ISET Journal of Earthquake Technology, Paper No. 513, Vol. 48, No. 1, March 2011, pp. 1–10

DEFORMATION OF A UNIFORM HALF-SPACE WITH RIGID BOUNDARY DUE TO A LONG TENSILE FAULT

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ABSTRACT

The Airy stress function for a long tensile fault of arbitrary dip and finite width buried in a homogeneous, isotropic, perfectly elastic half-space with rigid boundary is obtained. This Airy stress function is used to derive closed-form analytical expressions for the stresses at an arbitrary point of the half-space with rigid boundary caused by a long vertical tensile fault of finite width. The variation of the stress fields with distance from the fault and depth is studied numerically.

KEYWORDS: Half-Space, Long Tensile Fault, Airy Stress Function, Rigid Boundary

INTRODUCTION

Steketee (1958) first made use of the elastic theory of dislocations to obtain the displacement and stress fields of a strike-slip fault. Following this fundamental study, numerous theoretical formulations describing the deformation of an isotropic, homogeneous, semi-infinite medium have been developed. Okada (1985) presented analytical expressions for the surface displacements, strains and tilts due to inclined shear and tensile faults in a half-space for both point and finite rectangular sources. Okada (1992) extended these results to internal deformation.

The tensile-fault representation has several very important geophysical applications, such as modelling of the deformation field due to a dyke injection in the volcanic region, mine collapse, or fluid-driven cracks. Moreover, several studies have shown that a large number of earthquake sources cannot be represented by the double-couple source mechanism which models a shear fault. Dziewonski and Woodhouse (1983) obtained the ‘centroid-moment tensor’ solutions for 201 moderate and large earthquakes and observed that the shallow earthquakes north of New Guinea and along the Soloman Islands showed systematic and substantial departures from the double-couple mechanism. Barker and Langston (1983) observed that the inversion of long-period teleseismic P and SH waves, for the 25 and 27 May 1980 Mammoth Lakes, California, earthquakes, yielded moment tensors with large non-double-couple components. According to Julian (1983) and Sipkin (1986), the non-double-couple mechanism might be due to the tensile failure under high fluid pressure. Therefore, it is important to study the elastic field of a tensile dislocation. For a complete description of the elastic field of an earthquake source, using the elastic dislocation theory, it is necessary to consider all the three components, namely, strike-slip, dip-slip and tensile. While the strike-slip and dip-slip cases have been discussed extensively in the seismological literature, tensile dislocation has received comparatively less attention.

Maruyama (1964) obtained surface displacements due to vertical and horizontal rectangular tensile faults in a semi-infinite Poisson solid. Davis (1983) derived an expression of the vertical displacement due to an inclined tensile fault in a half-space and showed that this model can approximate well a tensile crack, just as shear dislocations are successfully used to approximate the deformation fields by shear cracks. Yang and Davis (1986) obtained closed analytical expressions for the displacements, strains and stresses due to a rectangular inclined tensile fault in an elastic half-space.

Singh and Garg (1986) obtained integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional sources. Beginning with these results, Rani et al. (1991) obtained closed-form analytical expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space due to an arbitrary line source. By integrating over the width of the fault, Rani and Singh (1992) obtained expressions for the Airy stress function, displacements and stresses in a uniform half-space due to a long dip-slip fault of finite width. Recently Piombo et al. (2007) studied the displacement, strain and stress fields due to shear and tensile dislocations in a viscoelastic half-space.

It is useful to consider the effect of material discontinuities on the elastic field due to a tensile dislocation. The simplest model to consider the effect of a material discontinuity is that of a tensile dislocation in an elastic half-space (say, medium 1 with rigidity μ_1) in contact with another elastic half-space (say, medium 2 with rigidity μ_2). If $m = \mu_1/\mu_2$, the two particular cases of special interest are for $m = 0$ and $m \to \infty$. In the case of $m = 0$, we have a tensile dislocation in an elastic half-space with free boundary, as discussed by Singh and Singh (2000). On the other hand, when $m \to \infty$, we have the case of tensile dislocation in an elastic half-space with a rigid boundary, as considered in the present study. This implies that our model consists of a tensile dislocation in an elastic half-space in contact with a rigid half-space. This model is useful when the medium on the other side of the material discontinuity is very hard. High-rigidity layers are generally present at depths below a volcanic edifice, covered by much softer volcanic-sedimentary layers composed of a mixture of ash, mud and lava (Bonafede and Revalta, 1999).

The aim of the present paper is to study the two-dimensional deformation of a uniform half-space with rigid boundary caused by a long tensile fault of finite width. Beginning with the closed-form expression for the Airy stress function for an arbitrary line source in a uniform half-space given by Rani et al. (1991) and following Singh and Singh (2000), we obtain Airy stress function for a long tensile fault of arbitrary dip and finite width and the expressions for the stresses at any point of the half-space with rigid boundary caused by a long vertical tensile fault. Analytic integration over the width of the fault yields the desired Airy stress function, and the expressions for the stresses at any point of the half-space follow immediately.

THEORY

Let the Cartesian coordinates be denoted by (x_1, x_2, x_3), with the x_3-axis oriented vertically downward. We consider a two-dimensional approximation, in which the displacement components u_1, u_2 and u_3 are independent of x_1 such that $\partial/\partial x_1 \equiv 0$. Under this assumption, the plane-strain problem ($u_1 = 0$) can be solved in terms of the Airy stress function U such that

$$\tau_{22} = \frac{\partial^2 U}{\partial x_3^2} \tag{1a}$$

$$\tau_{33} = \frac{\partial^2 U}{\partial x_2^2} \tag{1b}$$

$$\tau_{23} = -\frac{\partial^2 U}{\partial x_2 \partial x_3} \tag{1c}$$

and

$$\nabla^2 \nabla^2 U = 0 \tag{2}$$

where τ_{22}, τ_{33} and τ_{23} are the components of stress and

$$\nabla^2 \equiv \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \tag{3}$$

As shown by Singh and Garg (1986), the Airy stress function U_0 for a line source parallel to x_1-axis and passing through the point $(0, 0, h)$ in an infinite medium can be expressed in the form

$$U_0 = \int_0^\infty \left[\left(L_0 + M_0 k \left| x_3 - h \right| \right) \sin kx_2 + \left(P_0 + Q_0 k \left| x_3 - h \right| \right) \cos kx_2 \right] k^{-1} e^{-k\left| x_3 - h \right|} \mathrm{d}k \tag{4}$$

where the source coefficients L_0, M_0, P_0 and Q_0 are independent of the variable k. Singh and Garg (1986) and Singh and Rani (1991) have obtained these source coefficients for various seismic sources.

For a line source parallel to the x_1-axis and acting at the point $(0,0,h)$ of the half-space $x_3 \geq 0$, a suitable solution of the bi-harmonic equation (i.e., Equation (2)) is of the form

$$U = U_0 + \int_0^\infty \left[\left(L + Mkx_3 \right) \sin kx_2 + \left(P + Qkx_3 \right) \cos kx_2 \right] k^{-1} e^{-kx_3} \mathrm{d}k \tag{5}$$

where U_0 is given by Equation (4); and L, M, P and Q are unknowns to be determined from the boundary conditions. We assume that the surface of the half-space $x_3 \geq 0$ is with a rigid boundary. Therefore, the boundary conditions are

$$u_2 = u_3 = 0 \text{ at } x_3 = 0 \tag{6}$$

We have (Sokolnikoff, 1956)

$$2\mu u_2 = -\frac{\partial U}{\partial x_2} + \frac{1}{2\alpha} \int \nabla^2 U \mathrm{d}x_2 \tag{7a}$$

and

$$2\mu u_3 = -\frac{\partial U}{\partial x_3} + \frac{1}{2\alpha} \int \nabla^2 U \mathrm{d}x_3 \tag{7b}$$

with

$$\alpha = \frac{\lambda + \mu}{\lambda + 2\mu} = \frac{1}{2(1-\sigma)} \tag{8}$$

Using Equations (5)–(7), we get

$$L = -(L_0 + M_0 kh) e^{-kh} \tag{9a}$$

$$M = -M_0 e^{-kh} \tag{9b}$$

$$P = -(P_0 + Q_0 kh) e^{-kh} \tag{9c}$$

and

$$Q = -Q_0 e^{-kh} \tag{9d}$$

On putting the values of L, M, P and Q from Equations (9a)–(9d) in Equation (5) and integrating, the Airy stress function at any point of the half-space is obtained as

$$U = L_0 \left[\tan^{-1} \frac{x_2}{|x_3 - h|} - \tan^{-1} \frac{x_2}{(x_3 + h)} \right] + M_0 \left[\frac{x_2 |x_3 - h|}{R^2} - \frac{x_2 (x_3 + h)}{S^2} \right] + P_0 \log\left(\frac{S}{R} \right) + Q_0 \left[\frac{|x_3 - h|^2}{R^2} - \frac{(x_3 + h)^2}{S^2} \right] \tag{10}$$

where

$$R^2 = x_2^2 + (x_3 - h)^2 \tag{11a}$$

$$S^2 = x_2^2 + (x_3 + h)^2, \ x_3 \neq h \tag{11b}$$

Following Singh and Singh (2000), the Airy stress function due to a tensile dislocation on an inclined plane can be expressed in the form,

$$U = U_{\mathrm{VTF}} \sin^2 \delta - U_{\mathrm{VDS}} \sin 2\delta + U_{\mathrm{HTF}} \cos^2 \delta \tag{12}$$

where δ is the dip angle of fault. Further, with $\mathrm{d}s$ as the width of the line fault,

$$U_{\mathrm{VTF}} = b\mathrm{d}sU_{22} \tag{13}$$

is the Airy stress function for a vertical tensile fault (i.e., δ = 90°) with dislocation in the x_2-direction,

$$U_{\mathrm{VDS}} = b\mathrm{d}sU_{23} \tag{14}$$

is the Airy stress function for a vertical dip-slip fault,

$$U_{\mathrm{HTF}} = b\mathrm{d}sU_{33} \tag{15}$$

is the Airy stress function for a horizontal tensile fault (i.e., δ = 0°) with dislocation in the x_3-direction, and

$$U_{ij} = U_{ji} = \lambda\delta_{ij}\frac{\partial U_k}{\partial y_k} + \mu\left(\frac{\partial U_j}{\partial y_i} + \frac{\partial U_i}{\partial y_j}\right) \tag{16}$$

where U_i denotes the Airy stress function at an arbitrary point $P(x_2, x_3)$ for a unit concentrated force acting at the point $Q(y_2, y_3)$ in the x_i-direction.

On using the values of the source coefficients L_0, M_0, P_0 and Q_0 given in Appendix, Equations (10) and (12) yield the Airy stress function due to a long tensile fault of arbitrary dip δ as

$$U = \frac{\mu b\mathrm{d}s}{2\pi(1-\sigma)}\left[\log\left(\frac{S}{R}\right) + \cos 2\delta\left\{\frac{(x_3-h)^2}{R^2} - \frac{(x_3+h)^2}{S^2}\right\} - \sin 2\delta\left\{\frac{x_2(x_3-h)}{R^2} + \frac{x_2(x_3+h)}{S^2}\right\}\right] \tag{17}$$

where σ denotes Poisson's ratio and μ denotes rigidity. This equation gives the Airy stress function for a line tensile dislocation located at the point $(0, h)$. However, if the line source is located at an arbitrary point (y_2, y_3), x_2 and h in Equation (17) should be replaced by $(x_2 - y_2)$ and y_3, respectively. We thus obtain

$$U = \frac{\mu b\mathrm{d}s}{2\pi(1-\sigma)}\left[\log\left(\frac{S}{R}\right) + \cos 2\delta\left\{\frac{(x_3-y_3)^2}{R^2} - \frac{(x_3+y_3)^2}{S^2}\right\}\right.$$
$$\left. - \sin 2\delta\left\{\frac{(x_2-y_2)(x_3-y_3)}{R^2} + \frac{(x_2-y_2)(x_3+y_3)}{S^2}\right\}\right] \tag{18}$$

with

$$R^2 = (x_2-y_2)^2 + (x_3-y_3)^2 \tag{19a}$$

and

$$S^2 = (x_2-y_2)^2 + (x_3+y_3)^2 \tag{19b}$$

From Figure 1, we put $y_2 = s\cos\delta$ and $y_3 = s\sin\delta$ into Equation (18) and integrate over s between the limits (s_1, s_2). We thus obtain the following expression for the Airy stress function for a long tensile fault of finite width $L = s_2 - s_1$:

$$U = (x_2\sin\delta - x_3\cos\delta)\tan^{-1}\left[\frac{s-(x_2\cos\delta + x_3\sin\delta)}{(x_2\sin\delta - x_3\cos\delta)}\right]$$
$$\left. - (x_2\sin\delta + x_3\cos\delta)\tan^{-1}\left[\frac{s-(x_2\cos\delta - x_3\sin\delta)}{(x_2\sin\delta + x_3\cos\delta)}\right]\right|_{s_1}^{s_2} \tag{20}$$

where

$$f(s)\Big|_{s_1}^{s_2} = f(s_2) - f(s_1) \tag{21}$$

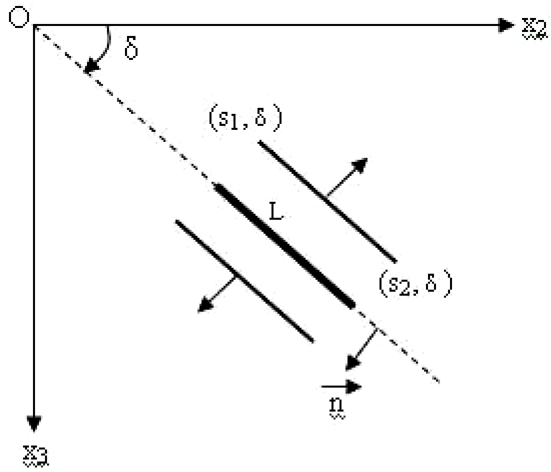


Fig. 1 Geometry of a long tensile fault of width $L = s_2 - s_1$ (the Cartesian coordinates of a point on the fault are (y_2, y_3) and its polar coordinates (s, δ), where δ is the dip angle and $s_1 \le s \le s_2$)

From Equations (1) and (21), the following expressions for stress components due to a vertical tensile fault (i.e., δ = 90°) are obtained

$$\tau_{22} = \frac{\mu b}{2\pi(1-\sigma)} \left[\frac{4x_2^6 x_3 + 4x_2^4 x_3 \left(2x_3^2 - s^2\right) + 4x_2^2 x_3 \left(x_3^4 + 2x_3^2 s^2 - 3s^4\right)}{\left[\left(x_2^2 + \left(s^2 - x_3^2\right)\right)^2 + 4x_2^2 x_3^2\right]^2} \right]\Bigg|_{s_1}^{s_2} \tag{22}$$

$$\tau_{23} = \frac{\mu b \left[\frac{\left[x_2^5 \left(20x_3^2 - 4s^2\right) + x_2^3 \left(8x_3^2 s^2 - 8s^4 - 16x_3^4\right) + x_2 \left(4x_2^2 s^4 + 4x_3^4 s^2 - 4s^6 - 4x_3^6\right)\right]}{\left[\left(x_2^2 + \left(s^2 - x_3^2\right)\right)^2 + 4x_2^2 x_3^2\right]^2} \right]\Bigg|_{s_1}^{s_2}}{2\pi(1-\sigma)} \tag{23}$$

$$\tau_{33} = \frac{\mu b \left[\frac{\left[4x_2^4 x_3 \left(3s^2 + x_3^2\right) + 8x_2^2 x_3 \left(s^4 - 2x_3^2 s^2 + x_3^4\right) + 4x_3 \left(3x_3^2 s^4 - 3x_3^4 s^2 + x_3^6 - s^6\right)\right]}{\left[\left(x_2^2 + \left(s^2 - x_3^2\right)\right)^2 + 4x_2^2 x_3^2\right]^2} \right]\Bigg|_{s_1}^{s_2}}{2\pi(1-\sigma)} \tag{24}$$

NUMERICAL RESULTS

To study the two-dimensional stress fields around a long vertical tensile fault of finite width L in a uniform half-space with rigid boundary, we put $s_1 = 0$ and $s_2 = L$, and assume $\sigma =$ 0.25. For numerical calculations, we define the following dimensionless quantities:

$$Y = \frac{x_2}{L} \tag{25a}$$

$$Z = \frac{x_3}{L} \tag{25b}$$

$$P_{ij} = \frac{\pi L}{\mu b} \tau_{ij} \tag{25c}$$

Thus, Y is the dimensionless distance from the fault trace, Z is the dimensionless depth, and P_{22}, P_{23} and P_{33} are the dimensionless stresses.

From Equations (22)–(25c), we obtain

$$P_{22} = \frac{8Y^2 Z\left[Y^4 + Z^4 + 2Z^2 + 2Y^2Z^2 - 2Y^2 - 3\right]}{3\left[\left\{Y^2 + \left(1 - Z^2\right)\right\}^2 + 4Y^2Z^2\right]^2} \tag{26}$$

$$P_{23} = \frac{8Y\left[5Y^4Z^2 - Y^4 + 2Y^2Z^2 - Y^2 - 4Y^2Z^4 + Z^4 - Z^6 - 1\right]}{3\left[\left\{Y^2 + \left(1 - Z^2\right)\right\}^2 + 4Y^2Z^2\right]^2} \tag{27}$$

$$P_{33} = \frac{8Z\left[3Y^4 + Y^4Z^2 + 2Y^2 - 4Y^2Z^2 + 2Y^2Z^4 + Z^6 - 3Z^4 + 3Z^2 - 1\right]}{3\left[\left\{Y^2 + \left(1 - Z^2\right)\right\}^2 + 4Y^2Z^2\right]^2} \tag{28}$$

DISCUSSION

Figures 2(a)–2(c) show the variations of the dimensionless stresses P_{22}, P_{23} and P_{33}, respectively, with dimensionless distance from the fault in the cases of x_3 = 0.1L, 0.3L and 0.5L. The patterns of P_{22} and P_{23} are similar with the only difference that P_{22} changes its sign from negative (i.e., compressive stress) to positive (i.e., tensile stress) for some values of x_2 lying between L and 2L. Both are zero at x_2 = 0. P_{33} has non-zero values at x_2 = 0 and it also changes its sign for some values of x_2 lying between 0 and L. All the three stresses tend to zero for the large values of x_2. The variation of stress P_{22} with distance from the fault (see Figure 2(a)) for x_3 = 0.1L is similar to that for P_{22} at x_3 = 0, as in Singh and Singh (2000), but for large values of depth, the pattern near the origin is significantly different due to the rigid boundaries.

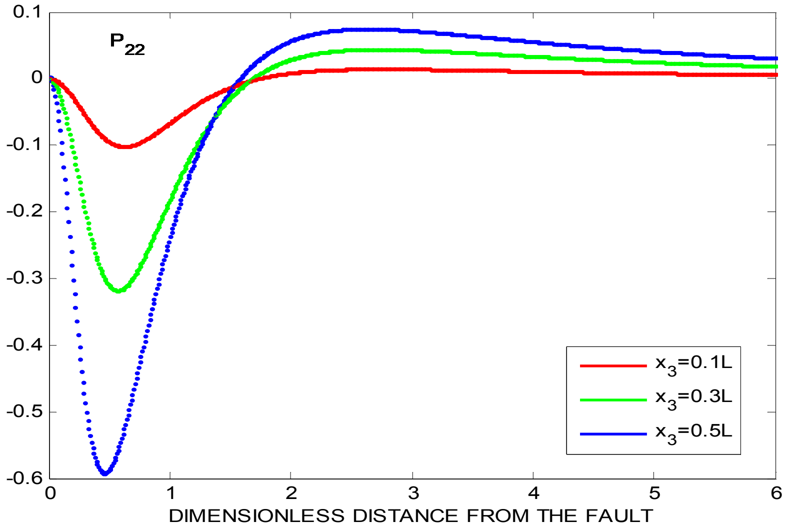
Figures 3(a)–3(c) show the variations of the dimensionless stresses P_{22}, P_{23} and P_{33}, respectively, with depth at x_2 = 0.3L, 0.8L and 1.3L. The variations of the normal and shear stresses are smooth for the large values of x_3. These variations are noteworthy near the fault, particularly at $x_3 = L$. The patterns of the stresses P_{22} and P_{23} in Figures 3(a) and 3(b) respectively are similar to those obtained by Singh and Singh (2000), except for the behaviour of stress components very near the origin.

ACKNOWLEDGEMENTS

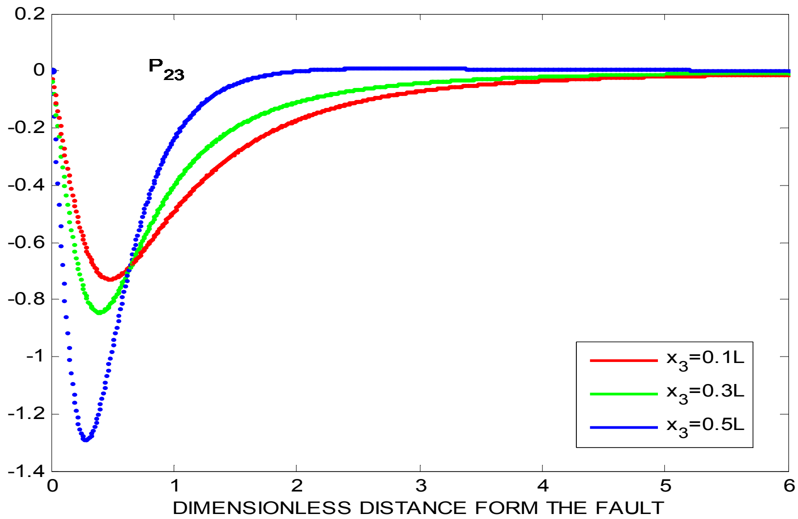
The authors are grateful to Professor Sarva Jit Singh for his valuable suggestions. The authors are thankful to the referees for their comments, which led to an improvement in the presentation of the paper.

APPENDIX: VALUES OF SOURCE COEFFICIENTS

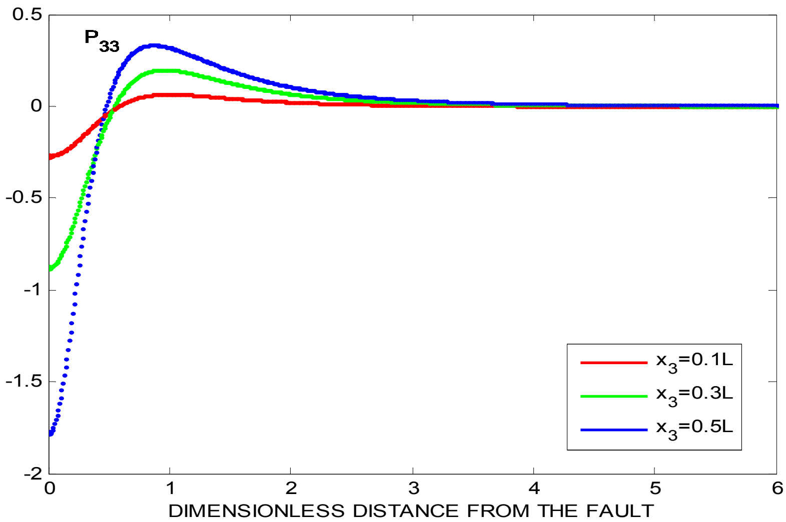
The values of source coefficients L_0, M_0, P_0 and Q_0 in three different cases are given below, where the upper sign is for $x_3 > h$, the lower sign is for $x_3 < h$, b is the magnitude of the displacement dislocation, and ds is the width of the line fault.



(a)



(b)



(c)

Fig. 2 Variations of the dimensionless (a) normal stress P_{22}, (b) shear stress P_{23}, (c) normal stress P_{33}, with distance from the fault for $x_3 = 0.1L$, $0.3L$ and $0.5L$

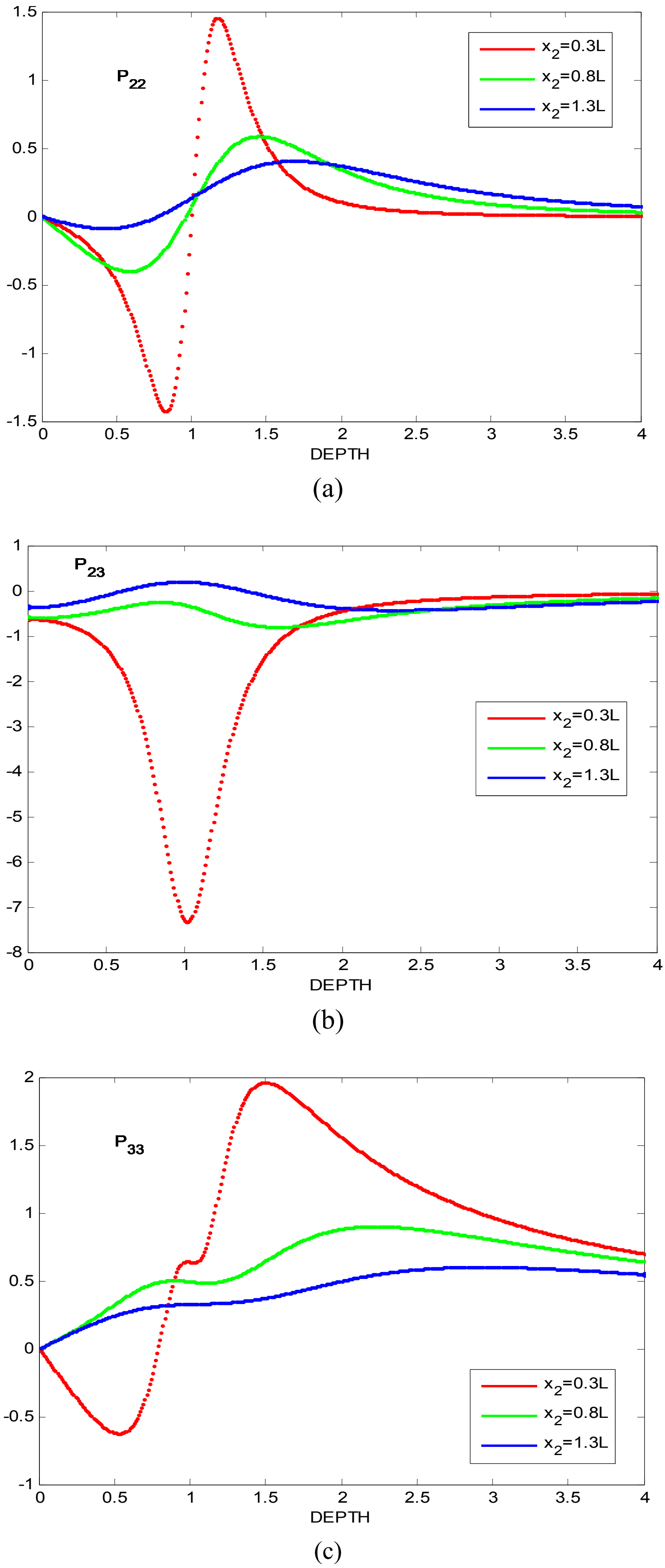


Fig. 3 Variations of the dimensionless (a) normal stress P_{22}, (b) shear stress P_{23}, (c) normal stress P_{33}, with depth for $x_2 = 0.3L$, $0.8L$ and $1.3L$

1. Vertical Dip-Slip Fault

$$L_0 = P_0 = Q_0 = 0 \tag{A.1}$$

$$M_0 = \pm \frac{\mu b \mathrm{d}s}{2\pi(1-\sigma)} \tag{A.2}$$

2. Vertical Tensile Fault

$$L_0 = M_0 = 0 \tag{A.3}$$

$$P_0 = -Q_0 = \frac{\mu b \mathrm{d}s}{2\pi(1-\sigma)} \tag{A.4}$$

3. Horizontal Tensile Fault

$$L_0 = M_0 = 0 \tag{A.5}$$

$$P_0 = Q_0 = \frac{\mu b \mathrm{d}s}{2\pi(1-\sigma)} \tag{A.6}$$

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