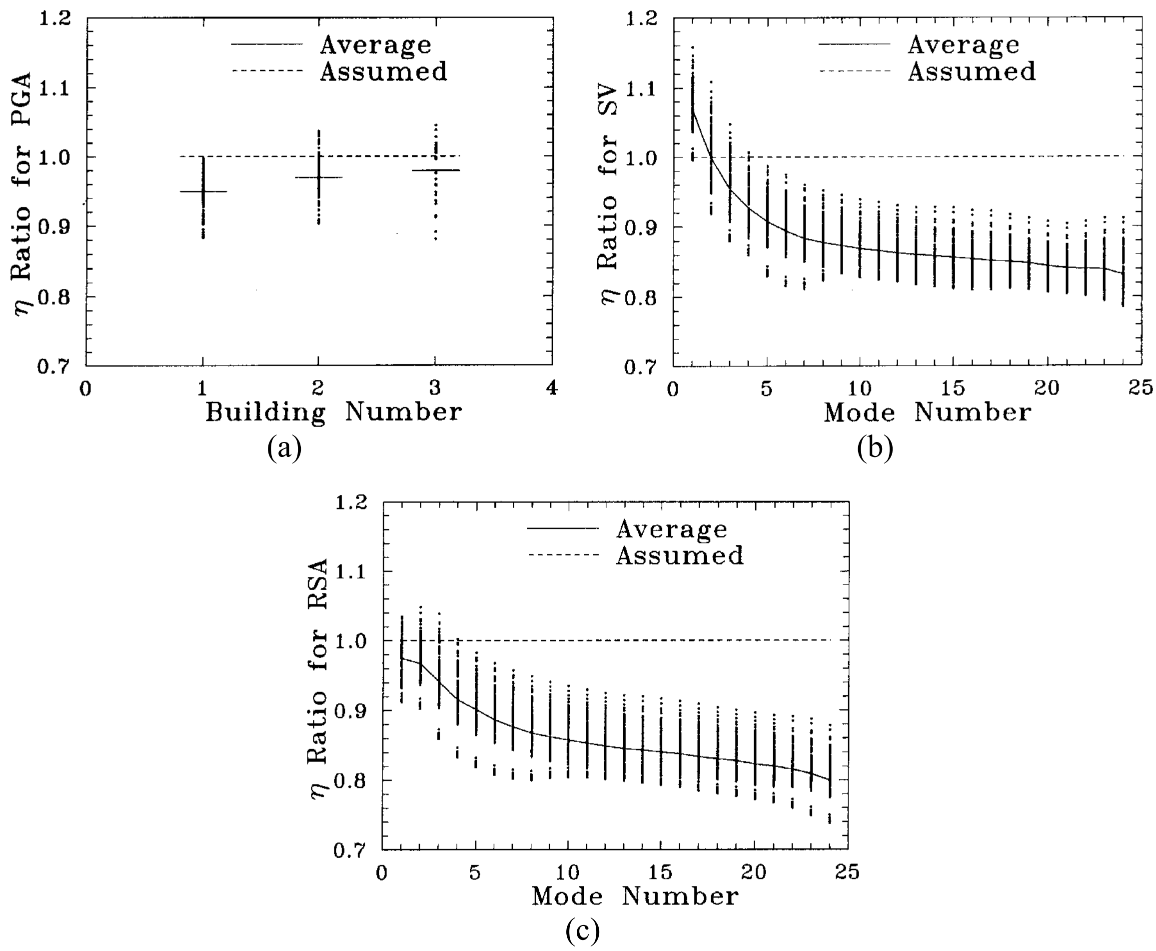

Fig. 3 (a) η^{a_i}/η^G ratio values for Building-1 (BD1), Building-2 (BD2) and Building-3 (BD3); (b) η^{a_i}/η_j^V ratio values for BD1; (c) η^{a_i}/η_j^A ratio values for BD1

It may be observed from Figures 3(a)–3(c) (and more such figures in Kumari (2007)) that all cases are associated with small scatters (within a range of about 0.15) in η ratios. Further, the average η ratio appears to depend on the mode number (in the cases of η^{a_i}/η_j^V and η^{a_i}/η_j^A), decreasing approximately in an exponential manner with an increase in the mode number; in the case of BD1 (see Figure 3(b)), it decreases from 1.07 to 0.83. This is expected as higher modes are associated with greater number of peaks and thus with higher peak factors (Gupta (1994)). In the case of η^{a_i}/η_j^V (see Figure 3(b)), it appears reasonable to assume the ratio as unity because the first mode that contributes maximum to the total response is associated with a ratio greater than or equal to unity. In the case of η ratios for PGA and RSA spectra, a value slightly less than unity could be assumed. However, for simplicity in the proposed rule, it has been preferred to continue with the value of unity.

In comparison with the η ratios, the β ratios are associated with much larger scatters as shown by Figures 4(a)–4(d). The range of scatter can be as large as 1.8 (see the 1st mode in Figure 4(c), and 20th mode in Figure 4(d)) and the use of mean value in such a case may not be justified. The observed large scatter is due to the results for different ground motions taken together, as the characteristics of ground motions seem to affect the β ratios significantly. To illustrate, if the results for each ground motion are considered separately in the case of the β^{a_i}/β_j^V ratio for BD1, the coefficient of variation (COV) in any mode can vary from 0.034 in the Michoacan motion to 0.125 in the Parkfield motion. This is much less than the minimum COV value of 0.236 (the maximum value is 0.516), obtained for all six ground motions taken together (as in Figure 4(b)). It is, therefore, clear that each of the β ratios should be correlated with the ground motion characteristics. Further, if we ignore scatter in the β ratios and look at their average values, it is observed that these ratios can be as large as 1.02 and as small as 0.79 in the case of PGA, as large as 1.18 and as small as 0.57 in the case of SV, and as large as 1.00 and as small as 0.48 in the case of RSA. Therefore, the β ratios should also be correlated with (i) the fundamental period of the building, and (ii) the mode number in the case of the β^{a_i}/β_j^V and β^{a_i}/β_j^A ratios (these ratios appear to decrease with increase in the mode number, though in an irregular fashion). However, considering that the average β^{a_i}/β^G ratio is close to unity and that β^{a_i}/β_j^V and β^{a_i}/β_j^A ratios are close to unity for the fundamental mode in each case, it has been decided to assume all β ratios to be uniformly equal to unity. These ratios could perhaps be assumed as 0.9 in the cases of β^{a_i}/β^G and η^{a_i}/η_j^A. However, this value is not very different from 1.0, and therefore, a uniform value of 1.0 has been assumed for the sake of simplicity.

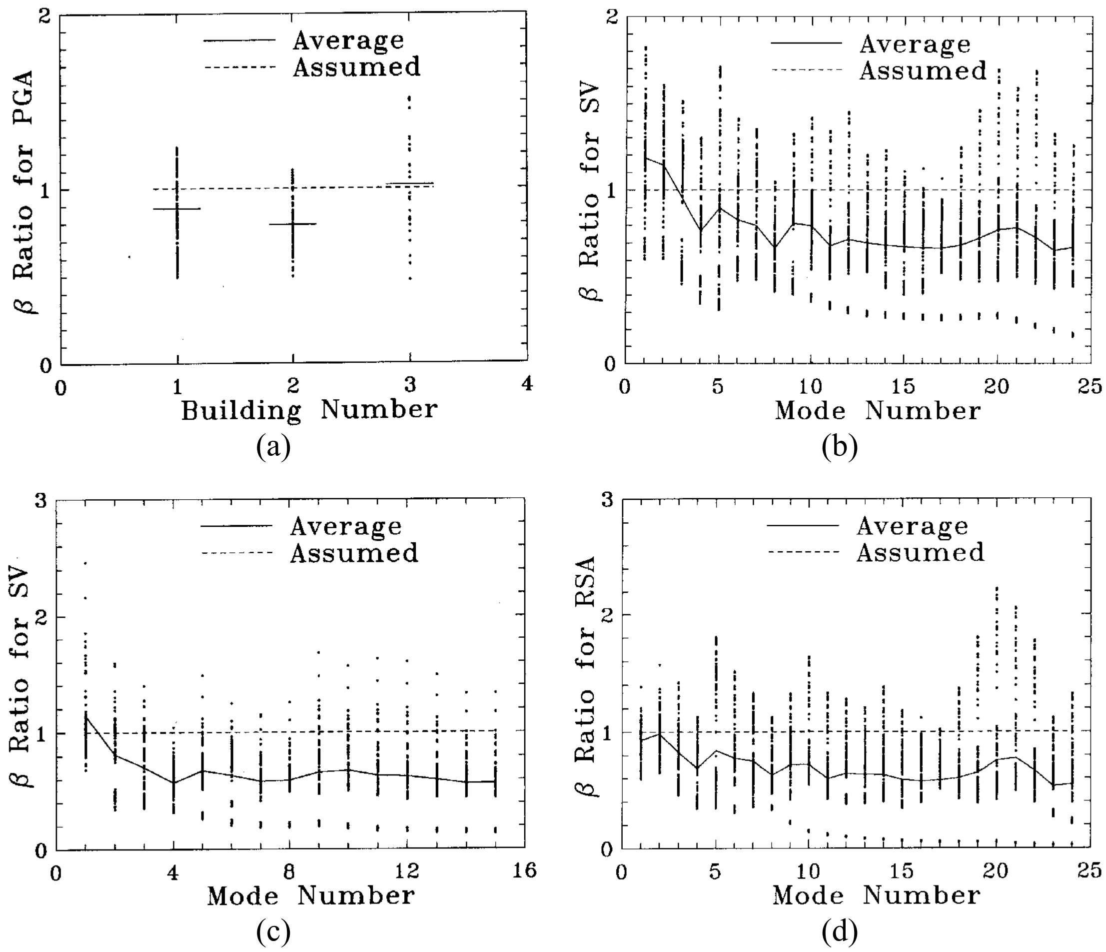

Fig. 4 (a) β^{a_i}/β^G ratio values for Building-1 (BD1), Building-2 (BD2) and Building-3 (BD3); (b) β^{a_i}/β_j^V ratio values for BD1; (c) β^{a_i}/β_j^V ratio values for BD2; (d) β^{a_i}/β_j^A ratio values for BD1

3. Results and Discussion: Proposed Rule and Its Variants

To illustrate and compare performances of the proposed rule and its variants (SRSS and quasi-SRSS), the three example buildings have been subjected to the six example ground motions and the estimates of peak floor accelerations obtained from (i) the (exact) time-history analysis, (ii) the proposed rule (see Equations (18) and (20)), (iii) the SRSS variant (see Equation (22)), and (iv) the quasi-SRSS variant (see Equations (20) and (21)). Absolute error averaged over all floors has been calculated for all 18 combinations of example buildings and ground motions for the 'proposed', 'SRSS' and 'quasi-SRSS' rules and shown in Table 4. It is clear from this table that the performance of the proposed rule is quite good with the average error being less than 10% in most cases. The maximum average error of 19.9% is observed in the case of BD1 subjected to the Parkfield motion, and the minimum average error of 2.26% is observed in the case of BD1 subjected to the Michoacan motion. Since BD1 is very flexible with respect to the Parkfield motion, it appears that the approximations made in developing the proposed rule are most inappropriate when the structural system is very flexible to the ground motion. Further, since BD1 is stiff with respect to the Michoacan motion, these approximations may be most appropriate when the system is relatively very stiff.

Table 4: Comparison of the Averaged Percentage Absolute Error (over Floors) in Peak Floor Acceleration for 5% Damping Ratio

Record No.	1	2	3	4	5	6
Example Building	BD1					
Proposed*	12.67	8.18	11.73	2.26	<u>19.88</u>	<u>6.07</u>
SRSS**	28.50	58.73	39.78	5.20	56.56	24.37
Quasi-SRSS***	27.50	52.19	33.92	8.35	47.58	24.02
Singh et al. (2006)	16.23	15.82	17.91	23.35	16.62	21.94
Example Building	BD2					
Proposed*	<u>15.75</u>	7.03	<u>11.81</u>	2.56	12.31	4.09
SRSS**	25.25	46.93	29.87	19.60	83.93	13.45
Quasi-SRSS***	28.97	49.42	30.91	20.85	84.00	14.29
Singh et al. (2006)	38.58	30.07	18.06	40.87	37.70	30.34
Example Building	BD3					
Proposed*	4.17	<u>9.29</u>	4.62	<u>3.56</u>	4.07	5.68
SRSS**	12.93	7.20	16.91	0.65	20.44	11.01
Quasi-SRSS***	15.44	10.51	12.98	9.05	15.73	7.63
Singh et al. (2006)	106.73	85.31	101.71	122.84	109.94	120.26
*Equations (18) and (20); **Equation (22); ***Equations (20) and (21)						

The envelopes of floor accelerations for the worst case for each ground motion (see the error figures underlined in Table 4) are compared in Figures 5(a)–5(f). Figures 5(e) and 5(f) show the comparisons for BD1 in the cases of Parkfield and San Fernando motions, respectively. Figures 5(a) and 5(c) show the comparisons for BD2 in the cases of Borrego Mountain and Kern County motions, respectively. Figures 5(b) and 5(d) show the comparisons for BD2 in the cases of Imperial Valley and Michoacan motions, respectively. Comparisons for the remaining cases may be seen in Kumari (2007). In each of Figures 5(a)–5(f), 'S' refers to the results from the SRSS variant of the proposed rule, 'E' refers to the exact results, 'P' refers to the results from the proposed rule, and 'Q' refers to the quasi-SRSS variant of the proposed rule. It is seen from these figures that the results of the proposed rule follow the exact results well despite these being the worst cases for each ground motion. The results of the SRSS and quasi-SRSS variants follow the exact results on the conservative side. The results in Figures 5(c)–5(f) do not also support the assumptions of ASCE (2003) regarding the (i) linear variation of peak floor acceleration with height, and (ii) peak roof accelerations as much as three times the peak ground accelerations. As shown by Figure 5(f), peak roof acceleration may even be greater than three times the peak ground acceleration.

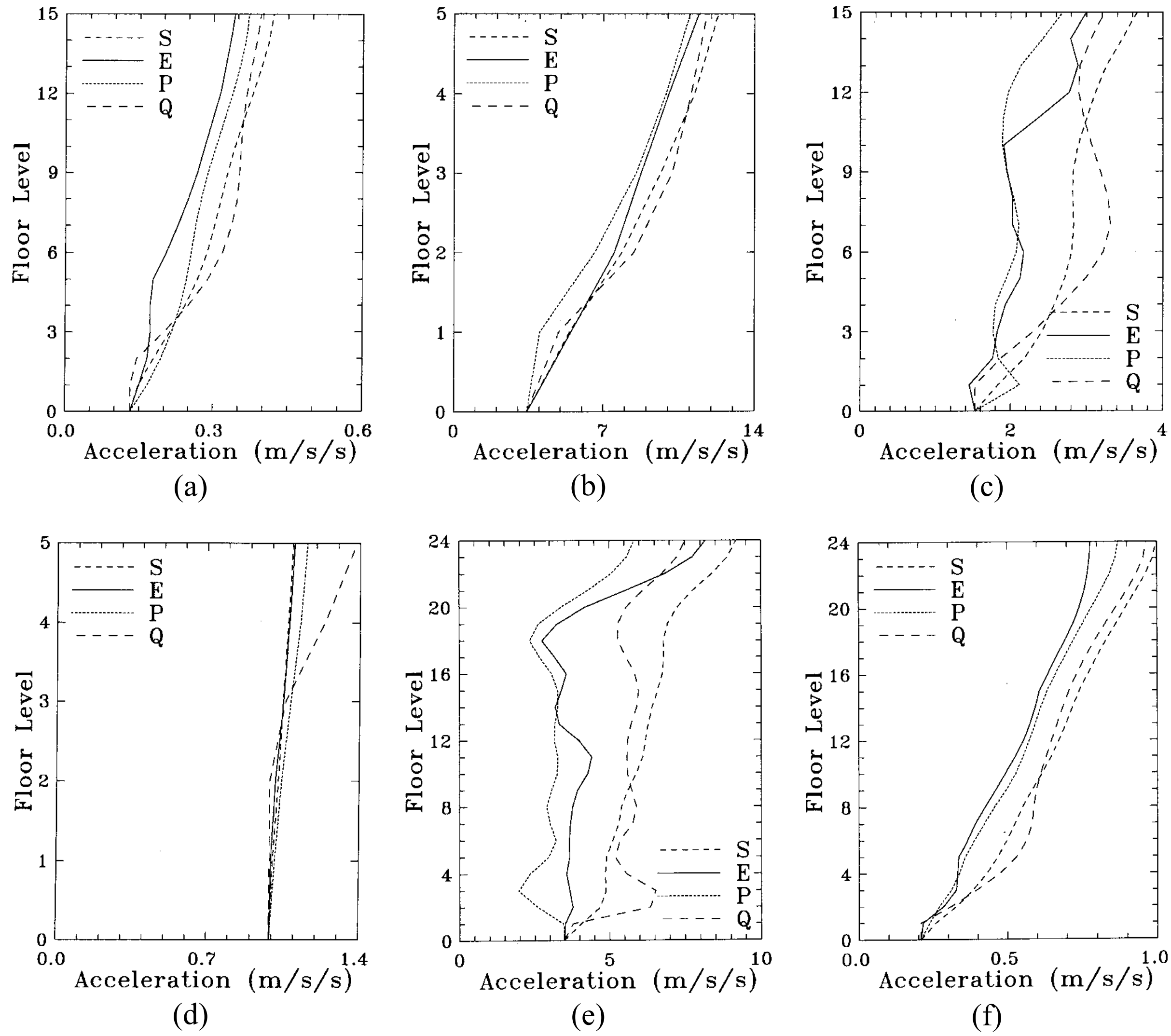

Fig. 5 Comparison of the floor acceleration envelopes for SRSS (S), exact (E), proposed (P) and quasi-SRSS (Q) estimates in the case of (a) BD2 and Borrego Mountain motion, (b) BD3 and Imperial Valley motion, (c) BD2 and Kern County motion, (d) BD3 and Michoacan motion, (e) BD1 and Parkfield motion, and (f) BD1 and San Fernando motion (see Equations (18) and (20) for the proposed estimates, Equation (22) for the SRSS estimates, and Equations (20) and (21) for the quasi-SRSS estimates)

On comparing the errors associated with the SRSS and quasi-SRSS variants of the proposed rule in Table 4, it is seen that both rules give comparable errors. This is expected since both rules differ from each other only in terms of the cross-modal correlation terms and since these terms do not play a significant role (the modes in all three example buildings are well-separated). Both SRSS and quasi-SRSS variants ignore correlation between the ground acceleration and relative floor acceleration, and this appears to lead to relatively larger errors in the cases like BD2 subjected to the Parkfield motion and BD1 and BD2 subjected to the Imperial Valley motion. This may be due to the building being very flexible with respect to the ground motion (fundamental period of the building becoming as large as 10 times the mean period of the ground motion) and periods of several significant modes exceeding the mean period. Both SRSS and quasi-SRSS variants of the proposed rule are thus likely to work well as long as the structural system is not flexible with respect to the ground motion. Based on the example buildings and motions considered in this study, it appears that a range of 0.5–3 times the mean period (of ground motion) for the fundamental period of the building would ensure an average absolute error within 20%. It is also to be noted that the errors associated with the two variants are usually on the conservative side (about 97% times in the case of SRSS and 91% times in the case of quasi-SRSS).

In order to see how the simple SRSS variant compares with the recent recommendation of Singh et al. (2006), absolute error (averaged over all floors) in the case of Singh et al. (2006) has also been shown in Table 4 for the 18 example cases. It may be observed from this table that the maximum average absolute error in the case of the SRSS variant of the proposed rule is 83.93%, while it is 122.84% in the case of Singh et al. (2006). Further, out of the 18 cases considered, there are 8 cases in which the recommendation of Singh et al. (2006) gives lesser errors. Based on this limited study, therefore, SRSS variant of the proposed rule may be expected to perform better than the recommendation of Singh et al. (2006), particularly when the structural system is not flexible with respect to the ground motion.

In order to see how damping affects the relative performance of the proposed rule and its variants, results have also been obtained for 2% modal damping (instead of 5%), and Table 5 shows the errors associated with all three approximate methods. The observations based on the 5%-damping results are found to be broadly applicable in the case of the 2%-damping results also. In order to see more closely how the error in peak floor acceleration is distributed in the case of the proposed rule, cumulative probability density function for percentage error has been estimated by considering all 264 peak floor acceleration results (for 44 floors of the three example buildings, each subjected to the six example motions) and by obtaining the fractions of those results that have percentage errors below different levels varying from –60 to 60. For example, there are 143 5%-damping results that have negative errors with respect to the exact results (i.e., the exact results are greater), and thus the cumulative probability for the zero percentage error works out to 54.2%. The cumulative probability density function is plotted in Figure 6 for the 2%- and 5%-damping results. It is clear from this figure that damping does not have significant influence on the way errors are distributed in the case of the proposed rule. However, errors are distributed more symmetrically around the zero value in the case of 5%-damping results, and therefore, one can perhaps assume a greater value (than unity) for various β ratios (or η ratios, or both) for a more symmetric error distribution in the case of 2%-damping results. Such an exercise may however disturb the symmetry in the case of 5%-damping results, except when we choose a value slightly greater than unity, and may thus cause the maximum 'average absolute error' (see Table 4) to go up.

Table 5: Comparison of the Averaged Percentage Absolute Error (over Floors) in Peak Floor Acceleration for 2% Damping Ratio

Record No.	1	2	3	4	5	6
Example Building	BD1					
Proposed*	7.84	14.09	16.37	3.02	25.31	3.38
SRSS**	16.16	31.72	10.32	3.57	29.35	13.06
Quasi-SRSS***	15.28	28.14	9.84	5.90	27.36	12.64
Example Building	BD2					
Proposed*	9.73	10.25	14.26	4.54	14.11	5.16
SRSS**	13.08	22.35	8.60	12.82	56.86	9.43
Quasi-SRSS***	15.19	24.31	14.85	15.74	57.02	8.50
Example Building	BD3					
Proposed*	1.98	8.94	6.47	6.22	8.46	3.70
SRSS**	10.83	2.72	14.10	2.14	10.42	10.11
Quasi-SRSS***	12.40	5.97	11.66	10.60	7.25	7.58
*Equations (18) and (20); **Equation (22); ***Equations (20) and (21)						

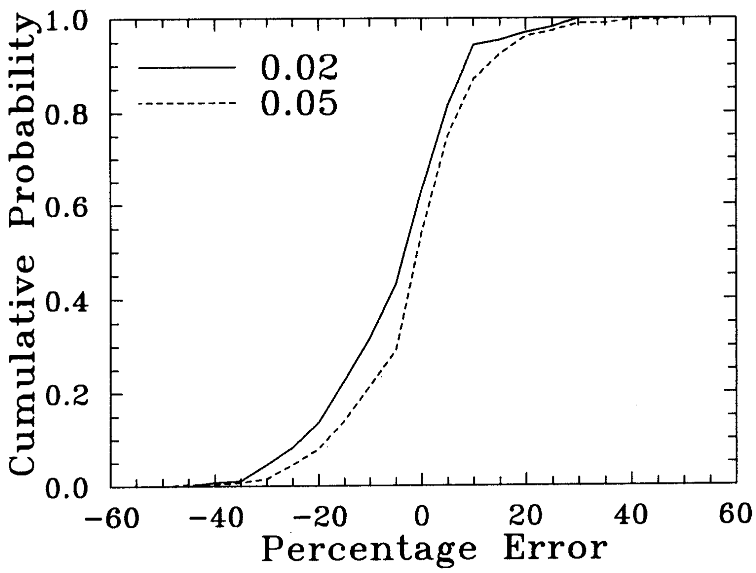

Fig. 6 Comparison of cumulative probability density functions for percentage error in the peak floor acceleration estimate from the proposed rule for 2% and 5% damping ratios

Though the cases considered in this study are not necessarily exhaustive or consider various possibilities in a balanced way, we can make following preliminary conclusions regarding the proposed rule. First, the absolute error in estimating absolute acceleration at the floor of a building does not exceed 50%. Second, the probability of a negative error in this estimation is about 55%. Third, the probability of an absolute error within 10% is about 65%. The proposed rule can be improved further by considering more realistic β ratios. This, however, requires an in-depth study on the correlations of these ratios with the ground motion characteristics and building periods.

CONCLUSIONS

A modal combination rule has been formulated to estimate maximum values of the absolute accelerations of floors in a multistoried shear building. The building is assumed to be a linear, lumped mass, classically damped, fixed-base system, which is excited at its base by the ground motion described by a given PSA spectrum. The proposed rule is based on assuming the excitation and responses processes to be stationary and on the use of nonstationarity and peak factors to relate the (stationary) r.m.s. response with the (nonstationary) largest peak response. It is further assumed for simplicity that the peak factors for the modal responses and the total responses are equal and that the nonstationarity factors for these responses are also equal. The spectral velocity and relative spectral acceleration spectra are assumed to be approximated by PSV and PRSA spectra, respectively. The proposed rule includes (i) correlations between the ground acceleration and the relative acceleration in each mode, and (ii) the correlations between the relative accelerations in various modes. This rule requires just the knowledge of the dynamic properties of the building (mode shapes, modal frequencies and modal participation factors), the PSA ordinates, and the mean period of the ground motion.

A numerical study carried out with the help of three example buildings and six example ground motions with widely different characteristics shows that the peak floor acceleration estimates of the proposed rule follow the (exact) time-history estimates reasonably well through the building height and that the maximum average absolute error in any combination of building and excitation is less than 20% in the case of 5% damping. It is also seen that the probability of absolute error at any floor of being less than 10% is about 65%. In any case, this error does not exceed 50%. The performance of the proposed rule is excellent particularly when the building is stiff or very stiff relative to the excitation. It is also observed that two simpler variants of the proposed rule: (i) SRSS, ignoring all cross-correlations, and (ii) quasi-SRSS, ignoring the correlation between the ground and relative accelerations, give comparable errors for the example buildings with well-separated modes. Both ignore correlation between the ground acceleration and relative floor acceleration, and give estimates greater than or equal to PGA. In comparison with the proposed rule, the estimates from these variants are associated with greater errors and those errors are usually on the conservative side. However, both variants are likely to work reasonably well provided the building is not flexible with respect to the ground motion.

The proposed rule is very convenient to apply as it uses the easily available PSA ordinates (including that at the zero period). For calculation of the mean period of the ground motion, it may be more convenient to use the PSV spectrum in place of the Fourier spectrum. The proposed rule can be made even more accurate by using more appropriate nonstationarity factor ratios. This however requires an in-depth study on how to account for the effects of building and modal periods and ground motion characteristics on these ratios.

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