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A NOTE ON SURFACE WAVE DISPERSION OF A 1-LAYER MICROPOLAR LIQUID-SATURATED POROUS HALF-SPACE

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ABSTRACT

A problem of surface wave propagation in a micropolar liquid-saturated porous layer over a micropolar liquid-saturated porous half-space of different elastic properties has been investigated. The frequency equation for surface wave propagation in the layer has been derived, and the effects of poroelasticity and micropolarity on phase velocity of surface waves have been studied in detail. Dispersion curves are computed numerically for a specific model and presented graphically. Some special cases have also been deduced.

KEYWORDS: Surface Wave Dispersion, Micropolarity, Poroelasticity, Frequency Equations

INTRODUCTION

A unified theory of wave propagation through a fluid-saturated porous medium was developed long back by Biot (1956a). In a subsequent paper, Biot (1956b) derived the general solution of the equations of elasticity and consolidation for a porous material. Since then, a number of papers appeared in the open literature related to wave propagation in porous media. A few of them are cited here, e.g., Deresiewicz and Skalak (1963), Sharma et al. (1990), Wang and Zhang (1998), Lauriks et al. (1998), Barry and Mercer (1999), Fellah and Depollier (2000), Cederbaum (2000), and Schanz and Cheng (2000).

The theory of micropolar continua was initiated by Suhubi and Eringen (1964) as a special case of their work on micro-elastic solid, and was renamed couple stress theory. Later, Eringen (1966) recapitulated and renamed it as micropolar theory. A similar theory appeared to be developed independently by Palmov (1964) for the linear elastic solid. Physically speaking, the theory of micropolar elasticity is concerned with those materials whose constituents are dumbbell molecules. These elements are allowed to rotate independently without stretch. The basic difference between the theory of micropolar elasticity and that of classical elasticity is the introduction of an independent microrotation vector. In classical elasticity, all other quantities can be obtained from the knowledge of three components of the displacement vector. In micropolar elasticity, we must also have knowledge of the three components of microrotation vector. In micropolar elastic bodies, the force at a point of a surface element is completely characterized by a stress vector and a couple stress vector at that point, while in classical elastic theory, the effect of couple stress is neglected.

Physically, solids that are composed of dumbbell molecules may be adequately represented by the model of micropolar elasticity. Fibrous materials and some granular and porous bodies may also fall in the category of this theory (Eringen, 1966). It is believed that “porous granular” material can be best approximated to soil (Deresiewicz, 1958). Thus, a peculiar type of soil/rocks whose molecules are granular, e.g., polycrystalline material, aluminum-epoxy, concrete, may be examples of micropolar solids.

It is believed that some soils whose molecules are granular, are very close to micropolar elastic porous medium. Hence, the present model is the motivation of the situation, when a micropolar liquid--saturated porous layer is resting on micropolar liquid-saturated porous elastic foundation. Since water, oil, chemicals, etc. are present inside the earth, the propagation of surface waves in such a layer may be relevant in the exploration of oils and other valuable liquids. The problem of surface wave propagation as a part of exploration seismology is also helpful in various economic activities, like tracing of hydrocarbons and other mineral ores. The present study, in fact, is a step to attempt a more realistic model of the earth's crust, and hence may be helpful in further investigation of exploration seismology, geophysics, earthquake engineering and soil dynamics, both theoretically and practically. We hope, this study will enhance the knowledge in better understanding the complexities of the earth medium.

Rao and Rao (1972) discussed the problem of a layered micropolar half-space. They obtained the frequency equation of surface wave propagation in a semi-infinite micropolar solid lying on another micropolar elastic solid. Ewing et al. (1957) studied the analogous problem in linear elasticity. Rao and Rao (1976) also discussed the problem of propagation of Rayleigh waves in a layer of micropolar elastic solid lying over a micropolar elastic half-space and under a uniform layer of inviscid liquid. Assaf and Jentsch (1992) worked on the elasticity theory of microporous solids. Rao and Reddy (1993) discussed Rayleigh type wave propagating on the surface of a micropolar elastic circular cylinder in an azimuthal direction. They have shown that due to the micropolar effect, there exists an extra wave, and the frequency of Rayleigh waves increases due to the micropolar effect. Kumar and Miglani (1996) studied the effect of pore alignment on surface wave propagation in a liquid-saturated porous layer lying on a liquid-saturated porous half-space with loosely bonded interface. Murad and Cushman (2000) studied thermomechanical theories for swelling porous media with microstructure. Deswal et al. (2000) discussed the effect of fluid viscosity on wave propagation in a cylindrical bore in micropolar elastic medium. Kumar and Deswal (2000) studied wave propagation in micropolar liquid-saturated porous solid.

The aim of the present study is to see the combined effect of micropolarity and porosity on surface wave dispersion for the wave propagating in a layer over half-space; both (layer and half-space) are taken as micropolar liquid-saturated porous solids of different elastic properties. The main contributions of the present paper are the following. (1) Dispersion equation is derived for Rayleigh type surface waves propagating in a micropolar liquid-saturated elastic layer lying over a similar half-space with different elastic properties. (2) Effect of micropolarity and porosity on surface wave dispersion is studied in two cases: (i) when the pores of the layer and the half-space are fully connected at their interface of separation, (ii) when there is no connection between the pores. (3) Dispersion curves are plotted for an elastic micropolar liquid-saturated porous layer over a micropolar liquid-saturated porous half-space and for three special cases: (a) an elastic micropolar layer over a micropolar half-space (porous effect is neglected), (b) an elastic porous layer over an elastic porous half-space (micropolar effect is neglected), and (c) an elastic layer over an elastic half-space (both micropolar and porous effects are neglected).

NOTATIONS

λ, μ	= elastic constants
α, β, γ, K	= micropolar constants
ρ	= density
J	= microinertia
$\rho_{11}, \rho_{12}, \rho_{22}$	= dynamical coefficients
R, Q	= material constants for solid-liquid aggregate
t_{ij}	= force stress tensor
m_{ij}	= couple stress tensor
σ	= stress in the liquid
$\vec{u}$	= displacement vector in the solid
$\vec{U}$	= displacement vector in the liquid
$\vec{\phi}$	= microrotation vector in the solid
χ	= coefficient of permeability
β^*	= porosity
η	= viscosity of the liquid
b	= dissipation coefficient
k	= wave number

c	= phase velocity
ω	= angular frequency
i	= $\sqrt{-1}$
$\vec{u}$	= (u, v, w)
$\vec{U}$	= (U, V, W)
δ_{ij}	= Kronecker's delta
ε_{ijk}	= permutation symbol
x, y, z	= space co-ordinates
$\vec{\Phi}$	= (ϕ_1, ϕ_2, ϕ_3)
ε	= pore alignment parameter

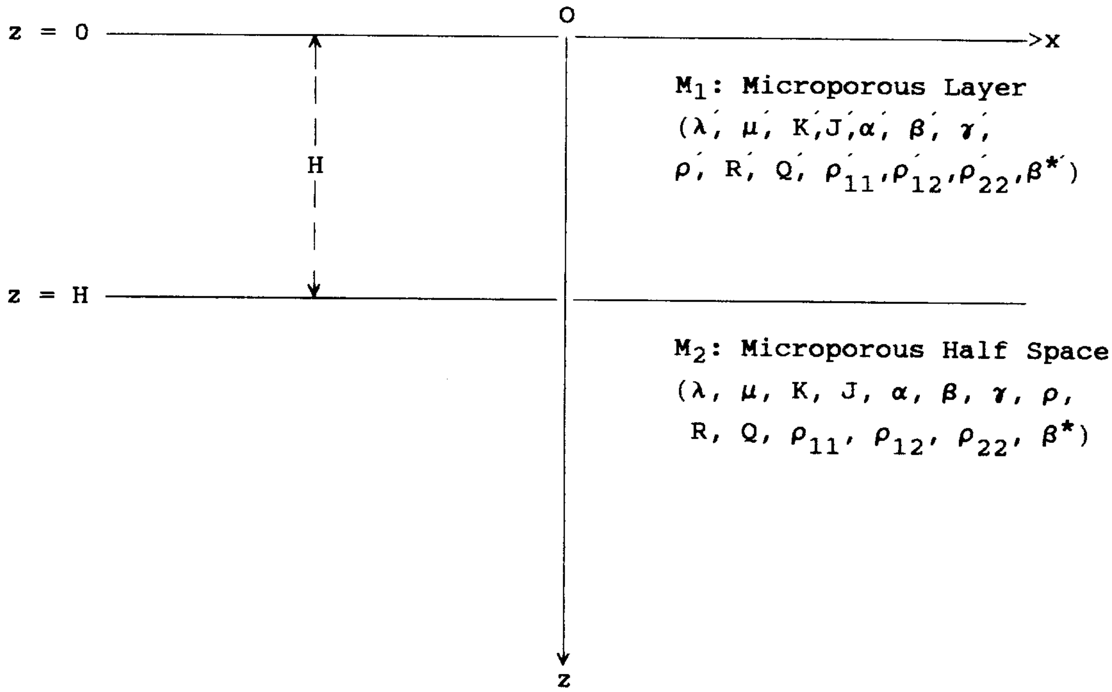

Fig. 1 Micropolar liquid-saturated porous layer over a micropolar liquid-saturated porous half-space

FORMULATION AND SOLUTION OF THE PROBLEM

We consider a micropolar liquid-saturated porous layer (M_1) of thickness H, lying over a micropolar liquid-saturated porous half-space (M_2) and separated by a plane interface. The $z-$axis is chosen in the direction of increasing depth, and $z = H$ is taken as the plane interface between the layer and half-space. The rectangular Cartesian co-ordinates (x, y, z) are used, and the complete geometry of the problem is shown in Figure 1. A two-dimensional problem of Rayleigh type surface wave propagation is considered, whose wave-front is parallel to $(x-z)$ plane. With this consideration, the quantities would remain independent of $y-$coordinate. We identify the field variables and constants, in the layer with superscript primes and that in the half-space without primes.

Following Konczak (1986, 1987), the vector form of equations of elastodynamics for a micropolar liquid-saturated porous solid without body forces and body couples in the presence of dissipation are given by

$$(\lambda+2\mu+K)\nabla\left(\nabla.\vec{u}\right) - (\mu+K)\,\nabla\times\left(\nabla\times\vec{u}\right) + K\,\nabla\times\vec{\phi} + Q\,\nabla\left(\nabla.\vec{U}\right)$$

$$=\frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{u}+\rho_{12}\vec{U}\right)+b\frac{\partial}{\partial t}\left(\vec{u}-\vec{U}\right) \tag{1}$$

$$\nabla\left(Qe+R\xi\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{u}+\rho_{22}\vec{U}\right)-b\frac{\partial}{\partial t}\left(\vec{u}-\vec{U}\right) \tag{2}$$

$$(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right) -\gamma\nabla\times\left(\nabla\times\vec{\phi}\right) + K\left(\nabla\times\vec{u}\right) -2K\vec{\phi} = \rho J\,\frac{\partial^2\vec{\phi}}{\partial t^2} \tag{3}$$

and the constitutive relations:

$$t_{ij}=\left(\lambda u_{r,r}+QU_{r,r}\right)\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{i,j}-\varepsilon_{ijr}\phi_r\right) \tag{4}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i} \tag{5}$$

$$\sigma=Qe+R\xi \tag{6}$$

where, $e = \nabla.\vec{u}$, $\xi=\nabla.\vec{U}$. Here, we employ rectangular co-ordinates $x_k\ (k=1,\ 2,\ 3)$ or $(x_1\equiv x,\ x_2\equiv y,\ x_3\equiv z)$ and usual summation convention on repeated indices. Also, indices following a comma indicate partial differentiation, e.g.,

$$u_{i,j}=\frac{\partial u_i}{\partial x_j}$$

The dissipation coefficient b is given by

$$b=\eta\beta^{*2}/\chi$$

Since we are considering two-dimensional problem in $x-z$ plane, we take

$$\vec{u}=(u,\ 0,\ w)\ \text{and}\ \vec{U}=(U,\ 0,\ W) \tag{7}$$

Assuming the time-harmonic variations, $e^{i\omega t}$, writing the Helmholtz representation of displacement vectors in scalar and vector potentials as

$$\vec{u}=\nabla q+\nabla\times\vec{H},\quad \nabla.\vec{H}=0$$

$$\vec{U}=\nabla\psi+\nabla\times\vec{G},\quad \nabla.\vec{G}=0 \tag{8}$$

in Equations (1) to (3), then eliminating ψ, $\nabla^2\psi$, ϕ_2 and $\nabla^2\phi_2$ from the resulting equations, we obtain the following equations

$$\left(A\nabla^4+B\omega^2\nabla^2+\omega^4C\right)\ q=0 \tag{9}$$

$$\left[A\nabla^2+\omega^2\left(R\rho_{11}-\rho_{12}Q\right)+i\omega b\left(R+Q\right)\right]\ q-\omega^2B^0\psi=0 \tag{10}$$

$$\left(\nabla^4+D\omega^2\nabla^2+E\omega^4\right)H^*=0 \tag{11}$$

$$\nabla^2\left(\nabla^2+\omega^2E_2+pr_2\right)H^*-p\left(-r_0+r_1\omega^2\right)\phi_2=0 \tag{12}$$

where

$$H^*=\left(-\vec{H}\right)_y\ \ \phi_2=\left(-\vec{\phi}\right)_y,\ A=PR-Q^2$$

$$P=\lambda+2\mu+K,\ p=\frac{K}{\mu+K}$$

$$B=P\left(\rho_{22}+\frac{ib}{\omega}\right)+R\left(\rho_{11}+\frac{ib}{\omega}\right)-2Q\left(\rho_{12}+\frac{ib}{\omega}\right)$$

$$C = \left(\rho_{11} + \frac{ib}{\omega}\right)\left(\rho_{22} + \frac{ib}{\omega}\right) - \left(\rho_{12} - \frac{ib}{\omega}\right)^2$$

$$B^o = \left(\rho_{22}Q - \rho_{12}R\right) + \frac{ib}{\omega}\left(R+Q\right), \quad D = E_2 + r_1 + \frac{\left(pr_2 - r_0\right)}{\omega^2}$$

$$r_2 = \frac{K}{\gamma}, \quad r_1 = \frac{\rho J}{\gamma}, \quad r_0 = 2r_2, \quad E = E_2\left(r_1 - \frac{r_0}{\omega^2}\right), \quad E_2 = \frac{E_1}{\mu + K}$$

$$E_1 = \left[\rho_{11} + \rho_{12}\frac{-\rho_{12}\omega + ib}{\rho_{22}\omega + ib} + ib\frac{\rho_{22} + \rho_{12}}{\rho_{22}\omega + ib}\right] \tag{13}$$

The solutions of Equations (9) to (12) in the layer M_1 are given by

$$q' = \left[A_1' e^{-\zeta_1' z} + B_1 e^{\zeta_1' z} + A_2' e^{-\zeta_2' z} + B_2 e^{\zeta_2' z}\right] e^{i(\omega t - kx)}$$

$$\psi' = \left[\mu_1'\left(A_1' e^{-\zeta_1' z} + B_1 e^{\zeta_1' z}\right) + \mu_2'\left(A_2' e^{-\zeta_2' z} + B_2 e^{\zeta_2' z}\right)\right] e^{i(\omega t - kx)}$$

$$H' = \left[A_3' e^{-\zeta_3' z} + B_3 e^{\zeta_3' z} + A_4' e^{-\zeta_4' z} + B_4 e^{\zeta_4' z}\right] e^{i(\omega t - kx)}$$

$$\phi_2' = \left[\mu_3'\left(A_3' e^{-\zeta_3' z} + B_3 e^{\zeta_3' z}\right) + \mu_4'\left(A_4' e^{-\zeta_4' z} + B_4 e^{\zeta_4' z}\right)\right] e^{i(\omega t - kx)} \tag{14}$$

and in the half-space M_2 are given by

$$q = \left[A_1 e^{-\zeta_1 z} + A_2 e^{-\zeta_2 z}\right] e^{i(\omega t - kx)}$$

$$\psi = \left[\mu_1 A_1 e^{-\zeta_1 z} + \mu_2 A_2 e^{-\zeta_2 z}\right] e^{i(\omega t - kx)}$$

$$H = \left[A_3 e^{-\zeta_3 z} + A_4 e^{-\zeta_4 z}\right] e^{i(\omega t - kx)}$$

$$\phi_2 = \left[\mu_3 A_3 e^{-\zeta_3 z} + \mu_4 A_4 e^{-\zeta_4 z}\right] e^{i(\omega t - kx)} \tag{15}$$

where, A_j, A_j' and B_j (j = 1, 2, 3, 4) are arbitrary constants and

$$\zeta_j = k\left(1 - c^2\lambda_j^2\right)^{1/2}, \quad j = 1, 2, 3, 4$$

$$\lambda_1^2 = \frac{1}{2A}\left[B - \left(B^2 - 4AC\right)^{1/2}\right], \qquad \lambda_2^2 = \frac{1}{2A}\left[B + \left(B^2 - 4AC\right)^{1/2}\right]$$

$$\lambda_3^2 = \frac{1}{2}\left[D - \left(D^2 - 4E\right)^{1/2}\right], \qquad \lambda_4^2 = \frac{1}{2}\left[D + \left(D^2 - 4E\right)^{1/2}\right] \tag{16}$$

$$\mu_i = \frac{1}{B^0}\left[-A\lambda_i^2 + \left(\rho_{11}R - \rho_{12}Q\right) + \frac{ib}{\omega}\left(R+Q\right)\right], \quad i = 1, 2$$

$$\mu_j = \frac{\omega^2\lambda_j^2\left(\omega^2\lambda_j^2 - \omega^2 E_2 - pr_2\right)}{p\left[-r_0 + r_1\omega^2\right]}, \quad j = 3, 4 \tag{17}$$

The corresponding quantities with primes will define the medium M_1.

BOUNDARY CONDITIONS

Following are the boundary conditions. (i) Vanishing of stresses at the free surface, i.e.,

$$t_{zz}' = t_{zx}' = m_{zy}' = \sigma' = 0 \tag{18}$$

and at the interface, $z = H$,

(ii) Continuity of normal stress, i.e.,

$$t_{zz} + \sigma = t'_{zz} + \sigma' \tag{19}$$

(iii) Continuity of shear stress, i.e.,

$$t_{zx} = t'_{zx} \tag{20}$$

(iv) Continuity of couple stress, i.e.,

$$m_{zy} = m'_{zy} \tag{21}$$

(v) Continuity of tangential displacement component, i.e.,

$$u = u' \tag{22}$$

(vi) Continuity of normal displacement component, i.e.,

$$w = w' \tag{23}$$

(vii) Conservation of mass of the liquid, i.e.,

$$\beta^*\left(\frac{\partial W}{\partial t} - \frac{\partial w}{\partial t}\right) = \beta^{*'}\left(\frac{\partial W'}{\partial t} - \frac{\partial w'}{\partial t}\right) \tag{24}$$

(viii) Continuity of microrotational displacement components, i.e.,

$$\phi_2 = \phi'_2 \tag{25}$$

(ix) When two liquid-saturated porous solids are in contact with each other through a plane boundary, then it may be possible that either the pores of two media are fully connected (open pore condition) or there is no connection between the interstices of the two media (sealed pore condition), and in the intermediate situations, the pores of the two media are partially connected. Following Deresiewicz and Skalak (1963), nonalignment of a portion of pores can produce an interfacial flow area which is smaller than when the pores are aligned. The effect of non-alignment of the portion of pores can be accomplished physically by inserting a porous membrane between the two poroelastic media with fully aligned pores. Flow through such an interface would result in a pressure drop across the interface. Therefore, with the assumed consistency between the pressure drop and normal component of filteration velocity, we can write the continuity requirement, regarding pressure drop, as

$$p - p' = \frac{1-\varepsilon}{\varepsilon}\beta^{*'}\left(\frac{\partial W'}{\partial t} - \frac{\partial w'}{\partial t}\right) \tag{26}$$

where, ε is defined as the pore-alignment parameter and $0 \le \varepsilon \le 1$. $\varepsilon = 1$ implies that the pores of the two media are completely connected, and $\varepsilon = 0$ corresponds to the case when there is no connection between the interstices of the two media. The intermediate values of ε $(0 < \varepsilon < 1)$ will correspond to the case when there is a partial alignment of pores at the interface of the two media. Making use of Equations (4) to (8), (15) and (16) in the above boundary conditions, i.e., in Equations (18) to (26), a system of twelve homogeneous equations in twelve unknowns, namely, A_j, A'_j, B_j (j = 1, 2, 3, 4) is obtained. The non-trivial solution of this system of equations requires that the determinant of coefficients of these unknowns must vanish, i.e.

$$|a_{ij}| = 0 \tag{27}$$

where, the non-zero entries of the twelfth order determinant are as given in Appendix I.

Now, if we consider the open and closed pore conditions, we have the following results.

(i) For $\varepsilon = 0$, i.e., when there is no connection between pores (closed pore condition), the elements are

$$a_{12,\,5} = a_{12,\,6} = -X'_1, \quad a_{12,\,7} = a_{12,\,8} = -X'_2$$

$$a_{12,9} = a_{12,10} = a_{12,11} = a_{12,12} = X$$

(ii) For $\varepsilon = 1$, i.e., when pores are completely connected (open pore condition), the elements are

$$a_{12,i} = (Q + \mu_i R)(\zeta_i^2 - k^2)$$

$$a_{12,\,5} = a_{12,\,6} = -V_1^{'},\;\; a_{12,\,7} = a_{12,\,8} = -V_2^{'}$$

where

$$S_i' = \left(\zeta_i'^2 - k^2\right)\ \left(\lambda' + Q'\mu_i'\right) + \left(2\mu' + K'\right)\zeta_i'^2$$

$$R_i' = k\zeta_i'\left(2\mu' + K'\right),\ Q_j' = k\zeta_j'\left(2\mu' + K'\right),\ V_i' = \left(Q' + R'\mu_i'\right)\ \left(\zeta_i'^2 - k^2\right)$$

$$T_j' = \left(\mu' + K'\right)\zeta_j'^2 + \mu'k^2 - K'\mu_j',\ U_j' = \mu_j'\zeta_j'$$

$$W_i' = \left(\zeta_i'^2 - k^2\right)\ \left(\lambda' + Q'\mu_i' + Q' + R'\mu_i'\right) + \left(2\mu' + K'\right)\zeta_i'^2$$

$$X_i' = \beta^{*'}\zeta_i'\left(1 - \mu_i'\right),\ X = k\beta^{*'}\left(\alpha_0' - 1\right),\ \alpha_0 = -\left(\frac{\rho_{12}\omega - ib}{\rho_{22}\omega + ib}\right) \tag{28}$$

$$(i = 1, 2\ \text{ and }\ j = 3, 4)$$

Equation (27) is the required frequency equation, relating the phase velocity c to the wavelength $2\pi / k$. The wave length is a multi-valued function of phase velocity (each value corresponding to the different mode of propagation), and hence indicates the dispersive nature of surface wave. Equation (27) is complex because of the dissipation of the system. The dispersion curves from these equations can be determined. However, the analytical solution of Equation (27) is impossible, and the numerical solution is also difficult. Thus, to solve Equation (27) numerically, we take the liquid-saturated porous media to be non-dissipative.

Special Cases

(i) Neglecting the micropolar effect, i.e., when $K = \alpha = \beta = \gamma = 0$, Equation (27) reduces to

$$\left|b_{ij}\right| = 0 \tag{29}$$

where, the non-zero entries of the determinant b_{ij} are as given in Appendix II.

(a) when $\varepsilon = 0$, we have

$$b_{94} = b_{95} = -X_5',\ b_{96} = b_{97} = -X_6',\ b_{98} = b_{99} = X$$

(b) when $\varepsilon = 1$, we have

$$b_{91} = \left(Q + \mu_5 R\right)\ \left(\zeta_5^2 - k^2\right),\ b_{92} = \left(Q + \mu_6 R\right)\ \left(\zeta_6^2 - k^2\right)$$

$$b_{94} = b_{95} = -V_5',\ b_{96} = b_{97} = -V_6'$$

where

$$S_i' = \left(\zeta_i'^2 - k^2\right)\ \left(\lambda' + Q'\mu_i'\right) + 2\mu'\zeta_i'^2$$

$$R_j' = 2k\mu'\zeta_j',\ Q_7' = \mu'\left(\zeta_7'^2 + k^2\right),\ V_i' = \left(Q' + R'\mu_i'\right)\ \left(\zeta_i'^2 - k^2\right)$$

$$W_i = \left(\lambda + Q\mu_i + Q + R\mu_i\right)\ \left(\zeta_i^2 - k^2\right) + 2\mu\zeta_i^2$$

$$W_i' = \left(\lambda' + Q'\mu_i + Q' + R'\mu_i'\right)\ \left(\zeta_i'^2 - k^2\right) + 2\mu'\zeta_i'^2$$

$$X_i' = \beta^{*'}\zeta_i'\left(1 - \mu_i'\right),\ \zeta_j = k\left(1 - c^2\lambda_j^2\right)^{1/2}$$

$$\lambda_5^2 = \frac{1}{2A^*}\left[B^* - \left(B^{*2} - 4A^*C\right)^{1/2}\right]$$

$$\lambda_6^2 = \frac{1}{2A^*}\left[B^* + \left(B^{*2} - 4A^*C\right)^{1/2}\right],\ \lambda_7^2 = \frac{c}{\mu\left[\rho_{22} + \dfrac{ib}{\omega}\right]}$$

$$A^* = P^* R - Q^2, \quad P^* = \lambda + 2\mu$$

$$B^* = P^*\left(\rho_{22} + \frac{ib}{\omega}\right) + R\left(\rho_{11} + \frac{ib}{\omega}\right) - 2Q\left(\rho_{12} - \frac{ib}{\omega}\right)$$

$$\mu_5 = \frac{1}{B^0}\left[-A^*\lambda_5^2 + \left(\rho_{11}R - \rho_{12}Q\right) + \frac{ib}{\omega}\left(R+Q\right)\right]$$

$$\mu_6 = \frac{1}{B^0}\left[-A^*\lambda_6^2 + \left(\rho_{11}R - \rho_{12}Q\right) + \frac{ib}{\omega}\left(R+Q\right)\right], \; i = 5, 6 \text{ and } j = 5, 6, 7 \tag{30}$$

Equation (29) is the frequency equation for surface wave propagation in a liquid-saturated porous layer lying over a liquid-saturated porous half-space.

(ii) Neglecting the porous effect, Equation (27) reduces to

$$\left|c_{ij}\right| = 0 \tag{31}$$

where, the non-zero entries are as given in Appendix III.

Thus, Equation (27) reduces to Equation (31) which corresponds to surface wave propagation in a micropolar elastic layer over a micropolar elastic half-space.

(iii) Neglecting both the micropolar and porous effects, we obtain the frequency equation in an elastic layer over an elastic half-space as

$$\left|d_{ij}\right| = 0 \tag{32}$$

where the non-zero entries are as given in Appendix IV.

The frequency equation, Equation (32), is same as obtained by Ewing et al. (1957), with slight change in notations. The difference of sign in some terms is due to the consideration of $H^* = \left(-\vec{H}\right)_y$.

NUMERICAL RESULTS AND DISCUSSION

In order to study the effect of micropolarity, porosity and pore alignment parameter on the dispersion curves, we solve the dispersion equation (Equation (27)) numerically for a specific model. The specific model assumed is based on the situation existing in reality, for example, that in the geological oil reservoirs. Since kerosene oil is lighter than water, we have considered the presence of kerosene oil in the layer and that of water in the half-space to satisfy the physically acceptable situation. Nevertheless, the analysis holds for other types of liquids existing in nature. Thus, the specific model for the purpose of numerical calculations consists of a microporous kerosene-saturated layer over a microporous water-saturated half-space. The non-dissipative assumption of the microporous media shall facilitate us to obtain the real wave velocity numerically.

Following the experimental results given by Yew and Jogi (1976) and earlier data given by Fatt (1959), the following values of relevant parameters are taken.

(i) For kerosene-saturated sandstone (medium M_1)

$\lambda = 0.4436\times10^{10}$ N/m^2, $\mu = 0.2765\times10^{10}$ N/m^2

$Q = 0.07635\times10^{10}$ N/m^2, $R = 0.0326\times10^{10}$ N/m^2

$\rho_{11} = 1.926137\times10^3$ kg/m^3, $\rho_{12} = -0.002137\times10^3$ kg/m^3

$\rho_{22} = 0.215337\times10^3$ kg/m^3

(ii) For water-saturated sandstone (medium M_2)

$\lambda = 0.306\times10^{10}$ N/m^2, $\mu = 0.922\times10^{10}$ N/m^2

$$Q = 0.013\times10^{10}\ \text{N/m}^2, \qquad R = 0.0637\times10^{10}\ \text{N/m}^2$$

$$\rho_{11} = 1.9032\times10^3\ \text{kg/m}^3, \qquad \rho_{12} = 0$$

$$\rho_{22} = 0.268\times10^3\ \text{kg/m}^3$$

and the porosity for the media M_1 and M_2 are taken to be $\beta^* = 0.26$ and $\beta^* = 0.268$ respectively.

Following Gauthier (1982), we take the following values of the relevant micropolar constants as

$$\rho = 2.19\times10^3\ \text{kg/m}^3, \qquad \gamma = 2.63\times10^3\ \text{N}$$

$$J = 0.00196\times10^{-2}\ \text{m}^2, \qquad K = 0.00149\times10^{10}\ \text{N/m}^2$$

Equation (27) is solved by using the above values of parameters in the media M_1 and M_2. It is found that there exist infinite number of modes of propagation of surface waves for various values of wave numbers; the variations of only fundamental modes are presented graphically for different theories. For the purpose of numerical calculations, we have calculated the normalized phase velocity c/c_1 against the values of normalized wave number *kH.* Because of the limitations of computer machine available with us, the values of phase velocity could not be calculated for $0 < kH < 0.01$.

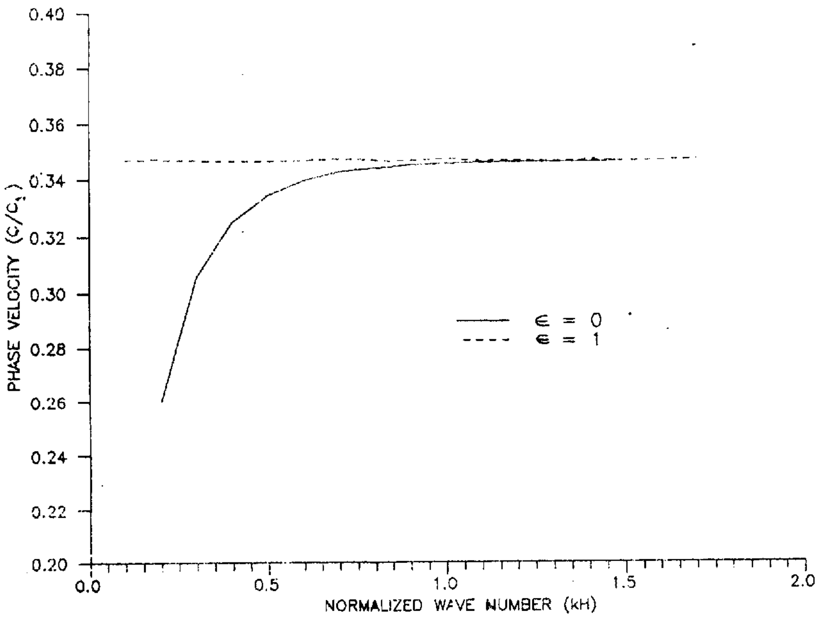

Fig. 2 Variation of phase velocity with wave number (MEP theory)

The variations of c/c_1 for different values of *kH* are shown in Figure 2 for micropolar elastic with porous (MEP) theory. The dispersion curve with solid line corresponds to the case when there is no connection between the pores of the media M_1 and M_2 at the interface, while the dashed curve corresponds to the case when there is total alignment in between the pores of the two media. It can be concluded from Figure 2 that phase velocity c/c_1 depends on the wave number *kH,* indicating that surface waves are dispersive. For the initial values of *kH,* the dispersion curves for $\varepsilon = 0$ and $\varepsilon = 1$ are seen to be different in nature. However, for larger values of *kH,* the dispersion curves have almost similar behaviour in two cases, i.e., $\varepsilon = 0$ and $\varepsilon = 1$. When the micropolarity is removed and media M_1 and M_2 are elastic porous (EP), the dispersion curves for $\varepsilon = 0$ and $\varepsilon = 1$ overlap in the range $0.01 < kH < 2.5$. However, for $kH \geq 2.5$, the values of phase velocity for $\varepsilon = 0$ are more in comparison to $\varepsilon = 1$, and the behaviour of dispersion curves for the two cases is similar, as shown in Figure 3.

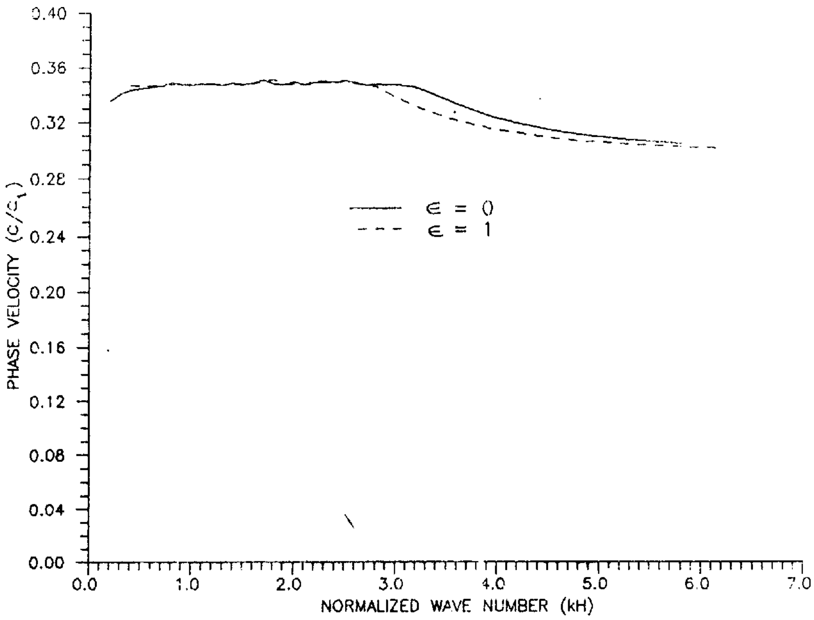

Fig. 3 Variation of phase velocity with wave number (EP theory)

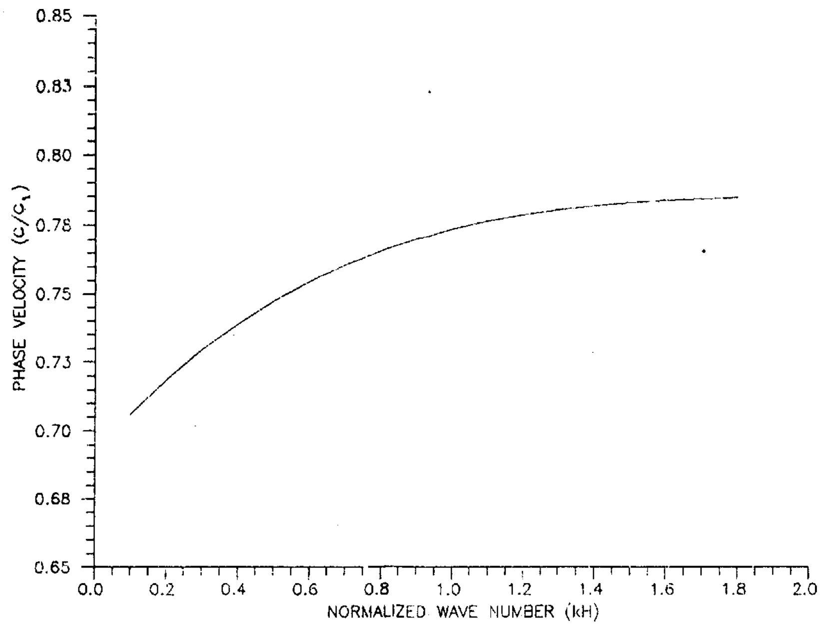

Fig. 4 Variation of phase velocity with wave number (ME theory)

Further, it can be noticed from Figure 4 that there is a smooth increase in the values of phase velocity with the increase in wave number for $0.1 < kH < 1.85$, and then those become almost constant with the increase in wave number, when the pores of the media are neglected (ME). The values of phase velocity for ME theory are more in comparison to MEP and EP theories due to porosity effect. The comparison of Figures 2 and 3 shows that the behaviour of dispersion curves is slightly different in the two figures due to effect of micropolarity, although the range of variation of phase velocity is almost similar for the two theories.

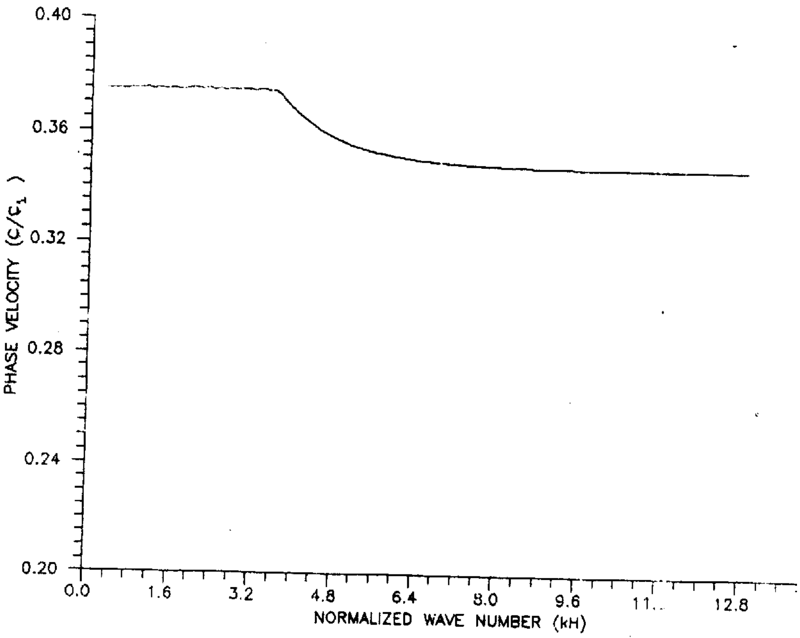

Fig. 5 Variation of phase velocity with wave number (Elastic theory)

Figure 5 contains the dispersion curve for the case when the media are free from micropolarity and porosity, and it is worth noticing that the values of phase velocity remain almost constant in the range, $0.4 < kH < 4.0$, thereafter decrease very smoothly with the increase in wave number in the range, $4.0 \le kH \le 9.6$, and then become constant for the further range of kH. Thus, the present study reveals that there is a significant effect of micropolarity as well as of porosity of the media on the surface wave propagation.

A mathematical study has been presented here to determine the effect of micropolarity, porosity, and pore alignment parameter on the dispersion curves. Numerical computations have been performed to solve the frequency equations (MEP, EP and ME theories), and the following inferences are made:

i. it is seen that the phase velocity of wave propagation depends on the wave number, showing that the frequency equation is dispersive,
ii. the effect of porosity is more significant than that of micropolarity.

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APPENDIX I

$$a_{15} = S_1' \; e^{\zeta_1' H}, \qquad a_{16} = S_1 \; e^{-\zeta_1' H}, \qquad a_{17} = S_2' \; e^{\zeta_2' H}$$

$$a_{18} = S_2' \; e^{-\zeta_2' H}, \qquad a_{19} = Q_3' \; e^{\zeta_3' H}, \qquad a_{1,10} = Q_3' \; e^{-\zeta_3' H}$$

$$a_{1,11} = Q_4' \; e^{\zeta_4' H}, \qquad a_{1,12} = Q_4' \; e^{-\zeta_4' H}$$

$$a_{25} = -R_1' \; e^{\zeta_1' H}, \qquad a_{26} = R_1' \; e^{-\zeta_1' H}, \qquad a_{27} = -R_2' \; e^{\zeta_2' H}$$

$$a_{28} = R_2' \; e^{-\zeta_2' H}, \qquad a_{29} = T_3' \; e^{\zeta_3' H}, \qquad a_{2,10} = T_3' \; e^{-\zeta_3' H}$$

$$a_{2,11} = T_4' \; e^{\zeta_4' H}, \qquad a_{2,12} = T_4' \; e^{-\zeta_4' H}$$

$$a_{39} = -U_3' \; e^{\zeta_3' H}$$

$$a_{3,10} = U_3' \; e^{-\zeta_3' H}, \qquad a_{3,11} = -U_4' \; e^{\zeta_4' H}, \qquad a_{3,12} = U_4' \; e^{-\zeta_4' H}$$

$a_{45} = V_1' \ e^{\zeta_1' H}$, $a_{46} = V_1' \ e^{-\zeta_1' H}$, $a_{47} = V_2' \ e^{\zeta_2' H}$

$a_{48} = V_2' \ e^{-\zeta_2' H}$

$a_{5i} = \left\{\lambda + (Q+R)\mu_i + Q\right\} \ \left(\zeta_i^{\,2} - k^2\right) + (2\mu + K)\zeta_i^{\,2}$

$a_{5j} = k\zeta_j (2\mu + K)$, $a_{55} = a_{56} = W_1'$, $a_{57} = a_{58} = W_2'$

$a_{59} = -Q_3'$, $a_{5,\,10} = Q_3'$, $a_{5,\,11} = -Q_4'$

$a_{5,\,12} = Q_4'$

$a_{6i} = k\zeta_i (2\mu + K)$, $a_{6j} = -\left[(\mu + K)\zeta_j^{\,2} + \mu k^2 - K\mu_j\right]$

$a_{65} = -R_1'$, $a_{66} = R_1'$, $a_{67} = -R_2'$

$a_{68} = R_2'$

$a_{69} = a_{6,\,10} = T_3'$, $a_{6,\,11} = a_{6,\,12} = T_4'$

$a_{7j} = \gamma\mu_j\zeta_j$, $a_{79} = -\gamma' U_3'$, $a_{7,\,10} = \gamma' U_3'$

$a_{7,\,11} = -\gamma' U_4'$, $a_{7,\,12} = \gamma' U_4'$

$a_{8i} = \zeta_i$, $a_{8j} = -k$, $a_{85} = -\zeta_1'$

$a_{86} = \zeta_1'$, $a_{87} = -\zeta_2'$, $a_{88} = \zeta_2'$

$a_{89} = a_{8,\,10} = a_{8,\,11} = a_{8,\,12} = k$

$a_{91} = a_{92} = -k$, $a_{9j} = \zeta_j$, $a_{95} = a_{96} = a_{97} = a_{98} = k$

$a_{99} = -\zeta_3'$, $a_{9,\,10} = \zeta_3'$, $a_{9,\,11} = -\zeta_4'$

$a_{9,\,12} = \zeta_4'$

$a_{10,\,i} = \beta^* (\mu_i - 1)\zeta_i$, $a_{10,\,j} = k\beta^* (1 - \alpha_0)$

$a_{10,\,5} = X_1$, $a_{10,\,6} = -X_1$, $a_{10,\,7} = X_2$

$a_{10,\,8} = -X_2$, $a_{10,\,9} = a_{10,\,10} = a_{10,\,11} = a_{10,\,12} = X$

$a_{11,j} = \mu_j$, $a_{11,\,9} = a_{11,\,10} = \mu_3'$, $a_{11,\,11} = a_{11,\,12} = \mu_4'$

APPENDIX II

$b_{14} = S_5' \ e^{\zeta_5' H}$, $b_{15} = S_5' \ e^{-\zeta_5' H}$, $b_{16} = S_6' \ e^{\zeta_6' H}$

$b_{17} = S_6' \ e^{-\zeta_6' H}$, $b_{18} = -R_7' \ e^{\zeta_7' H}$, $b_{19} = R_7' \ e^{-\zeta_7' H}$

$b_{24} = -R_5' \ e^{\zeta_5' H}$, $b_{25} = R_5' \ e^{-\zeta_5' H}$, $b_{26} = -R_6' \ e^{\zeta_6' H}$

$b_{27} = R_6' \ e^{-\zeta_6' H}$, $b_{28} = -Q_7' \ e^{\zeta_7' H}$, $b_{29} = Q_7' \ e^{-\zeta_7' H}$

$b_{34} = V_5' \ e^{\zeta_5' H}$, $b_{35} = V_5' \ e^{-\zeta_5' H}$, $b_{36} = V_6' \ e^{\zeta_6' H}$

$b_{37} = V_6' \ e^{-\zeta_6' H}$, $b_{41} = -W_5$, $b_{42} = -W_6$

$b_{43} = 2\mu k \ \zeta_7$, $b_{44} = b_{45} = W_5'$, $b_{46} = b_{47} = W_6'$

$b_{48} = -R'_7$, $b_{49} = R'_7$

$b_{51} = 2\mu k\zeta_5$, $b_{52} = 2\mu k\zeta_6$, $b_{53} = -\mu\left(\zeta_7^2 + k^2\right)$

$b_{54} = -R'_5$, $b_{55} = R'_5$, $b_{56} = -R'_6$

$b_{57} = R'_6$, $b_{58} = b_{59} = Q'_7$, $b_{61} = \zeta_5$

$b_{62} = \zeta_6$, $b_{63} = -k$, $b_{64} = -\zeta'_5$

$b_{65} = \zeta'_5$, $b_{66} = -\zeta'_6$, $b_{67} = \zeta'_6$

$b_{68} = b_{69} = k$, $b_{71} = b_{72} = -k$, $b_{73} = \zeta_7$

$b_{74} = b_{75} = b_{76} = b_{77} = k$, $b_{78} = -\zeta'_7$

$b_{79} = \zeta'_7$, $b_{81} = \beta^*\left(\mu_5 - 1\right)\zeta_5$, $b_{82} = \beta^*\left(\mu_6 - 1\right)\zeta_6$

$b_{83} = k\beta^*\left(1 - \alpha_0\right)$, $b_{84} = X'_5$, $b_{85} = -X'_5$

$b_{86} = -X'_6$, $b_{87} = X'_6$, $b_{88} = b_{89} = X$

APPENDIX III

$c_{14} = S'_8 \;\; e^{\zeta'_8 H}$, $c_{15} = S'_8 \;\; e^{-\zeta'_8 H}$, $c_{16} = -R'_9 \;\; e^{\zeta'_9 H}$

$c_{17} = R'_9 \;\; e^{-\zeta'_9 H}$, $c_{18} = -R'_{10} \;\; e^{\zeta'_{10} H}$, $c_{19} = R'_{10} \;\; e^{-\zeta'_{10} H}$

$c_{24} = -R'_8 \;\; e^{\zeta'_8 H}$, $c_{25} = R'_8 \;\; e^{-\zeta'_8 H}$, $c_{26} = T'_9 \;\; e^{\zeta'_9 H}$

$c_{27} = T'_9 \;\; e^{-\zeta'_9 H}$, $c_{28} = T'_{10} \;\; e^{\zeta'_{10} H}$, $c_{29} = T'_{10} \;\; e^{-\zeta'_{10} H}$

$c_{36} = -U'_9 \;\; e^{\zeta'_9 H}$, $c_{37} = U'_9 \;\; e^{-\zeta'_9 H}$, $c_{38} = -U'_{10} \;\; e^{\zeta'_{10} H}$, $c_{39} = U'_{10} \;\; e^{-\zeta'_{10} H}$

$c_{41} = S_8$, $c_{42} = k\zeta_9\left(2\mu + K\right)$, $c_{43} = k\zeta_{10}\left(2\mu + K\right)$

$c_{44} = c_{45} = S'_8$, $c_{46} = -R'_9$, $c_{47} = R'_9$

$c_{48} = -R'_{10}$, $c_{49} = R'_{10}$, $c_{51} = k\zeta_8\left(2\mu + K\right)$

$c_{52} = -T_9$, $c_{53} = -T_{10}$, $c_{54} = -R'_8$

$c_{55} = R'_8$, $c_{56} = c_{57} = T'_9$, $c_{58} = c_{59} = T'_{10}$

$c_{62} = \gamma\mu_9\zeta_9$, $c_{63} = \gamma\mu_{10}\zeta_{10}$, $c_{66} = -\gamma' U'_9$

$c_{67} = \gamma' U'_9$, $c_{68} = -\gamma' U'_{10}$, $c_{69} = \gamma' U'_{10}$

$c_{71} = \zeta_8$, $c_{72} = c_{73} = -k$, $c_{74} = -\zeta'_8$

$c_{75} = \zeta'_8$, $c_{76} = c_{77} = c_{78} = c_{79} = k$

$c_{81} = -k$, $c_{82} = \zeta_9$, $c_{83} = \zeta_{10}$

$c_{84} = c_{85} = k$, $c_{86} = -\zeta'_9$, $c_{87} = \zeta'_9$

$c_{88} = -\zeta'_{10}$, $c_{89} = \zeta'_{10}$, $c_{92} = -\mu_9$

$c_{93} = -\mu_{10}$, $c_{96} = c_{97} = \mu'_9$, $c_{98} = c_{99} = \mu'_{10}$

where

$$S'_8 = \lambda'\left(\zeta_8'^2 - k^2\right) + \left(2\mu' + K'\right)\zeta_8'^2, \qquad S_8 = \lambda\left(\zeta_8^2 - k^2\right) + \left(2\mu + K\right)\zeta_8^2$$

$$R'_j = k\zeta'_j\left(2\mu' + K'\right), \qquad T'_i = \left(\mu' + K'\right)\zeta_i'^2 + \mu' k^2 - K'\mu'_i$$

$$U'_i = \mu'_i\,\zeta'_i, \qquad T_i = \left(\mu + K\right)\zeta_i^2 + \mu k^2 - K\mu_i$$

$$\zeta_j = k\left(1 - c^2\lambda_j^2\right)^{1/2} \qquad \lambda_8^2 = \frac{1}{c_1^2}, \qquad c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$$

$$\lambda_9^2 = \frac{1}{2}\left[D^* - \left(D^{*2} - 4E^*\right)^{1/2}\right], \; \lambda_{10}^2 = \frac{1}{2}\left[D^* + \left(D^{*2} - 4E^*\right)^{1/2}\right]$$

$$D^* = E_2^* + r_1 + \frac{\left(pr_2 - r_0\right)}{\omega^2}, \qquad E^* = E_2^*\left(r_1 - \frac{r_0}{\omega^2}\right)$$

$$E_2^* = \frac{\rho}{\mu + K}, \qquad \mu_9 = \frac{\omega^2\lambda_9^2\left(\omega^2\lambda_9^2 - \omega^2 E_2^* - pr_2\right)}{p\left[-r_0 + r_1\omega^2\right]}$$

$$\mu_{10} = \frac{\omega^2\lambda_{10}^2\left(\omega^2\lambda_{10}^2 - \omega^2 E_2^* - pr_2\right)}{p\left[-r_0 + r_1\omega^2\right]} \qquad (i = 9,\ 10 \text{ and } j = 8,\ 9,\ 10)$$

APPENDIX IV

$$d_{11} = \mu' T'_{12}\, e^{\zeta'_{11}H}, \qquad d_{12} = \mu' T'_{12}\, e^{-\zeta'_{11}H}, \qquad d_{13} = -T'_{12}\, e^{\zeta'_{12}H}$$

$$d_{14} = T'_{12}\, e^{-\zeta'_{12}H}, \qquad d_{21} = -T'_{11}\, e^{\zeta'_{11}H}, \qquad d_{22} = T'_{11}\, e^{-\zeta'_{11}H}$$

$$d_{23} = T'_{12}\, e^{\zeta'_{12}H}, \qquad d_{24} = T'_{12}\, e^{-\zeta'_{12}H}, \qquad d_{31} = d_{32} = \mu' T'_{12}$$

$$d_{33} = -S'_{12}, \qquad d_{34} = S'_{12}, \qquad d_{35} = -\mu\left(2k^2 - k_{\alpha 12}^2\right)$$

$$d_{36} = 2k\mu\zeta_{12}, \qquad d_{41} = -S'_{11}, \qquad d_{42} = \delta'_{11}$$

$$d_{43} = d_{44} = \mu' T'_{12}, \qquad d_{45} = 2k\mu\zeta_{11}, \qquad d_{46} = d_{35}$$

$$d_{51} = d_{52} = k, \qquad d_{53} = -\zeta'_{12}, \qquad d_{54} = \zeta'_{12}$$

$$d_{55} = -k, \qquad d_{56} = \zeta_{12}, \qquad d_{61} = -\zeta'_{11}$$

$$d_{62} = \zeta'_{11}, \qquad d_{63} = d_{64} = k, \qquad d_{65} = \zeta_{11}$$

$$d_{66} = -k$$

where

$$T'_{12} = \left(2k^2 - k_{\alpha 12}'^2\right), \qquad S'_i = 2k\mu'\zeta'_i, \qquad k_{\alpha 12}^2 = \omega^2\lambda_{12}^2$$

$$\zeta_i = k\left(1 - c^2\lambda_i^2\right)^{1/2}, \qquad \lambda_{11}^2 = \frac{\rho}{\lambda + 2\mu}, \qquad \lambda_{12}^2 = \frac{\rho}{\mu}, \qquad i = 11,\ 12$$

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