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INCLUSION OF $P-\Delta$ EFFECTS IN THE ESTIMATION OF HYSTERETIC ENERGY DEMAND BASED ON MODAL PUSHOVER ANALYSIS

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ABSTRACT

The estimation of hysteretic energy demand is the first significant step in the energy-based seismic design of structures. The present paper extends a modal pushover analysis (MPA)-based energy demand estimation method to include the $P-\Delta$ effects in structures. The efficiency of the extended procedure is tested on three standard steel moment-resisting frames by comparing the estimates based on this method with the results from the nonlinear dynamic analyses of MDOF systems for several earthquakes. In addition, three non-standard frames with artificially increased susceptibility to $P-\Delta$ effects are also considered. Bias statistics are presented to show the effectiveness of the proposed method on including the $P-\Delta$ effects for both the standard and the non-standard designs. The $P-\Delta$ effects on the actual hysteretic energy demand of a structure are also studied. The MPA-based method including the $P-\Delta$ effects remains less demanding on computations and is also suitable for adopting in design guidelines.

KEYWORDS: Hysteretic Energy Demand, Modal Pushover Analysis, $P-\Delta$ Effect, Dynamic Stability, Energy-Based Seismic Design

INTRODUCTION

In recent years, many researchers have identified hysteretic energy demand or its equivalent parameters as the demand parameters that are most closely correlated to the seismic damage of structures (Zahrah and Hall, 1984; Fajfar, 1992; Manfredi, 2001). The hysteretic energy demand takes into account the effects of the duration of the earthquake ground motion and the cyclic-plastic deformation behavior of the structure. A monotonic demand parameter, such as peak inelastic drift or displacement, cannot represent this cumulative cyclic damage. A design approach based on hysteretic energy demand, thus, has the potential to account for the damage potential explicitly.

The necessity of an energy-based design procedure for future seismic design guidelines has been emphasized by many researchers, including a few attempts at providing a framework for such design procedures. The discussions of these efforts can be found in Ghosh and Collins (2006) and Prasanth et al. (2008). The estimation of hysteretic energy demand on structures is the first significant step in an energy-based design method. With the computing facilities available today, such estimation for a specific structure under certain earthquake ground motion is not difficult, even though this may be computation intensive. However, one has to apply the detailed method, i.e., the nonlinear response history analysis (NL-RHA) of a multi-degree-of-freedom (MDOF) model, for each individual structure separately. In addition, this method cannot use a single-degree-oscillator-based design/response spectrum, thus making the direct method unsuitable for incorporating in a general purpose design methodology based on hysteretic energy demand.

This paper investigates a simpler method for estimating hysteretic energy demand. It is an extension of the work by Prasanth et al. (2008) that used the concepts of modal pushover analysis (MPA) (Chopra and Goel, 2002; Goel and Chopra, 2004) for estimating hysteretic energy demand on MDOF systems. Prasanth et al. (2008) used multiple equivalent single-degree-of-freedom (ESDOF) systems for the modes of a structure. This MPA-based approximate method of estimating hysteretic energy demand was found to be very efficient for steel frame structures. However, this method did not consider the effects of gravity load and $P-\Delta$.

The $P-\Delta$ effects (i.e., the global/structure P-delta effects) are the second-order effects arising due to geometric nonlinearity in the static and dynamic analyses of structures under lateral loads, such as those due to earthquakes. These are secondary moment effects due to gravity loads combined with large inter-story deformations. These effects may significantly alter the response of an inelastic system susceptible to large deformations during the course of an earthquake. Gravity loads, acting through the large inter-story deformations, may even cause dynamic instability by reducing the lateral stiffness in a severe ground motion scenario (Bernal, 1998). The $P-\Delta$ effect can be very severe on flexible structures, such as steel moment-resisting frames, because those are subjected to large lateral displacements during a seismic shaking (Gupta and Krawinkler, 1999). A detailed review of research work on the various aspects of $P-\Delta$ effects on the seismic response of building structures is avoided in this article. Asimakopoulos et al. (2007) compiled a list of works available on $P-\Delta$ effects in structures. The present article proposes a modification on the method proposed by Prasanth et al. (2008) to account for the $P-\Delta$ effects and studies the effectiveness of the modified method for representative low-, mid- and high-rise steel frame building structures, by comparing MPA estimates with the results from the NL-RHA of a MDOF system. In addition, the effectiveness of the modified method is checked for non-standard designs that are specifically vulnerable to the $P-\Delta$ effects due to very high gravity loads. The primary focus of this paper is on measuring the effectiveness and on checking the robustness of the approximate method through various case studies. The $P-\Delta$ effects on the computed hysteretic energy demand are also studied. It may be noted that the objective here is not to find out how damage (in terms of hysteretic energy demand) is distributed in the structure. This paper follows the concept of using the overall energy demand in a structure as a design criterion, as proposed by previous researchers (Fajfar, 1992; Ghosh and Collins, 2006).

INCLUDING $P-\Delta$ EFFECTS IN AN MPA-BASED ANALYSIS

If large inelastic deformations occur during an earthquake, the $P-\Delta$ effects in a structure become significant as those further increase the displacements and may reduce the lateral load carrying capacity. Those may even result in the dynamic instability or the collapse of a story. Based on these considerations, the inclusion of $P-\Delta$ effects becomes necessary while estimating inelastic force and deformation parameters through nonlinear analysis (Gupta and Krawinkler, 1999).

In MPA (Chopra and Goel, 2002) the elastic mode shapes and frequencies are used to formulate a nonlinear ESDOF model for each mode. The nonlinear characteristics of each mode, such as yield point and strain hardening stiffness ratio, are obtained through a nonlinear static pushover analysis using a mode-specific lateral force distribution. The use of multiple modal ESDOF systems in MPA overcomes the limitation of traditional pushover analysis of not being able to account for the higher-mode effects. MPA was used for estimating seismic force and displacement demands on nonlinear (inelastic) as well as linear elastic MDOF systems with sufficient closeness to the results obtained from a response history analysis. The advantage of MPA is that it achieves this degree of accuracy without losing the conceptual simplicity and computational attractiveness of the traditional pushover procedure. Goel and Chopra (2004) modified this by including $P-\Delta$ effects in the nonlinear static pushover analyses. The modified method was tested on the 9- and 20-story SAC steel frames from Boston, Los Angeles and Seattle in USA. It was observed that $P-\Delta$ effects increased bias in the MPA-based estimation of story drift ratios (to over 40% for the 20-story building at Los Angeles), where bias was defined as the ratio of the NL-RHA-based estimation of a parameter to the MPA-based estimation of that parameter.

A similar modification (Goel and Chopra, 2004) is attempted here for the MPA-based estimation of hysteretic energy demand. The $P-\Delta$ effects are included in the nonlinear pushover analysis corresponding to each mode. The lateral force distribution $\{f_n\}$ for the nth-mode pushover analysis is obtained based on the nth mode shape, after normalizing $[m]\{\phi_n\}$ to a unit base shear (i.e., $\{f_n\}$ = $[m]\{\phi_n\}/\{\iota\}^T[m]\{\phi_n\}$), where $[m]$ is the mass matrix, $\{\phi_n\}$ is the nth mode shape normalized to a unit roof displacement component, and $\{\iota\}$ is the influence vector. For each mode, the pushover analysis is carried out to achieve a maximum interstory drift of 2.5%. As mentioned in similar works earlier (Ghosh and Collins, 2006; Prasanth et al., 2008), the results do not change significantly if a higher value of maximum drift is considered in the pushover analyses including $P-\Delta$ effects. The base shear (i.e., V_n) versus roof displacement (i.e., D_n) "pushover" curve is approximated by a bilinear function by equating the areas underneath the two curves. This bilinear curve gives the elastic stiffness K_{pon}, the yield displacement D_{yn} (= V_{yn}/K_{pon}) and the strain hardening stiffness ratio α_{kn}, from which critical parameters for the *n*th-mode equivalent system are obtained as described by Prasanth et al. (2008). The inclusion of $P-\Delta$ effects in the pushover analysis changes the parameters for the corresponding modal ESDOF system. It primarily changes the strain-hardening stiffness ratio α_{kn} and yield force V_{yn}, though it may also affect the elastic parameters, such as stiffness K_n and period T_n (or frequency ω_n). The governing equation of motion for the *n*th modal ESDOF system subjected to the ground acceleration $\ddot{u}_g$ is written as

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 G_n\left(q_n, \operatorname{sgn}\dot{q}_n\right) = -\Gamma_n\ddot{u}_g \tag{1}$$

where ξ_n is the modal damping ratio, G_n expresses the force-deformation relation based on the bilinearized pushover plot, and Γ_n is the participation factor for the *n*th mode given by

$$\Gamma_n = \frac{\{\phi_n\}^{\mathrm{T}}[m]\{\iota\}}{\{\phi_n\}^{\mathrm{T}}[m]\{\phi_n\}} \tag{2}$$

The hysteretic energy demand E_{nh} in each mode is obtained by solving the nonlinear dynamic relation of Equation (1). Since E_{nh} is a cumulative (and non-decreasing) function in time, the peak hysteretic energy will always occur at the end of the analysis. A simple way to combine the individual E_{nh} values is to add those together. However, this is an approximation because this ignores any coupling that may occur in the inelastic domain (Prasanth et al., 2008).

The geometric nonlinearity due to flexural deformations within a member, or $P-\delta$ effects (i.e., the member P-delta effects), are not considered here because the focus is on the overall building response. Adam and Krawinkler (2004) found this phenomenon to be mostly insignificant for the overall seismic response of building structures.

CASE STUDY 1: SAC STEEL FRAMES IN LOS ANGELES

In order to test the effectiveness of the modified MPA-based hysteretic energy demand estimation method, it is used to predict energy demands for the 3-, 9- and 20-story "pre-Northridge" SAC steel moment frame buildings in Los Angeles, USA (Gupta and Krawinkler, 1999), subjected to 21 ground motion records. These buildings are selected for this case study because they represent standard earthquake-resistant designs and have been used in numerous research studies. Based on linear elastic static considerations, these buildings are not expected to show significant susceptibility to $P-\Delta$ effects. A story-specific stability coefficient is used, similar to Equation (4.1) in the report by Gupta and Krawinkler (1999), to measure this susceptibility based on a linear static pushover analysis with the IBC 2006-based lateral force distribution (ICC, 2006):

$$\theta_i = \frac{P_i\Delta_i}{V_i h_i} \tag{3}$$

Here, P_i is the total vertical load above the floor level i, Δ_i is the relative deformation at the *i*th story, V_i is the *i*th story shear, and h_i is the *i*th story height. The maximum stability coefficients obtained for the example 3-, 9- and 20-story buildings are 0.030, 0.069 and 0.112, respectively. A value of 0.1 is often cited as a limit above which $P-\Delta$ effects should be considered in design. The effectiveness of the method proposed by Prasanth et al. (2008) is also tested for another set of buildings where significant $P-\Delta$ effects are expected. This will be discussed in the next section.

It may be mentioned that the use of a stability coefficient similar to Equation (3) to predict the vulnerability of a structure to $P-\Delta$ effects is highly questionable. FEMA-355C (FEMA, 2000) states that it "provides inadequate information on the occurrence of a negative post-mechanism stiffness and against excessive drifting of the seismic response." This coefficient cannot properly account for the inelastic and dynamic effects. Gupta and Krawinkler (1999) pointed out that $P-\Delta$ effects are very sensitive to ground motion characteristics, once negative post-yield stiffness is attained. Despite these shortcomings, it is used in this paper because of its familiarity to engineers and researchers.

The details of Los Angeles SAC buildings, including gravity loads, can be obtained from the previous publications (Gupta and Krawinkler, 1999; Ghosh, 2003) and are, therefore, avoided here. The details of the ground motion records considered, which include the 18 records used by Prasanth et al. (2008) and three additional records (from 1971 San Fernando earthquake, 1978 Tabas earthquake and 1989 Loma Prieta earthquake), are available in a detailed report (Roy Chowdhury, 2008). Only the North-South moment frames of the (symmetric) Los Angeles buildings are considered for static pushover and nonlinear response history analyses. These analyses are performed using the software DRAIN-2DX (Prakash et al., 1993). The analysis methods and considerations are the same as those adopted by Prasanth et al. (2008), except for including $P-\Delta$ effects in the pushover analyses as discussed in the previous section. The nonlinear response-history analyses are carried out by applying the earthquake accelerations at the base of the 2-D frames. The frame members are modeled by using the plastic hinge beam-column element (of Type 02), with rigid-plastic hinges at their ends. The P-M interaction is considered for the hinge capacity. The material behaviour is assumed to be bilinear with 5% strain-hardening stiffness ratio. The strength and stiffness degradations are neglected in the hysteretic behaviour. The flexibility of the joint panel zones and the lateral stiffness of the gravity frames are not considered. A Rayleigh damping of 5% is assumed (for the first two modes) for the NL-RHA. Although Prasanth et al. (2008) recommended the use of only the first three modes, even for the 9- and 20-story buildings, here the first five modes are considered for energy estimations while including $P-\Delta$ effects. Various results from this set of case studies are discussed next.

The $P-\Delta$ effects are found to be significant on some ESDOF parameters, but primarily on the strain-hardening stiffness ratio α_{kn}. Those also affect the other parameters, such as the yield force V_{yn} and stiffness K_n. Table 1 presents, for example, the changes in some of these parameters due to the inclusion of $P-\Delta$ effects in pushover analyses for the first five modes of the 20-story frame. The $P-\Delta$ effects are most significant for the first mode. These observations are similar to those reported by Goel and Chopra (2004).

Table 1: ESDOF Parameters for First Five Modes of 20-Story SAC Steel Frame

Mode	Without $P-\Delta$ Effects			With $P-\Delta$ Effects		
	α_{kn} (%)	V_{yn} (kN)	T_n (s)	α_{kn} (%)	V_{yn} (kN)	T_n (s)
1	13.4	2821	3.81	–9.20	2439	3.99
2	3.00	8200	1.32	3.74	6562	1.32
3	4.76	10220	0.766	3.24	9889	0.771
4	5.85	14370	0.543	4.87	14010	0.552
5	9.10	19490	0.414	8.18	19030	0.414

Table 2: Bias Statistics in Estimation of Hysteretic Energy Demand for Original SAC Steel Frames

	3-Story		9-Story		20-Story	
	Without $P-\Delta$	With $P-\Delta$	Without $P-\Delta$	With $P-\Delta$	Without $P-\Delta$	With $P-\Delta$
Mean	1.08	1.13	1.11	1.12	1.38	1.05
SD	0.057	0.066	0.150	0.147	0.412	0.228
CoV	0.053	0.059	0.135	0.131	0.299	0.218
Maximum	1.18	1.25	1.44	1.44	2.56	1.81
Minimum	0.954	1.00	0.939	0.958	0.906	0.767

The accuracy of the MPA-based estimation is measured by using the statistics of a bias factor defined as

$$N_{\text{MPA}} = \frac{E_{\text{NL-RHA}}}{E_{\text{MPA}}} \tag{4}$$

where $E_{\text{NL-RHA}}$ is the hysteretic energy demand based on the nonlinear response history analysis of the MDOF model of the actual structure, and E_{MPA} is the hysteretic energy demand based on the MPA-based method. This bias factor is calculated for each earthquake, and its statistics (i.e., mean, standard deviation, coefficient of variation, and maximum and minimum values) are provided in Table 2. Table 2 also presents the results for the same buildings when no $P-\Delta$ effects are considered (neither in NL-RHA nor in MPA). In addition, Figure 1 provides simple scatter plots that compare the values of $E_{\text{NL-RHA}}$ and E_{MPA} for each earthquake, when $P-\Delta$ effects are included. The diagonal line across a scatter plot represents the perfect agreement between the NL-RHA and the approximate analysis technique. These scatter plots provide an easy estimation of how effective the MPA method is when $P-\Delta$ effects are considered.

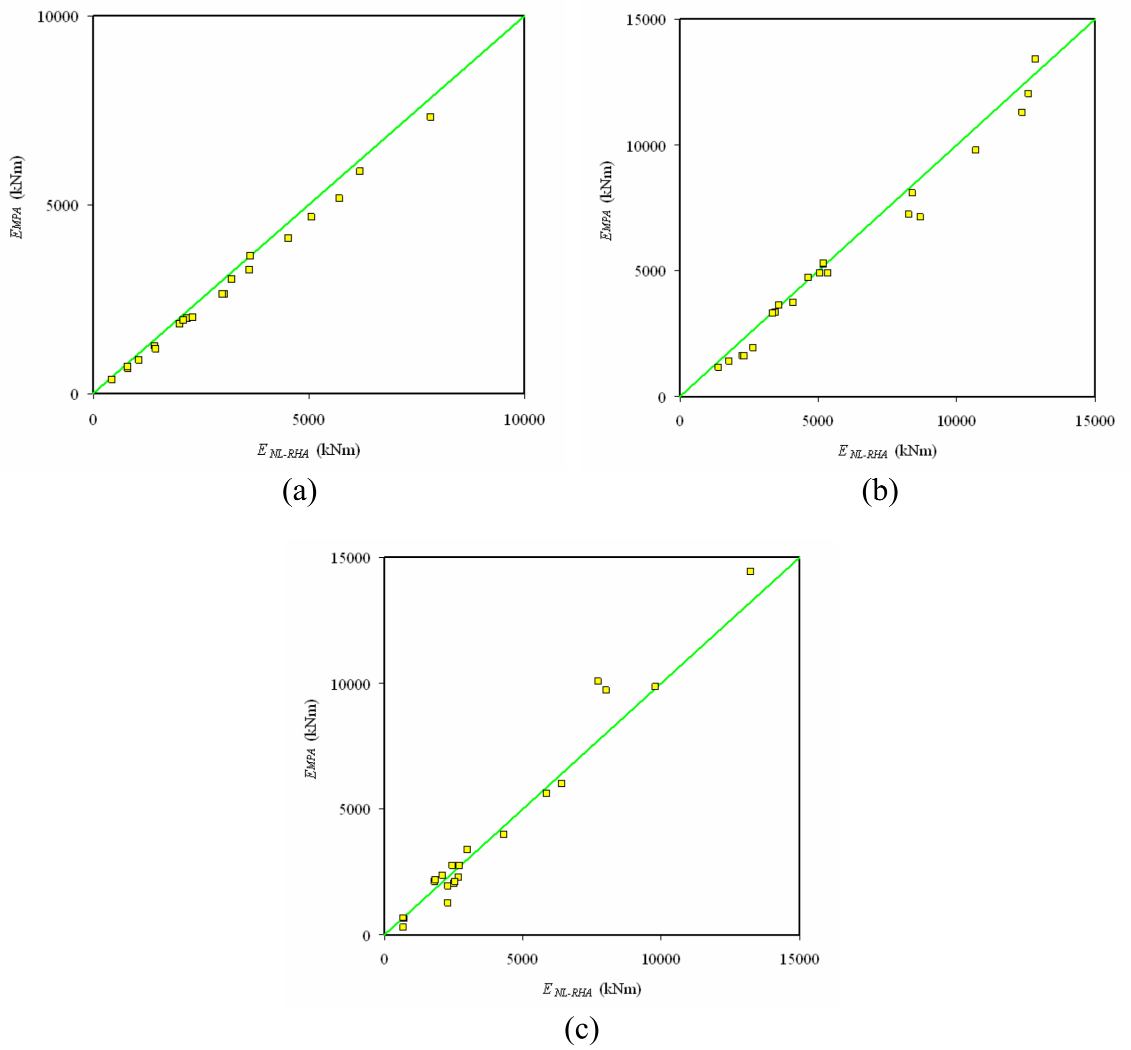


Fig. 1 Scatter plots comparing E_{MPA} with $E_{\text{NL-RHA}}$, with $P-\Delta$ effects included, for the original SAC steel (a) 3-story, (b) 9-story and (c) 20-story buildings

The scatter plots and Table 2 show very clearly that the MPA-based method of hysteretic energy demand, on including the $P-\Delta$ effects, provides estimates that are comparable to the results from the "exact" NL-RHA for most of the cases considered. The mean bias is close to its ideal value of 1.0 and the scatter is also low for all the three frames. The largest discrepancies occur for the 20-story building, as also observed by Goel and Chopra (2004) for the MPA-based displacement estimation.

Table 3 provides a summary of the modal contributions of each ESDOF system to the E_{MPA} estimates presented in Figure 1(c) for the 20-story building. These results show that considering only the first three modes is sufficient even for the 20-story building. For many records, only the first mode contributes significantly. However, for a small number of records, the 2nd- and 3rd-mode contributions are also significant. In fact, the 2nd-mode contribution is more than the 1st-mode contribution in some cases. Prasanth et al. (2008) also observed similar results for these specific records while estimating the hysteretic energy demand for these frames without the $P-\Delta$ effects. As discussed in their paper, these interesting results are attributed to the unique characteristics and frequency content of the input ground motion records.

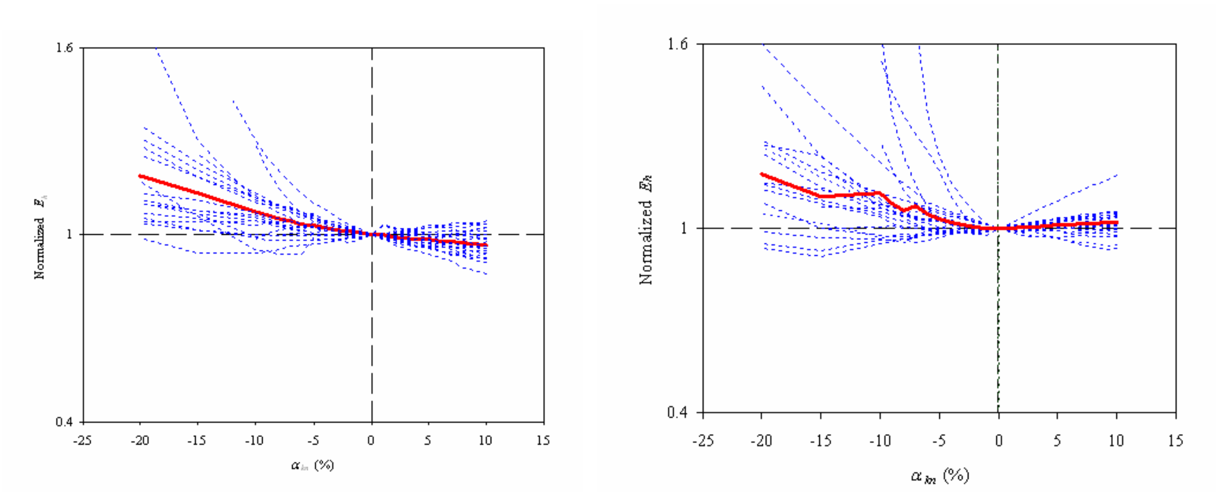
The level of accuracy, as measured in terms of the mean bias presented in Table 2, reduces on including the $P-\Delta$ effects by 4.8% and 0.89% for the 3- and 9-story frames, respectively. This may be due to the fact that $P-\Delta$ effects, based on their stability coefficients, are expected to be insignificant for these buildings. For the 20-story frame, on the other hand, the level of accuracy improves by 24%. It is difficult to ascertain any specific reason for this improvement. Prasanth et al. (2008) observed that when no $P-\Delta$ effects are considered, the MPA-based method underestimates the energy demand (on average), and the error (or the degree of underestimation) increases for higher buildings. It was suspected that the inability of the MPA-based method to account for the inelastic coupling of modes, which become more significant for the 20-story frame, is the reason for this underestimation. As shown later in Figure 4(a), the actual hysteretic energy demand $E_{\text{NL-RHA}}$ on these frames does not change significantly on the inclusion or exclusion of the $P-\Delta$ effects. On considering the improvements in the MPA-based estimation, while including $P-\Delta$ effects for the 20-story frame, it can be conjectured that the effect of accounting for the $P-\Delta$ phenomenon in the MPA-based method is additive in terms of hysteretic energy demand on this structure (i.e., the hysteretic energy demand increases if $P-\Delta$ effects are included in the MPA-based method).

Table 3: Mode-wise Distribution of Hysteretic Energy Demand for SAC Steel 20-Story Building

Ground Motion Record	E_{nh}/E_{MPA} (with $E_{\mathrm{MPA}}=\sum_{n=1}^{5}E_{nh}$)				
	Mode 1 (%)	Mode 2 (%)	Mode 3 (%)	Mode 4 (%)	Mode 5 (%)
s640r005	100	0	0	0	0
s503r005	100	0	0	0	0
s065r005	97.9	2.06	0	0	0
s621r004	100	0	0	0	0
s050r005	89.9	10.1	0	0	0
s212r008	100	0	0	0	0
s305r008	72.1	27.9	0	0	0
s549r009	94.3	5.70	0	0	0
nr	41.4	53.2	5.38	0	0
ns	30.4	67.1	2.49	0	0
chy08036	16.2	81.5	2.27	0	0
chy0809	0	74.4	25.6	0	0
tcu0659	92.7	7.27	0	0	0
syl90	100	0	0	0	0
newh360	62.9	37.1	0	0	0
nh	67.1	32.9	0	0	0
syl360	67.6	32.4	0	0	0
tcu06536	90.9	9.07	0	0	0

lgp00	82.5	14.0	3.45	0	0
pcd164	54.3	45.7	0	0	0
tabln	84.6	15.4	0	0	0

The mean ratio of E_{MPA} with and without $P-\Delta$ effects is found to be 0.979, 0.867 and 1.16, respectively, for the 3-, 9-, and 20-story frames. The primary reason for this increase in E_{MPA} for the 20-story frame is the level of reduction in the ESDOF parameter α_{kn}. For example, for the 1st-mode ESDOF system, α_{kn} changes from 13.4% to –9.20% for the 20-story frame, from 11.8% to 2.88% for the 9-story frame, and from 11.4% to 8.91% for the 3-story frame. A simple study is performed to monitor the effect of change in α_{kn} on the hysteretic energy demand on an inelastic SDOF system. For this, two SDOF systems (one having the properties corresponding to the 1st modal ESDOF system of the 20-story frame, and the other corresponding to the 1st modal ESDOF system of the 9-story frame) subjected to the selected set of 21 earthquake records are analyzed and the hysteretic energy demand E_h is monitored with varying α_{kn} values. Figure 2 illustrates the variation in E_h (as normalized by E_h at α_{kn} = 0) with α_{kn} (the thick unbroken lines represent the mean values for all the records). This illustration shows that the variation in E_h changes with the earthquake record and with the SDOF system considered. As mentioned earlier, the $P-\Delta$ effects become very sensitive to ground motion characteristics, once the negative post-yield stiffness is attained. The mean curves show that, on average, E_h increases when there is a decrease in α_{kn} in the range of negative post-yield stiffness. For the 1st modal ESDOF of the 20-story frame, this increase occurs for a decrease in α_{kn} from 10% to –20%. With the change in the other ESDOF parameters being almost insignificant, a definite increase in hysteretic energy demand occurs when the reduction in the post-yield stiffness is more and to the range of 0% and below, i.e., for the high-rise frames having high stability coefficients.



(a) (b)

Fig. 2 Variation of hysteretic energy demand (as normalized by E_h at $\alpha_{kn} = 0$) with α_{kn} under 21 records (and the mean variation) for the 1st modal ESDOF systems of the (a) 20-story and (b) 9-story frames (the dashed lines represent results for the individual earthquakes and the solid line represents their mean)

CASE STUDY 2: BUILDINGS WITH INCREASED $P-\Delta$ EFFECTS

The MPA-based method proposed by Prasanth et al. (2008) for estimating hysteretic energy demand is also tested for the building frames susceptible to increased $P-\Delta$ effects during large earthquakes. For this case study of non-standard designs, the 3-, 9- and 20-story "pre-Northridge" SAC steel frames in Los

Angeles are again considered, but with artificially increased gravity loads (along with a suitable increase in inertial masses for the response-history analyses). The modified 3-, 9- and 20-story frames have maximum stability coefficients of 0.060, 0.137 and 0.168, respectively, as per Equation (3). It may be noted that the modified 3-story frame with the increased mass has a stability coefficient less than what is conventionally accepted as the minimum value (= 0.1) for having significant $P-\Delta$ effects. The 9- and 20-story frames with artificially increased stability coefficients become unstable (and the DRAIN-2DX solver fails to converge) for several large-magnitude earthquakes considered in the previous section. Also, the MPA-based method fails to deliver any results for some earthquakes as a very high negative strain-hardening stiffness of the modal ESDOF system causes the collapse of the frames. For example, the 1st-mode ESDOF system for the modified 20-story frame has α_{kn} = −30%, leading to a zero force carrying capacity under several earthquake records. These unstable cases are excluded from the results to follow. Similar cases of instability were also reported by previous researchers (Goel and Chopra, 2004) for the MPA-based displacement estimation.

The bias statistics summary for the above estimates, based on all 21 records for the 3-story frame, 14 records for the 9-story frame, and 9 records for the 20-story frame, is presented in Table 4. These results (see Table 4 and the scatter plots in Figure 3) show that the MPA-based method of estimating hysteretic energy works quite well even for those systems where $P-\Delta$ effects are exaggerated. The mean bias for the modified 9-story frame is at the same level as that for the original 9-story frame discussed in the previous section. For the modified 3-story frame, the estimates are good, with the level of accuracy slightly deteriorating from that of the original SAC frame. However, for the modified 20-story frame (with a very high stability coefficient), the mean bias goes down to 0.789. This may be due to the additive effect of significant $P-\Delta$ phenomenon on the SDOF energy demand for the MPA-based method, as discussed in the previous section (α_{kn} was observed to be −30% for the 1st modal ESDOF system of the modified 20-story frame). Overall, the results (with mean bias close to 1.0 and low coefficients of variation) show that the MPA-based method is effective even for the non-standard designs with very high stability coefficients. The discrepancies introduced by the inclusion of the $P-\Delta$ effects in the MPA-based hysteretic energy demand are lower than those for the MPA-based displacement estimation (Goel and Chopra, 2004). It may be mentioned here that the number of samples considered for these results, specifically for the modified 20-story frame, is small and that conclusions drawn here may need modifications based on a larger sample size.

Table 4: Bias Statistics in Estimation of Hysteretic Energy Demand for Modified 3-, 9- and 20-Story Frames

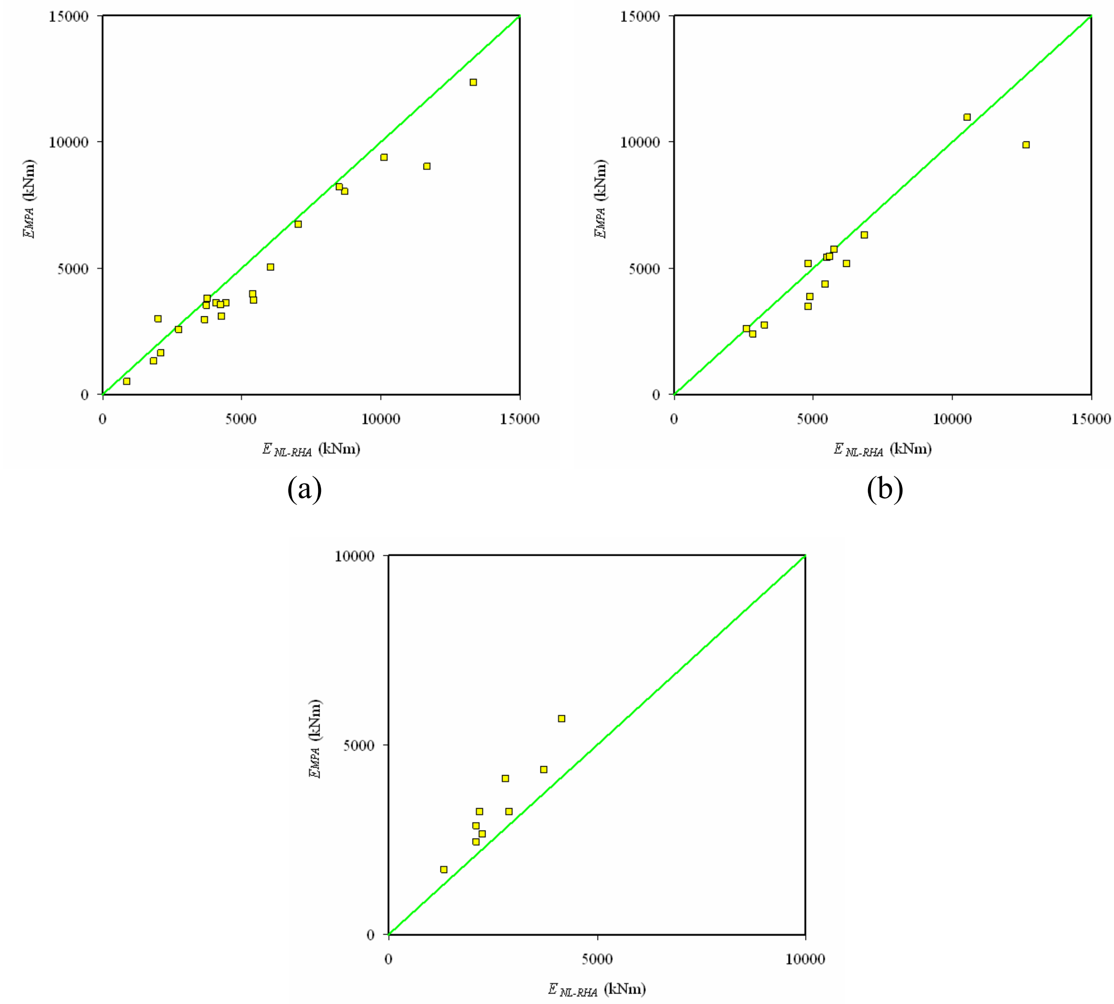
	3-Story[a]		9-Story[b]		20-Story[c]	
	Without $P-\Delta$	With $P-\Delta$	Without $P-\Delta$	With $P-\Delta$	Without $P-\Delta$	With $P-\Delta$
Mean	1.21	1.20	1.18	1.13	1.13	0.789
SD	0.164	0.223	0.159	0.141	0.275	0.085
CoV	0.136	0.186	0.135	0.125	0.244	0.107
Maximum	1.76	1.78	1.47	1.38	1.81	0.895
Minimum	1.05	0.67	1.02	0.935	0.903	0.674

[a] Based on 21 records
[b] Based on 14 records (7 records excluded due to instability)
[c] Based on 9 records (12 records excluded due to instability)

$P-\Delta$ EFFECTS ON HYSTERETIC ENERGY DEMAND

The effects of including the $P-\Delta$ behaviour on the actual hysteretic energy demand on a structure, as obtained from the NL-RHA of the MDOF model, are also studied for the three original SAC steel frames and the corresponding modified frames for the same set of acceleration records. Gupta and

Krawinkler (1999), based on their study of SAC steel frames, concluded that the dynamic effects of $P-\Delta$ behaviour can be additive or subtractive in terms of inelastic displacement demand (unlike the static effects which are always additive). However, there is no specific data available in the published literature on quantifying $P-\Delta$ effects in terms of hysteretic energy demand. Based on the results for the three original SAC steel frames, the $P-\Delta$ effects have almost no impact (as the mean ratio of $E_{\text{NL-RHA}}$ with $P-\Delta$ effects to that without $P-\Delta$ effects is 0.97 for the three frames together, with a standard deviation of 0.057) on the hysteretic energy demand for the whole frame. The scatter plot in Figure 4(a) illustrates this fact for all the three frames. For the modified 3-, 9- and 20-story frames with higher stability coefficients, the $P-\Delta$ effects are slightly more significant; however, there is no specific trend of increase or decrease in the energy demand, with the average demand remaining almost the same (see Figure 4(b); the mean ratio of $E_{\text{NL-RHA}}$ with $P-\Delta$ effects to that without $P-\Delta$ effects is 0.962 for the three frames together, with a standard deviation of 0.117).

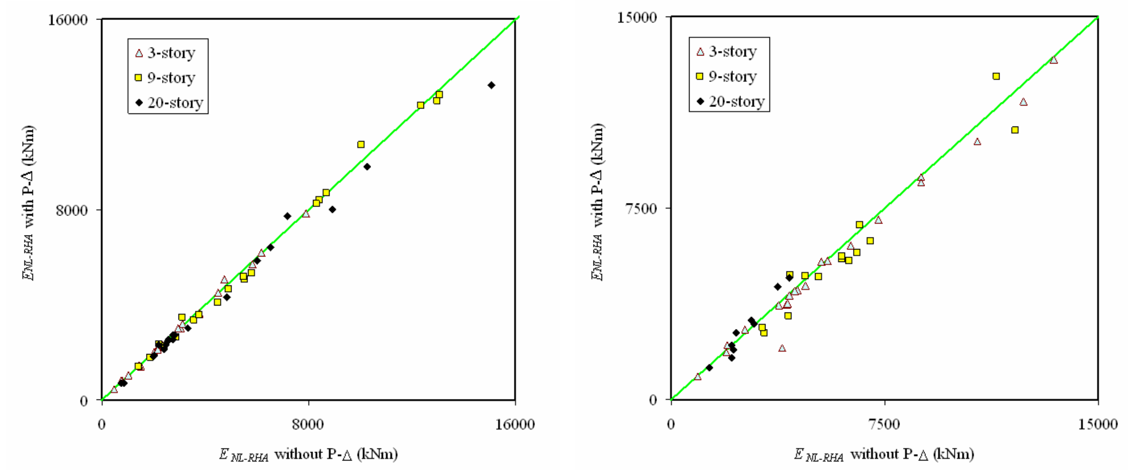


(c)

Fig. 3 Scatter plots comparing E_{MPA} with $E_{\text{NL-RHA}}$ for the (a) 3-story, (b) 9-story and (c) 20-story buildings with increased stability coefficient

SUMMARY AND CONCLUSIONS

A MPA-based hysteretic energy demand estimation technique has been extended in this paper to include the $P-\Delta$ effects in structures. The modified method has been tested on three low-to-high-rise steel frames conforming to design standards and on three non-standard low-to-high-rise steel frames with relatively high susceptibility to $P-\Delta$ effects. The $P-\Delta$ effects on the hysteretic energy demand (as obtained from the NL-RHA of the MDOF system) have also been studied.



(a) (b)

Fig. 4 Comparison of $E_{\text{NL-RHA}}$ with and without $P-\Delta$ effects for the (a) original SAC steel frames, and the (b) modified frames with increased stability coefficients

The following general conclusions have been drawn based on the study presented in this article:

- The procedure proposed by Prasanth et al. (2008) remains a simple and effective method of estimating hysteretic energy demand on a structure even when the $P-\Delta$ effects are included.
- Based on the analyses of the 3-, 9- and 20-story SAC steel buildings, this procedure has been found to provide consistently good estimates of hysteretic energy demand, with the level of accuracy slightly increasing for taller frames.
- The MPA-based method also works well for non-standard designs where very high $P-\Delta$ effects are expected. The level of accuracy however goes down when the stability coefficient is increased to a very high value.
- The $P-\Delta$ effects may increase or decrease the value of energy demand estimated by using MPA (i.e., E_{MPA}), depending on the amount of negative post-yield stiffness caused by those effects.
- Based on NL-RHA, the $P-\Delta$ effects do not appear to significantly affect the hysteretic energy demand of a frame. The $P-\Delta$ effects on hysteretic energy demand increase for the buildings with large stability coefficients; however, there is no specific trend of increase or decrease in the demand.

These conclusions are based on the two sets of case studies conducted herein. The future extensions of this work may primarily focus on (a) estimating member/story-level hysteretic energy demands, and on (b) estimating energy demands for other structures with different hysteretic behavior (such as reinforced concrete buildings).

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