ISET Journal of Earthquake Technology, **Technical Note**, *Vol. 37, No. 1-3, March-Sept. 2000, pp. 39-47*

EQUIVALENT MECHANICAL MODEL FOR HORIZONTAL VIBRATION OF RIGID INTZE TANKS

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ABSTRACT

Equivalent mechanical model for rigid intze type tanks for horizontal vibration is developed. Parameters of the model are evaluated for a wide range of shapes of the tank and compared with those of the equivalent cylindrical tanks. It is shown that the errors associated with the use of equivalent cylindrical tank model in place of the intze tank model are small. Therefore, for design purpose the values available for equivalent cylindrical tank may be used.

KEYWORDS: Intze Tank, Mechanical Model, Hydrodynamic Forces

INTRODUCTION

Equivalent mechanical model offers a convenient approach for estimating hydrodynamic forces on containers, and hence the approach has become an integral part of the routine design procedures for liquid storage tanks. Although the early investigations considered tanks to be rigid (Jacobsen, 1949; Housner, 1957), later investigations showed that the hydrodynamic pressure is significantly influenced by the flexibility of the tank walls, and the mechanical model, originally developed for rigid tanks, was subsequently modified to account for the effect (Veletsos, 1974; Fischer, 1979; Haroun, 1983; Haroun and Housner, 1981a, 1981b, 1982; Gupta and Hutchinson, 1988, 1989). However, tanks with regular shapes only, rectangular and cylindrical, have been studied extensively and containers with irregular shapes have received little attention. Abramson (1966) reviews models for tanks with various shapes for aerospace applications, and therefore, shapes frequently used for elevated water storage tanks do not appear in his work.

Intze tanks, because of their optimal load balancing shape, are extensively used for storing water for civic purposes. The objective of this paper is to develop a mechanical model for intze type tanks and explore the possibility of using for their design the procedure already available for cylindrical tanks with necessary modifications. To this end, mechanical model parameters for intze tanks of different shapes are evaluated and compared with those of the equivalent cylindrical tanks, and the associated error is quantified.

While evaluating model parameters, following assumptions are made. The tank is assumed rigid. Fluid pressure is evaluated using linearized potential flow theory in which the fluid is assumed inviscid and incompressible and the sloshing wave height is assumed small. Only first sloshing mode is considered in the mechanical model, and the mass values and other parameters of the mechanical model are not adjusted to account for the missing mass corresponding to higher sloshing modes.

PROBLEM FORMULATION

1. Boundary Value Problem

The small amplitude, irrotational motion of an incompressible and inviscid fluid is governed by the Laplace equation (Lamb, 1945),

$$\nabla^2 \bar{p}(\mathbf{x}, \omega) = 0 \tag{1}$$

subject to boundary conditions

$$\frac{\partial}{\partial n}\bar{p}(\mathbf{x},\omega) = -\rho\bar{a}(\mathbf{x}) \tag{2}$$

on the tank-water interface Γ_w and

$$\frac{\partial}{\partial n}\bar{p}(\mathbf{x},\omega) = \frac{\omega^2}{g}\bar{p}(\mathbf{x},\omega) \tag{3}$$

on the free surface Γ_f. In Equations (1) to (3), $\bar{p}(\mathbf{x},\omega)$ is the frequency response function for hydrodynamic pressure at point $\mathbf{x}$ in the three dimensional fluid domain Ω, $\bar{a}(\mathbf{x},\omega)$ is the amplitude of harmonic acceleration of the tank wall-fluid interface Γ_w along its normal direction, ω is the excitation frequency, n is the outward normal direction to Γ_w, ρ is mass density of fluid, and g is the gravitational acceleration.

According to linearized potential flow theory, the hydrodynamic pressure can be expressed as a sum of "convective" and "impulsive" pressures. Convective pressure results from vibration of the free surface and is frequency-dependent. Motion of the fluid that vibrates with the container walls gives rise to impulsive pressure. Convective pressure can be further expanded into contributions from different natural vibration modes of free surface which are evaluated by solution of the eigenvalue problem associated with the above boundary value problem. For large values of excitation frequency ω, the oscillations of the free surface diminish, and in the limit as $\omega \to \infty$, the convective pressure disappears completely and only frequency independent impulsive pressure remains. Thus, the impulsive pressure distribution is obtained by solution of the boundary value problem with the boundary condition of Equation (3) modified as

$$\bar{p} = 0 \qquad on \quad \Gamma_f \tag{4}$$

2. Numerical Evaluation Procedure

The fluid domain is axisymmetric about z axis. Additionally, the acceleration $\bar{a}$ along the normal direction of the tank wall-fluid interface due to rigid body motion of the tank along horizontal direction varies as cosine of the angle along the circumferential direction of the tank. These facts have been used to advantage to convert the original three-dimensional problem into a two-dimensional problem in r-z plane. Definitions of the quantities used hereafter remain same except that they are now defined on r-z plane and have one dimension less.

The natural frequencies ω_n and the pressure distribution p_n associated with the n-th natural vibration mode render the following functional minimum.

$$\Pi = \int_{\Omega} r\,\nabla p_n \cdot \nabla p_n \,\mathrm{d}\Omega + \int_{\Omega} \frac{(p_n)^2}{r}\mathrm{d}\Omega - \frac{(\omega_n)^2}{2g}\int_{\Gamma_f} r\,(p_n)^2 \mathrm{d}\Gamma \tag{5}$$

Similarly, p_0, the pressure due to unit rigid body harmonic acceleration of the tank along horizontal direction renders the following functional minimum.

$$\Pi = \int_{\Omega} r\,\nabla p_0 \cdot \nabla p_0 \,\mathrm{d}\Omega + \int_{\Omega} \frac{(p_0)^2}{r}\mathrm{d}\Omega + \int_{\Gamma_w} \rho\, r\, p_0\, n_r \,\mathrm{d}\Gamma \tag{6}$$

In the above equation, n_r is the direction cosine of the normal n with respect to direction r. Standard finite element analysis is used to evaluate the pressure distributions (Zienkiewicz and Taylor, 1989). Discretization of the fluid domain using nine-node element is shown in Figure 1(b). While evaluating p_0, shape functions that satisfy the Dirichlet condition of Equation (4) explicitly are used.

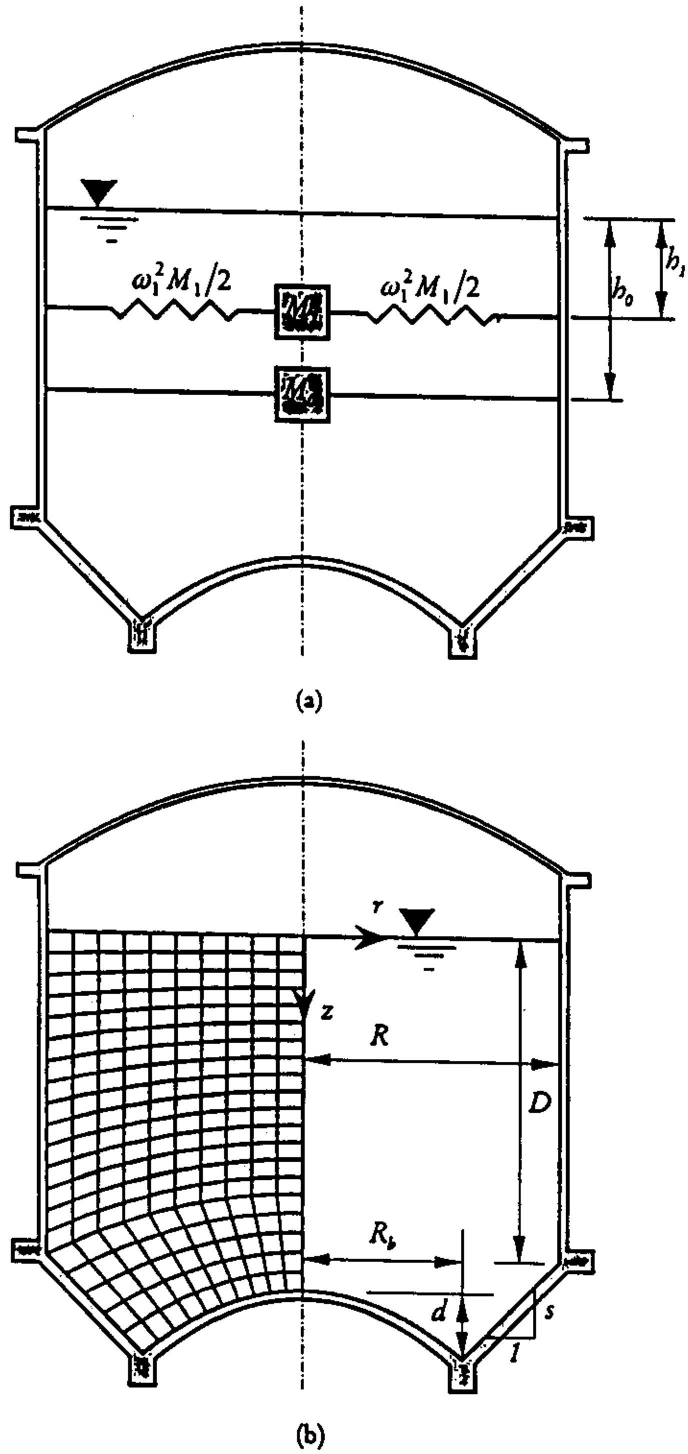

Fig. 1 Equivalent mechanical model and FEM discretization of fluid

3. Equivalent Mechanical Model

The mechanical behavior of a fluid domain in rigid containers can always be represented by a series of spring connected masses (Abramson, 1966). In an equivalent mechanical model, each sloshing mode is represented by a spring-mass system whereas a rigidly connected mass represents impulsive pressure. In this paper, only two-mass model is considered, each, respectively, corresponding to the impulsive pressure and the fundamental sloshing mode (Figure 1(a)). The mechanical model parameters are evaluated by equating force exerted by mechanical system on the tank walls to the hydrodynamic force evaluated by analysis of fluid domain using linearized potential flow theory. If ω_n and p_n are the natural frequency and pressure distribution corresponding to n-th sloshing modes, the mass and spring stiffness in the mechanical model and their location are given by the following expressions.

$$M_n = \frac{\rho\pi g}{\omega_n^2}\left[\int_{\Gamma_w} r\, p_n\, n_r\, \mathrm{d}\Gamma\right]^2 \Bigg/ \int_{\Gamma_f} r(p_n)^2 \mathrm{d}\Gamma \tag{7}$$

$$K_n = (\omega_n)^2 M_n \tag{8}$$

$$h_n = \frac{1}{\pi}\int_{\Gamma_w} r\, p_n\left(\pi z n_r + 4 r n_z\right)\mathrm{d}\Gamma \Bigg/ \int_{\Gamma_w} r\, p_n\, n_r\, \mathrm{d}\Gamma \tag{9}$$

In the above equation, n_z is the direction cosine of the normal n with respect to z-axis. Similarly, if p_0 is the pressure distribution due to unit horizontal acceleration of the tank with free surface waves ignored, the rigidly connected mass and its location are given by the following equations.

$$M_0 = -\pi \int_{\Gamma_w} r\, p_0\, n_r\, \mathrm{d}\Gamma \tag{10}$$

$$h_0 = \frac{1}{\pi}\int_{\Gamma_w} r\, p_0\left(\pi z n_r + 4 r n_z\right)\mathrm{d}\Gamma \Bigg/ \int_{\Gamma_w} r\, p_0\, n_r\, \mathrm{d}\Gamma \tag{11}$$

MECHANICAL MODEL PARAMETERS

As mentioned earlier, the objective of the paper is to quantify the errors associated with the use of cylindrical tank results for design of intze tanks. To this end, mechanical model parameters for a wide range of shapes of intze tank are evaluated and compared with those of the equivalent cylindrical tank. The shape of the intze tank considered here can be uniquely defined using four parameters: depth of the tank D, slope of the bottom slab {s in 1}, radius R_b and rise d of the bottom slab (Figure 1(b)). The equivalent cylindrical tank is defined as a cylindrical tank having radius and volume equal to that of an intze tank. Thus, an equivalent cylindrical tank has only one shape parameter: equivalent depth D_e.

As a first step, impulsive and convective pressure distributions for an intze tank of typical dimensions and the equivalent cylindrical tank are plotted for comparison in Figure 2. It is seen that the two pressure distributions are almost identical for the vertical portion of the tank walls.

Next, mechanical model parameters are evaluated for a wide range of shapes, and results are compared with those of equivalent cylindrical tanks. The range of shape parameters considered is as follows: $D/R = 0.5$ to 2, $s = 0.6$ to 1.7, $R_b/R = 0.5$ to 1.0, $d/R = 0$ to 0.4. The mechanical model parameters are plotted against normalized effective depth using scatter plots in Figure 3. Thus, each shape of the tank is represented by a 'dot' in the plots. Similar analysis is performed for equivalent cylindrical tanks and the results are plotted in the same figure using solid lines. Thus, any departure of the scattered points from the solid line is attributed to the *nonflatness* of the intze tank bottom.

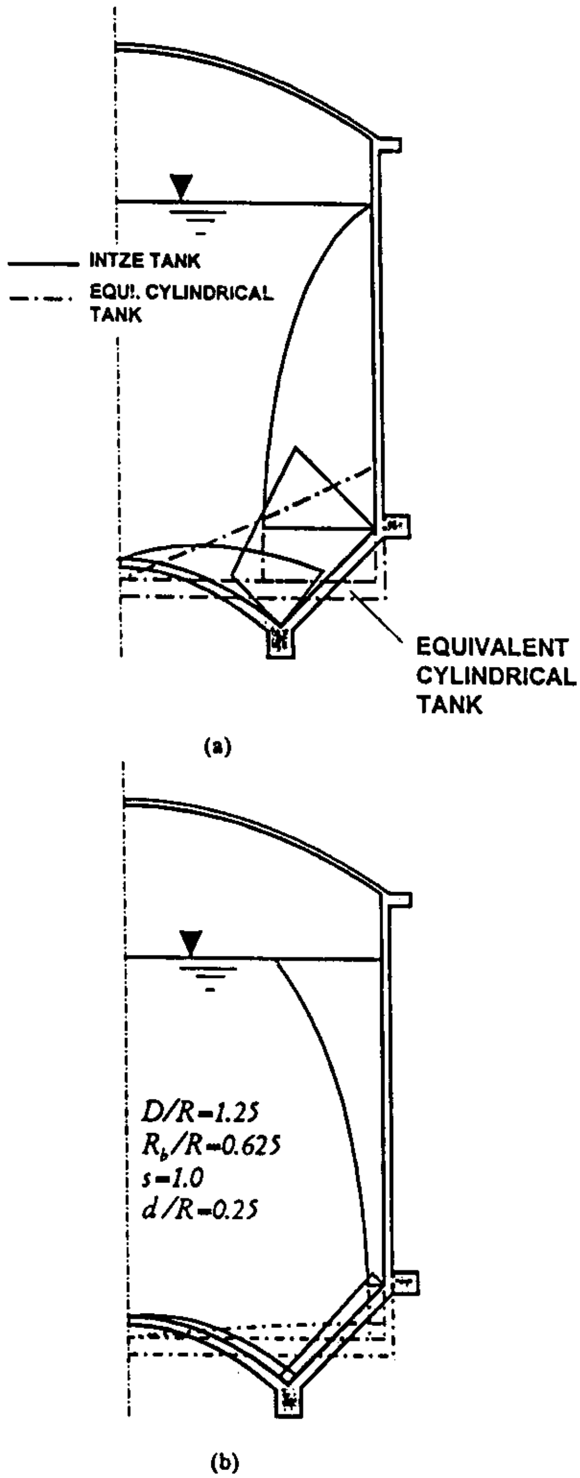

Fig. 2 Pressure distributions for intze and equivalent cylindrical tanks
(a) impulsive and (b) convective

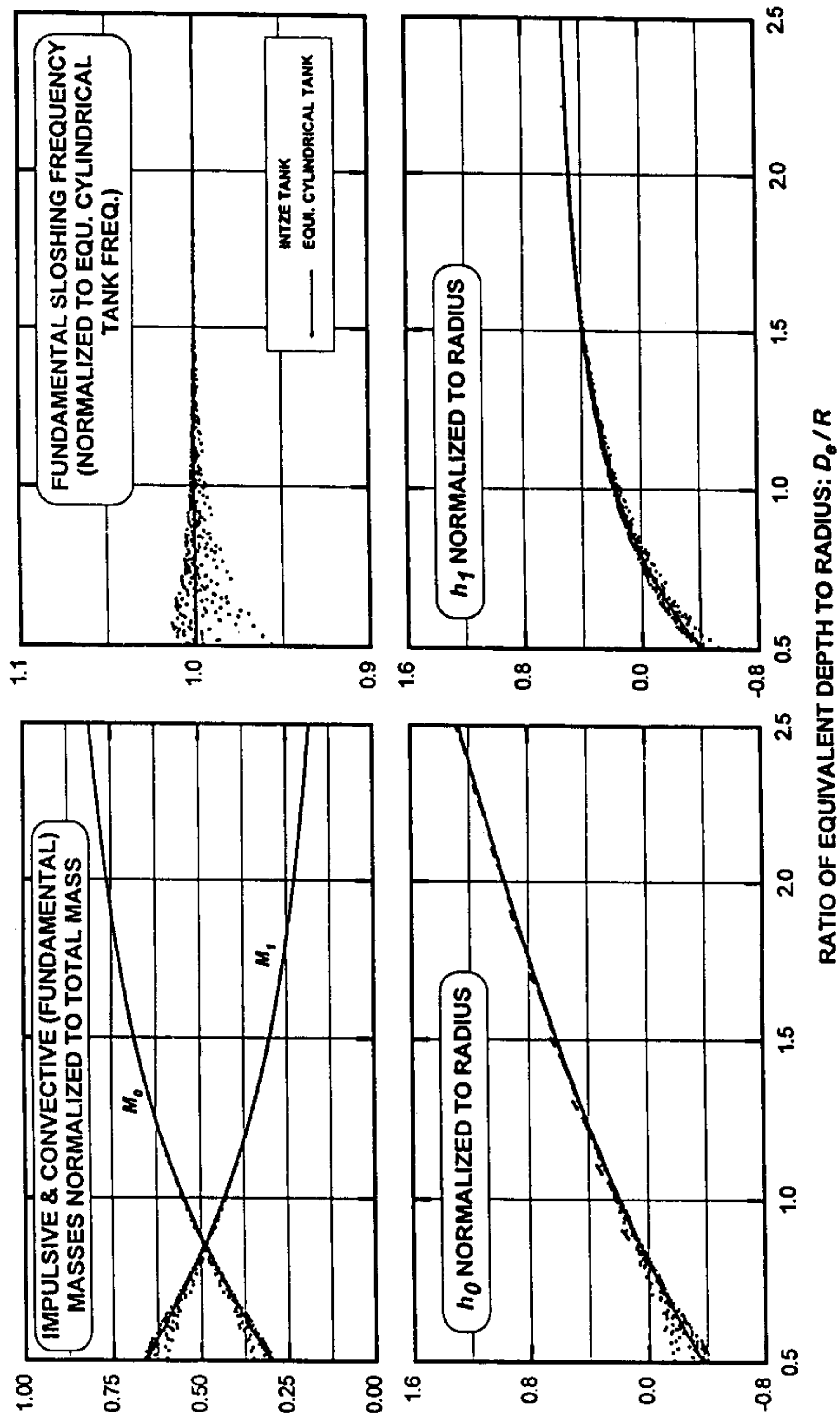

Fig. 3 Comparison of equivalent mechanical model parameters of intze and equivalent cylindrical tanks

Maximum absolute values of the departures from equivalent cylindrical tank results are presented in Table 1 for extremely low value of $D_e / R = 0.5$ as well as for moderate values of $D_e / R = 1.0, 1.5, 2.0$. It can be seen that for the latter cases, the error in both the mass values, impulsive and convective, is less than one percent of the total mass of the fluid. Departure from the fundamental sloshing frequency of the equivalent cylindrical tank is also of the same order. Similarly, maximum difference of 7% of the radius is found between the locations of masses in intze tank and the equivalent cylindrical tank. Therefore, since the two models do not differ significantly, the hydrodynamic forces for design of intze tank may be evaluated using equivalent cylindrical container without introducing significant error.

Table 1: Model Parameters for Intze and Equivalent Cylindrical Tanks - Maximum Absolute Error

	Maximum Absolute Error				
	M_0	h_0	M_1	ω_1	h_1
$D_e / R > 0.5$	$0.048\, M^{tot}$	0.198 R	$0.057\, M^{tot}$	$0.043\, \omega_1^{cyl}$	0.112 R
$D_e / R > 1.0$	$0.006\, M^{tot}$	0.061 R	$0.006\, M^{tot}$	$0.007\, \omega_1^{cyl}$	0.034 R
$D_e / R > 1.5$	$0.001\, M^{tot}$	0.040 R	$0.001\, M^{tot}$	$0.001\, \omega_1^{cyl}$	0.015 R
$D_e / R > 2.0$	$0.001\, M^{tot}$	0.026 R	$0.001\, M^{tot}$	$0.000\, \omega_1^{cyl}$	0.006 R

M^{tot} = Total mass of water; ω_1^{cyl} = Fundamental sloshing frequency for equivalent cylindrical tank

CONCLUSIONS

Equivalent mechanical model for the horizontal motion of an intze tank has been evaluated. By analyzing intze tanks of a wide range of shapes, it has been shown that the values of mechanical model parameters for intze tank and equivalent cylindrical tank are only slightly different. Therefore, for design purpose, the values available for equivalent cylindrical tank may be used.

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