MODE-BASED PROCEDURE TO INTERPOLATE STRONG MOTION RECORDS OF INSTRUMENTED BUILDINGS

Rakesh K. Goel
Department of Civil & Environmental Engineering
California Polytechnic State University
San Luis Obispo, CA 93407-0353, U.S.A.

ABSTRACT

This paper examines the accuracy of a commonly used piece-wise cubic polynomial interpolation (PWCPI) procedure to estimate the motions of non-instrumented floors in buildings with significant stiffness discontinuities. It is shown that results from the PWCPI procedure may depend on the locations of instrumented floors. While the PWCPI procedure may provide good estimates of floor displacements, this procedure may not accurately predict story drifts and floor accelerations. Therefore, a mode-based interpolation (MBI) procedure is presented as an enhancement to the PWCPI procedure. The MBI procedure is shown to provide accurate estimates of the floor displacements, story drifts, and floor accelerations. Furthermore, results from the MBI procedure are shown to be much less dependent on the locations of instrumented floors. The mode shapes needed in the MBI procedure may be computed from the eigenanalysis of the building, estimated from the system identification of recorded motions, or computed from the approximate formulas for mode shapes.

KEYWORDS: Acceleration, Buildings, Earthquake Response Interpolation, Modal Analysis, Structural Dynamics

INTRODUCTION

Recorded motions of buildings during strong shaking provide a unique opportunity to evaluate the current analytical procedures—nonlinear static analysis and nonlinear response history analysis—and to develop improved procedures. Needed for this purpose are the height-wise variations of displacement demands—floor displacements and story drifts—as well as the variations of acceleration demands during the ground shaking. Since buildings are typically instrumented at a limited number of floors, motions of the remaining (or non-instrumented) floors are estimated by an interpolation procedure. Typically, a piece-wise cubic polynomial interpolation (PWCPI) procedure is used for the conventional buildings (Naeim, 1997; De la Llera and Chopra, 1997; Goel, 2005, 2007; Limongelli, 2003, 2005), and a combination of cubic-linear interpolation is recommended for the base-isolated buildings (Naeim et al., 2004). It is generally believed that the PWCPI procedure provides reasonable estimates of motions at non-instrumented floors (Naeim, 1997; Naeim et al., 2004; De la Llera and Chopra, 1997). However, previous investigations on the accuracy of the PWCPI procedure have been limited to the estimation of floor displacements and floor accelerations in buildings without significant stiffness discontinuities. Many real buildings contain significant stiffness discontinuities over the building height, such as stiff shear walls in the basement and flexible moment-resisting frames in the upper stories, or soft-story condition. The accuracy of the PWCPI procedure has not been verified for such buildings.

The objective of this investigation is to re-examine the accuracy of the PWCPI procedure in providing accurate estimates of the floor displacements, story drifts, and floor accelerations for buildings with significant stiffness discontinuities. Since the PWCPI procedure is found not to be always accurate, a mode-based interpolation (MBI) procedure is presented as an enhancement to the PWCPI procedure. It is demonstrated that the MBI procedure provides very good estimates of the motions at non-instrumented floors and is much less sensitive, compared to the PWCPI procedure, to the locations of instrumented floors.

THEORETICAL BACKGROUND

Let an N-story building be instrumented at J locations with sensors at the base and roof levels. Also, let r_j be the response—displacement or acceleration—recorded by the jth sensor located at height h_j from the building base, and let r be the desired response at height h (or at a non-instrumented location) of the building. The response r is to be computed by interpolation of the recorded responses r_j. Following is the theoretical background of the commonly used PWCPI procedure and the proposed MBI procedure.

1. Piece-Wise Cubic Polynomial Interpolation (PWCPI) Procedure

For a building with sensors at J locations and sub-divided into $J-1$ sub-intervals, the response r at height h, within the jth sub-interval, is given by

$$r(h) = a_j\left(h-h_j\right)^3 + b_j\left(h-h_j\right)^2 + c_j\left(h-h_j\right) + d_j \tag{1}$$

in which a_j, b_j, c_j, and d_j are the constants for the cubic-polynomial to be fitted in the jth sub-interval. Since Equation (1) for each sub-interval involves four constants, $4\left(J-1\right)$ constants are needed to completely define the response of this building, which in turn implies that $4\left(J-1\right)$ equations are required to uniquely solve for these constants. For this purpose, $2\left(J-1\right)$ equations are obtained by matching the response from Equation (1) to the recorded response at the $2\left(J-1\right)$ locations. The remaining $2\left(J-1\right)$ equations may be obtained by forcing continuity conditions at the junctions of two adjacent sub-intervals and by utilizing the boundary conditions (or the known values of derivatives of the response) at the base and top of the building. Since the derivatives of the response at the bottom and top of the building are usually not available, one of the most commonly used boundary condition is the "not-a-knot" condition. With this boundary condition, the remaining $2\left(J-1\right)$ equations are obtained as follows: (a) $2\left(J-2\right)$ equations by forcing the first and second derivatives of the response to be equal at the $\left(J-2\right)$ junctions of the $\left(J-1\right)$ sub-intervals; (b) one equation by forcing the third derivative of the response to be equal at the top of the first sub-interval and bottom of the second-sub-interval; and (c) one equation by forcing the third derivative of the response to be equal at the top of the last-but-one sub-interval and bottom of the last sub-interval. The computation of the needed constants then requires solution of a tri-diagonal system of linear equations (see Appendix I).

A summary of the PWCPI procedure, adopted from Beatson (1986), is presented in Appendix I for estimating the motions at non-instrumented floors. This procedure may be used to interpolate the floor displacements (e.g., Naeim, 1997; De la Llera and Chopra, 1997) or floor accelerations (Limongelli, 2003). A convenient implementation of the PWCPI procedure is possible in MATLAB (MathWorks, 2006) with the use of "spline" function.

2. Mode-Based Interpolation (MBI) Procedure

For a linearly elastic building, idealized as an N degree-of-freedom system, the floor displacements and accelerations can be calculated by the superposition of modal displacements and accelerations (Chopra, 2007) as

$$\left\{u(t)\right\} = \sum_{i=1}^{N}\Gamma_i\left\{\phi_i\right\}D_i(t) \quad \text{and} \quad \left\{\ddot{u}(t)\right\} = \sum_{i=1}^{N}\Gamma_i\left\{\phi_i\right\}\ddot{D}_i(t) \tag{2}$$

in which $\left\{u(t)\right\}$ and $\left\{\ddot{u}(t)\right\}$ are the vectors containing the time-variations of relative floor displacements and accelerations, respectively; $\left\{\phi_i\right\}$ is the ith mode-shape vector; $\Gamma_i = \left\{\phi_i\right\}^T\left[m\right]\left\{\iota\right\}/\left\{\phi_i\right\}^T\left[m\right]\left\{\phi_i\right\}$ is the ith modal participation factor with $\left[m\right]$ being the mass matrix and $\left\{\iota\right\}$ being the influence vector describing the influence of support displacements on structural displacements; and $D_i(t)$ and $\ddot{D}_i(t)$ are the time-variations of displacement and acceleration of the *i*th-mode single-degree-of-freedom system, as computed from

$$\ddot{D}_i(t) + 2\zeta_i \omega_i \dot{D}_i(t) + \omega_i^2 D(t) = -\ddot{u}_g(t) \tag{3}$$

with ω_i and ζ_i being the *i*th-mode frequency and damping ratio, and $\ddot{u}_g(t)$ being the time-variation of ground acceleration. Denoting $q_i(t) = \Gamma_i D_i(t)$ as the modal displacement and $\ddot{q}_i(t) = \Gamma_i \ddot{D}_i(t)$ as the modal acceleration, Equation (2) leads to

$$\{u(t)\} = \sum_{i=1}^{N} q_i(t)\{\phi_i\} \quad \text{and} \quad \{\ddot{u}(t)\} = \sum_{i=1}^{N} \ddot{q}_i(t)\{\phi_i\} \tag{4}$$

Let us now consider a building in which recorded motions are available at *K* floors, in addition to the base. Let the locations of floors with recorded motions be designated as $k_1, k_2, \cdots, k_K$. Considering the first *K* modes in the modal superposition, the motions at the instrumented floors may be described as

$$\{\overline{u}(t)\} = [\Psi]\{\overline{q}(t)\} \quad \text{and} \quad \{\ddot{\overline{u}}(t)\} = [\Psi]\{\ddot{\overline{q}}(t)\} \tag{5}$$

in which $\{\overline{u}(t)\} = \{u_{k_1}\ u_{k_2} \ldots u_{k_K}\}^T$ and $\{\ddot{\overline{u}}(t)\} = \{\ddot{u}_{k_1}\ \ddot{u}_{k_2} \ldots \ddot{u}_{k_K}\}^T$ are the vectors of relative displacements and relative accelerations, respectively, at the *K* floors; $\{\overline{q}(t)\} = \{q_1(t)\ q_2(t) \ldots q_K(t)\}^T$ is the vector of modal displacements for the modes 1 to *K*; $\{\ddot{\overline{q}}(t)\} = \{\ddot{q}_1(t)\ \ddot{q}_2(t) \ldots \ddot{q}_K(t)\}^T$ is the vector of modal accelerations for the modes 1 to *K*; and $[\Psi]$ is the matrix containing mode-shape components at the instrumented floors. It may be noted that the matrix $[\Psi]$ differs from the complete mode-shape matrix $[\Phi]$, as the former contains the components of mode shape only at the instrumented floors. For the selected system, the matrix $[\Psi]$ of dimensions $K \times K$ is given by

$$[\Psi] = \begin{bmatrix} \phi_{k_1,1} & \phi_{k_1,2} & \cdots & \phi_{k_1,K} \\ \phi_{k_2,1} & \phi_{k_2,2} & \cdots & \phi_{k_2,K} \\ \vdots & \vdots & & \vdots \\ \phi_{k_K,1} & \phi_{k_K,2} & \cdots & \phi_{k_K,K} \end{bmatrix} \tag{6}$$

If $\{\overline{u}(t)\}$ or $\{\ddot{\overline{u}}(t)\}$, and the mode-shape component matrix $[\Psi]$ are available, the modal displacements $\{\overline{q}(t)\}$ and modal accelerations $\{\ddot{\overline{q}}(t)\}$ can be computed by solving Equation (5) at each time-instant. The relative displacements and accelerations at any non-instrumented floor *j* can then be computed from

$$u_j(t) = \sum_{i=1}^{K} q_i(t)\phi_{ji} \quad \text{and} \quad \ddot{u}_j(t) = \sum_{i=1}^{K} \ddot{q}_i(t)\phi_{ji} \tag{7}$$

The total displacements and accelerations at any non-instrumented floor can thus be computed from

$$u_j^t(t) = u_j(t) + u_b(t) \quad \text{and} \quad \ddot{u}_j^t(t) = \ddot{u}_j(t) + \ddot{u}_b(t) \tag{8}$$

in which $u_b(t)$ and $\ddot{u}_b(t)$ are the displacements and accelerations, respectively, recorded at the base of the building. Equations (5) to (8) represent the MBI procedure.

The MBI procedure requires that mode-shape component matrix $[\Psi]$ be well conditioned. Since the mode shape matrix of all modes is rank-deficient by one, the MBI procedure cannot include all modes, even if it is possible, in Equation (5). In fact, the largest number of modes that can be included in the MBI procedure is equal to the total number of instrumented floors excluding the base. Furthermore, the modes which have a "node" (or zero mode-shape component) at or near one or more of the floors with recorded motions must be avoided in the MBI procedure, because including those modes would lead to poorly conditioned $[\Psi]$ matrix. For the buildings considered in this investigation, three to four modes were found to be sufficient for the MBI of recorded motions.

The MBI procedure described so far is for estimating the translational displacement and accelerations at non-instrumented floors from the recorded translational displacements and accelerations at limited number of instrumented floors. However, this procedure could easily be extended to estimate the rotational (or torsional) displacements and accelerations as well if recorded torsional motions were available. It is useful to note that torsional motions are almost never directly recorded; they are typically computed from the differential translational motions at two ends of the building. Implementation of the MBI procedure including torsional motions would require that the matrix $[\Psi]$ (see Equation 6) also contains the mode-shape components corresponding to the torsional degrees of freedom, and relative torsional displacements and accelerations be computed from Equation (7), with ϕ_{ji} representing the torsional component of mode shape at the *i*th floor. However, care must be exercised in attempting to estimate torsional motions of a building with little or no coupling between the translational and torsional modes because of the possibility of poorly conditioned $[\Psi]$.

The MBI procedure requires mode shapes of the building. These mode shapes may either be computed from the eigenanalysis of a computer model of the building, estimated from the system identification methods (e.g., Mau and Aruna, 1994), or be computed from the approximate formulas for mode shapes (Miranda and Taghavi, 2005; Miranda and Akkar, 2006; Miranda and Reyes, 2002; Taghavi and Miranda, 2005). Among these three procedures to compute the mode shapes, the eigenanalysis is obviously the most complicated and time-consuming procedure. However, computer models of many instrumented buildings have already been developed for the purposes other than the eigenanalysis alone (e.g., to evaluate the current analytical procedures). These models have been carefully calibrated against the recorded data. Mode shapes from the eigenanalysis are therefore readily available for such buildings and have been utilized in the MBI procedure implemented in this investigation. If such computer models were not available, the other two procedures might be used to estimate the mode shapes needed in the MBI procedure.

The above development of the MBI procedure assumes complete uncoupling between the modal responses. This assumption is strictly valid for the buildings remaining in the linearly elastic range. However, the MBI procedure may be extended to estimate the response of buildings deformed slightly beyond the linearly elastic range because of weak modal coupling (Chopra, 2007). Clearly, the MBI procedure should not be applied to interpolate motions of buildings deformed far beyond the linearly elastic range, where significant modal coupling may occur. But application of an interpolation procedure is not necessary for the performance evaluation of such buildings, because performance may be evaluated visually in the case of large damage.

3. Error Function

The effectiveness of the PWCPI or MBI procedure to provide accurate estimates of the motions at non-instrumented floors is quantitatively measured by an error function defined by Limongelli (2003) as

$$\varepsilon_j = \sqrt{\frac{\sum_{i=1}^{\mathrm{NPT}} \left(r_{r,j}(t_i) - r_{c,j}(t_i) \right)^2}{\sum_{i=1}^{\mathrm{NPT}} \left(r_{r,j}(t_i) \right)^2}} \tag{9}$$

In this ε_j is the error function at the *j*th floor; $r_{r,j}(t_i)$ and $r_{c,j}(t_i)$ are the recorded (or exact) and interpolated values, respectively, of a response—floor displacement, story drift, or floor acceleration—at the *j*th location at the *i*th time-instant; and NPT is the number of time-instants for which the response is available. A lower value of the error function for a selected interpolation procedure is indicative of better estimate of the interpolated motion compared to the other interpolation procedure(s).

The error function defined by Equation (9) is the sum of squares of differences between the "exact" and interpolated responses at each time-instant. Therefore, the value of the error function is expected to be larger for rapidly varying functions, such as acceleration, with many more peaks and valleys (and thus with the possibility of a much larger numerator in Equation (9)) compared to the less rapidly varying functions, such as displacement, with fewer peaks and valleys.

SELECTED BUILDINGS

Two reinforced-concrete (RC) buildings have been selected for this investigation. The first building is a 13-story commercial building in Sherman Oaks, California. This building has two basements. Designed in 1964, its vertical load carrying system consists of 2.4-in (61-mm) thick one-way slabs supported by concrete beams, girders, and columns. The lateral load system consists of moment-resisting concrete frames in the upper stories and concrete shear walls in the basement. The foundation system consists of concrete piles. This building is instrumented by the California Strong Motions Instrumentation Program (CSMIP) to measure horizontal accelerations at the 2nd sub-basement level, ground level, 2nd floor, 8th floor, and at the roof level (see Figure 1).

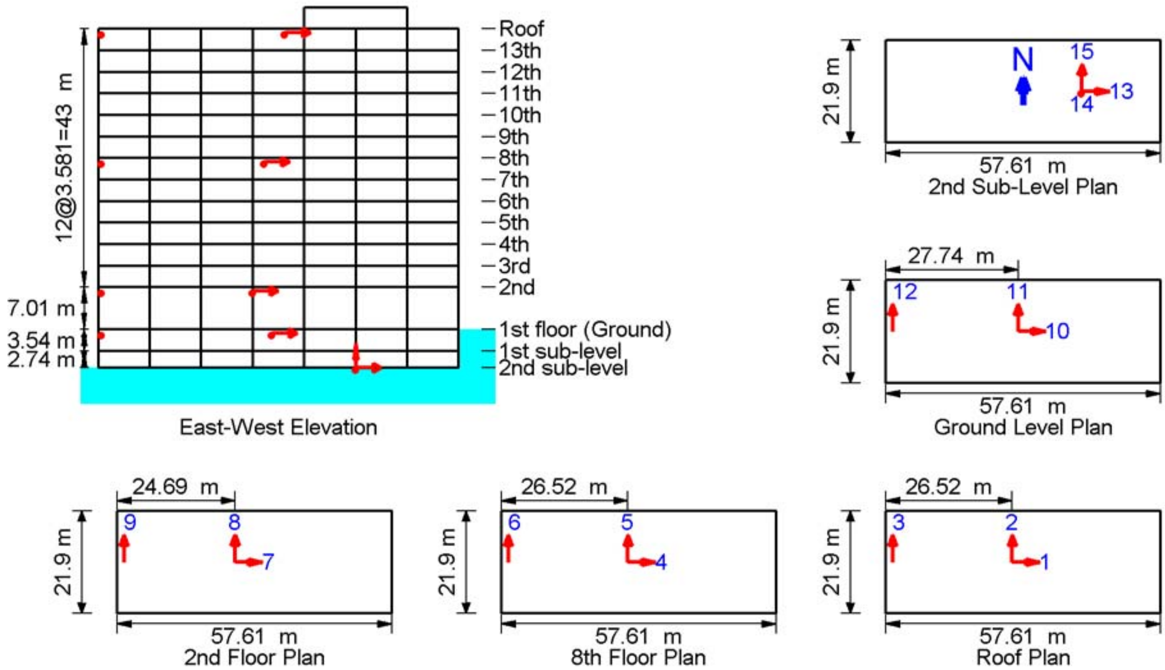

Fig. 1 The Sherman Oaks 13-Story Commercial Building

The second building selected is a 20-story hotel in North Hollywood (see Figure 2). This building has just one basement. Designed in 1966, its vertical load carrying system consists of 4.5–6 in (115–152 mm) thick RC slabs supported by concrete beams and columns. The lateral load system consists of moment-resisting concrete frames in the upper stories and concrete shear walls in the basement. The foundation system consists of spread footing below the columns. This building was instrumented by the CSMIP in 1983 with 16 sensors on five levels of the building. The sensors in the building measure horizontal accelerations at the basement level, 3rd floor, 9th floor, 16th floor, and at the roof level.

These two buildings have been selected in this investigation because of significant stiffness discontinuities and because three-dimensional computer models of these buildings are available from a recently completed study (Goel and Chadwell, 2007). Both of these buildings have very stiff shear walls in the basement and flexible moment-resisting frames in the upper stories. Furthermore, the Sherman Oaks building has soft-story condition between the ground and 2nd floor because the height of this story is much higher compared to those of the other stories (see Figure 1).

Motions of the two selected buildings are available from the Center for Engineering Strong Motion Data[1]. The motions used in this investigation are the translational motions recorded during the 1994 Northridge earthquake in the longitudinal (east-west) direction of the Sherman Oaks 13-Story

[1] Website of the Center for Engineering Strong Motion Data, http:// www.strongmotioncenter.org

Commercial Building, and in the transverse (north-south) direction of the North Hollywood 20-Story Hotel. Of the two selected buildings, the Sherman Oaks 13-Story Commercial Building was deformed slightly beyond the linearly-elastic range and the North Hollywood 20-Story Hotel remained within the linearly-elastic range during the 1994 Northridge earthquake (Goel and Chadwell, 2007).

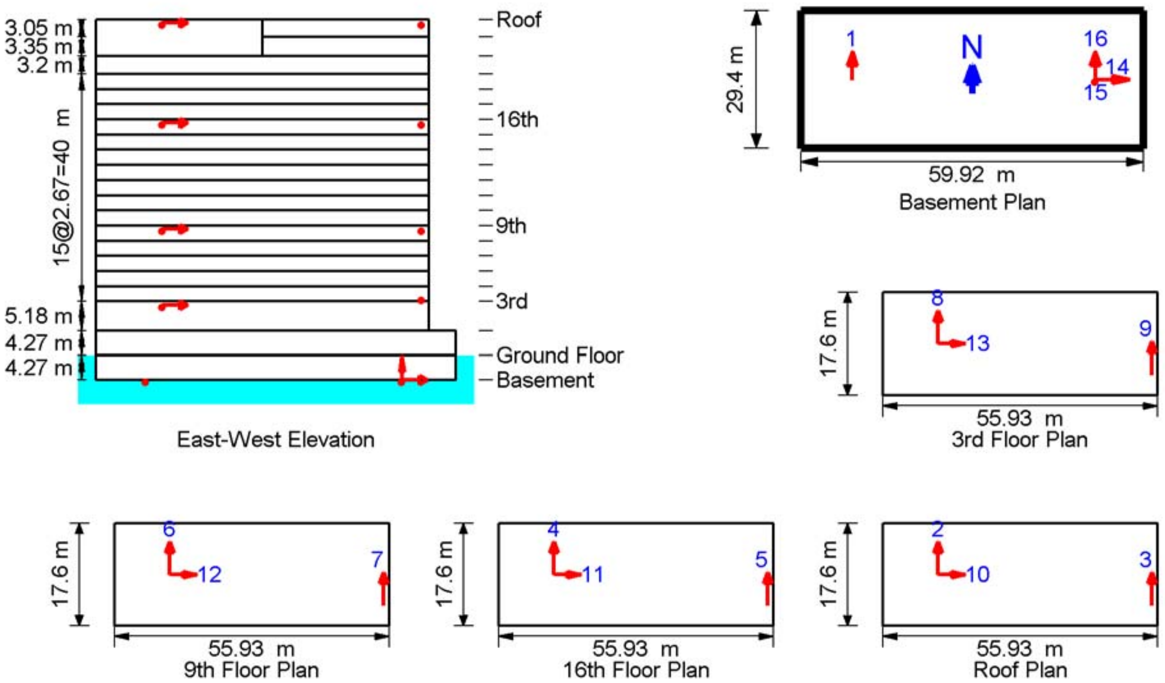

Fig. 2 The North Hollywood 20-Story Hotel

The relative floor motions obtained by subtracting the base motion from the floor motions recorded during the 1994 Northridge earthquake are shown in Figures 3 and 4 for the Sherman Oaks and North Hollywood buildings, respectively; motions recorded at the 2nd sub-level of the Sherman Oaks building (see Figure 1) and at the basement of the North Hollywood Hotel (see Figure 2) are considered as the base motions.

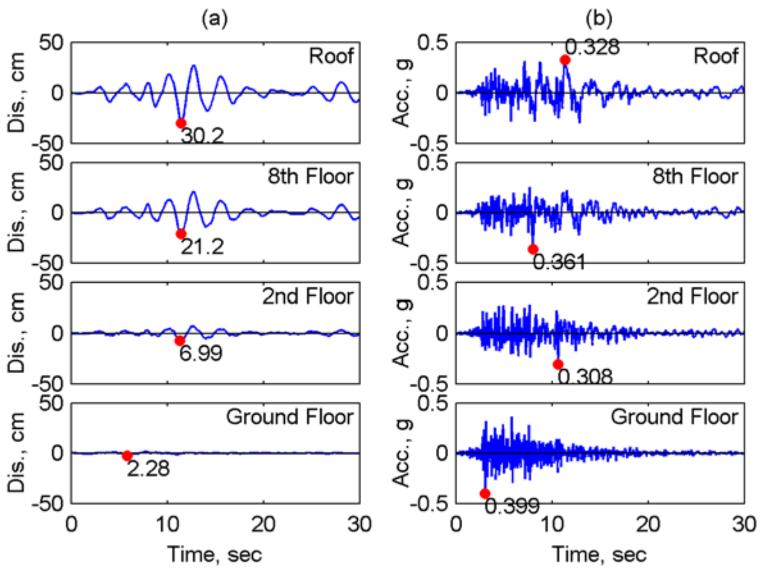

Fig. 3 Motions of the Sherman Oaks 13-Story Commercial Building in the longitudinal (east-west) direction during the 1994 Northridge earthquake: (a) Floor displacements relative to the base; and (b) Floor accelerations relative to the base

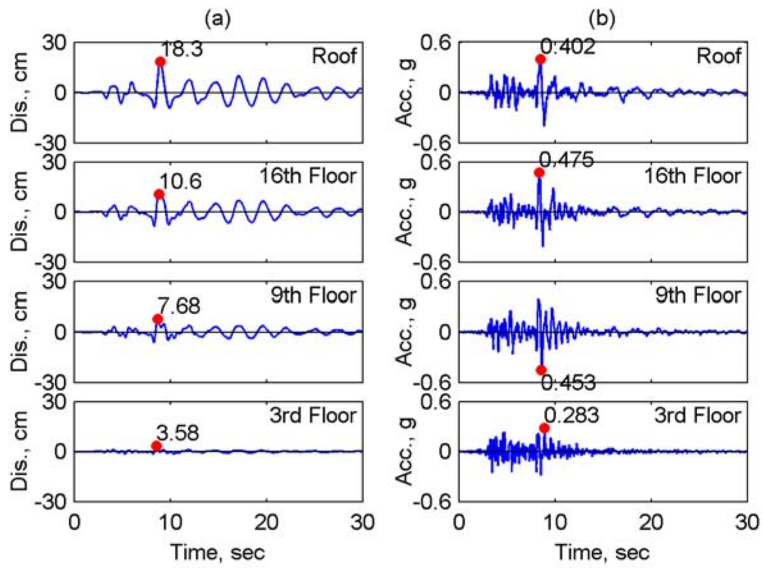

Fig. 4 Motions of the North Hollywood 20-Story Hotel in the transverse (north-south) direction during the 1994 Northridge earthquake: (a) Floor displacements relative to the base; and (b) Floor accelerations relative to the base

The motions used in this investigation are the corrected (or processed) strong motions recorded during the 1994 Northridge earthquake. The data processing procedure (see Shakal et al., 2003) attempts to eliminate the errors associated with noise in the actual recorded (or unprocessed) data. However, some errors may still remain in the processed data and may thus affect the accuracy of various interpolation procedures.

RESPONSE QUANTITIES

Response quantities investigated here are the floor displacement and floor acceleration at the center of the building relative to its base. Also considered is the story drift—defined as the relative displacement between two adjacent floors. It is useful to emphasize that while floor displacements and floor accelerations are recorded, albeit at a limited number of floors, story drift is not directly recorded but is "derived" from the recorded floor displacements. Torsional motions were not considered because the selected buildings are essentially symmetric; recorded motions indicated that these buildings experienced some torsional motions but those were very small.

MODE SHAPES

Needed in the MBI procedure are the mode shapes of the selected buildings. These mode shapes are computed by the eigen-value analyses of the linearly elastic models of the selected buildings. For this purpose, three-dimensional linear-elastic models of the two buildings were developed in Open System for Earthquake Engineering Simulation (OpenSees) (McKenna and Fenves, 2001). The effective flexural and shear stiffnesses of beams and columns were specified initially according to the FEMA-356 recommendations (FEMA, 2000). The first three mode shapes of the two selected buildings from the eigen-value analyses are shown in Figure 5. It may be noted that only the longitudinal component (in the east-west direction) of the mode shapes for the Sherman Oaks building and transverse component (in the north-south direction) of the mode shapes for the North Hollywood building are included in Figure 5. These components are extracted from the three-dimensional mode shapes of these buildings as in Goel and Chadwell (2007).

ACCURACY OF THE PWCPI PROCEDURE

The PWCPI procedure may be sensitive to the relative size of the interpolation segment (Beatson, 1986). Furthermore, the interpolated motions obtained from the PWCPI procedure may be sensitive to the significant stiffness discontinuities in buildings. This section examines sensitivity of the interpolated motions from the PWCPI procedure to such conditions. It is useful to emphasize that this investigation is not concerned about a complete sensitivity analysis of the PWCPI interpolation with the number and combinations of instrumented floors but with demonstrating the possible limitations of the PWCPI procedure and presenting an improved procedure. A detailed sensitivity analysis of the PWCPI procedure and optimal location of sensors for interpolation is available elsewhere (Limongelli, 2003).

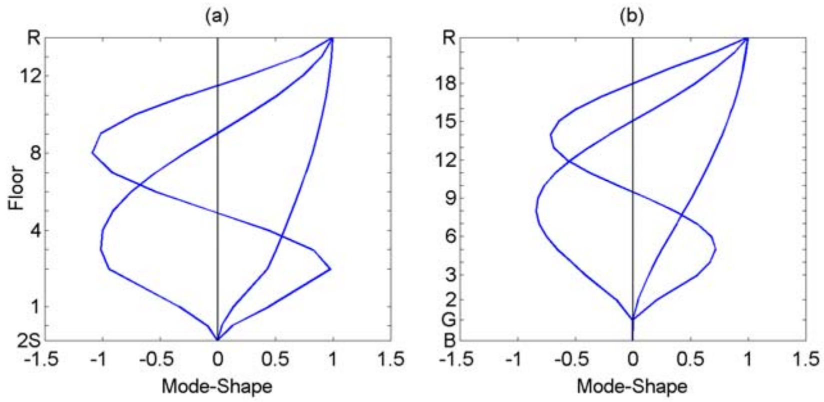

Fig. 5 Mode shapes of the selected buildings: (a) Sherman Oaks 13-Story Commercial Building; and (b) North Hollywood 20-Story Hotel

1. Interpolation of the Recorded Motions

Motions of the two selected buildings are available at five floors: motions of the Sherman Oaks building in the longitudinal direction were recorded at the 2nd sub-level (2S), ground floor (G), 2nd floor, 8th floor, and at the roof by the sensors, 13, 10, 7, 4, and 1, respectively (see Figure 1); and motions of the North Hollywood 20-Story Hotel in the transverse direction at the east edge were recorded at the basement (B), 3rd floor, 9th floor, 16th floor, and at the roof by the sensors, 16, 9, 7, 5, and 3, respectively (see Figure 2). The motions at the non-instrumented floors were first interpolated by the PWCPI procedure considering motions at all the five floors. The motions were then interpolated by dropping one of the intermediate instrumented floors; motions at the base and the roof of the building were always included in the interpolation. For the Sherman Oaks building, either the G or the 2nd floor was dropped from the set of instrumented floors; dropping the 8th floor led to unacceptable motions at the non-instrumented floors and is, therefore, not considered here. For the North Hollywood building, either the 3rd floor or the 9th floor was dropped from the set of instrumented floors. The peak floor displacements, story drifts, and floor accelerations obtained by these three interpolations are presented in Figures 6 and 7. Also included are the values of the error functions for the floors, which were dropped in the interpolation procedure, i.e., ε_G and ε_2 for the Sherman Oaks building, and ε_3 and ε_9 for the North Hollywood building. It may be noted that error function is computed only for those floor displacements and floor accelerations, for which the recorded data is available at the floor dropped in the interpolation procedure. Error function is not computed for the story drifts because recorded story drift data is not directly available.

The results presented in Figure 6 for the Sherman Oaks building indicate that peak values of interpolated motions depend noticeably on the instrumented floors selected in the interpolation procedure. The differences appear to be much larger for the story drifts (see Figure 6(b)) and floor accelerations (see Figure 6(c)) compared to the floor displacements (see Figure 6(a)).

When compared with the interpolated motions obtained from considering all the five floors, ignoring recorded motion of G or 2nd floor leads to an under-prediction of displacements at the floors 2 to 7 and to an over-prediction of displacements at the floors 9 to 13 (see Figure 6(a)). However, the discrepancy for the first case is smaller compared to that for the second, as indicated by a slightly lower value of ε_G compared to that of ε_2 (see Figure 6(a)). It may be recalled that ignoring the 2nd floor implies ignoring motion at a location of the soft-story condition. The floor displacements are identical for the three cases at 2S, 8th floor, and roof, because these three floors are included in each of the three sets of floors considered during the interpolation procedure.

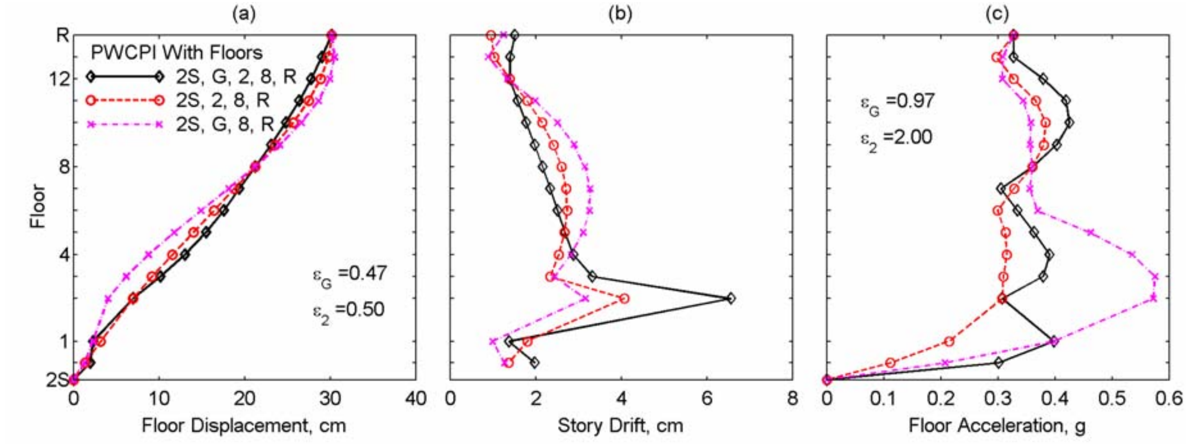

Fig. 6 Height-wise distribution of the peak response from the PWCPI procedure using recorded motions of the Sherman Oaks 13-Story Commercial Building for three different sets of sensors

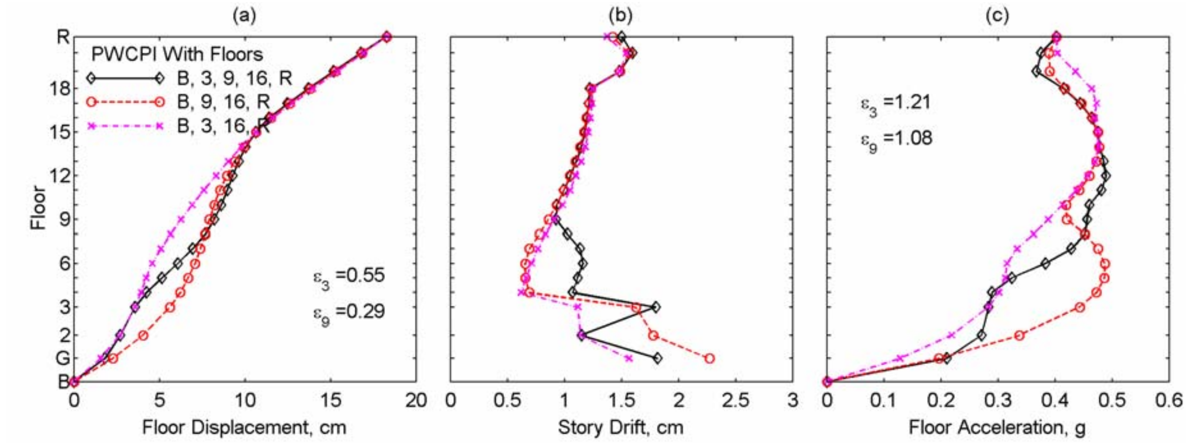

Fig. 7 Height-wise distribution of the peak response from the PWCPI procedure using recorded motions of the North Hollywood 20-Story Hotel for three different sets of sensors

The peak story drifts from the three sets of interpolation also differ significantly (see Figure 6(b)). When compared to the values from considering all the floors, ignoring the G or 2nd floor leads to a significant under-prediction for the stories 1–3 and an over-prediction for the stories 5–12. In particular, the differences are extremely large for the 2nd story drift—location of the soft-story condition. Unlike the floor displacements, which match at the common instrumented floors for the three cases (see Figure 6(a)), the story drifts appear to have the largest discrepancy at or near these floors (see Figure 6(b)). This is the case because story drifts, which are indicative of the slope of the displacement profile, differ the most at the common instrumented floors even though displacements match at these floors.

Similar to the story drift, peak floor accelerations from the three sets of interpolation also differ significantly (see Figure 6(c)). In particular, dropping either the G or 2nd floor leads to a significant under- or over-prediction of the floor accelerations, when compared with the interpolation using all the five floors. These differences tend to be larger in the lower floors compared to the upper floors.

The error functions for interpolated accelerations are much larger than those for the interpolated displacements: ε_G = 0.47 for the displacements, and ε_G = 0.97 for the accelerations; and ε_2 = 0.5 for the displacements, and ε_2 = 2.0 for the accelerations (see Figures 6(a) and 6(c)). As mentioned previously, a larger value of the error for accelerations is in part due to the rapidly varying nature of the acceleration function.

The results for the North Hollywood building (see Figure 7) indicate that the interpolation using three sets of instrumented floors provides almost identical floor displacements and story drifts in the upper part of the building—above floor 15 for the displacements (see Figure 7(a)), and above story 9 for the drifts (see Figure 7(b)). In the lower part of the building, however, both interpolated displacements and drifts depend on the selection of instrumented floors. Compared to the case in which all the five floors are considered, ignoring the 3rd floor tends to over-predict displacements in the floors 2–6 (see Figure 7(a)), and over-predict drift in the story 1 and under-predict drifts in the stories 3–8 (see Figure 7(b)). Ignoring the 9th floor under-predicts the displacements of floors 1–13 (see Figure 7(a)) and the drifts of stories 3–8 (see Figure 7(b)). The floor accelerations, however, differ throughout the building height (see Figure 7(c)). For this building, ignoring the recorded motions at the 9th floor leads to a lower error function for both displacements and accelerations compared to the values when recorded motions at the 3rd floor are ignored. As noted previously, error function for the interpolated accelerations is much larger than that for the interpolated displacements (see Figures 7(a) and 7(c)).

2. Interpolation of Simulated Motions

The results presented in the preceding section indicate that interpolated motions may depend noticeably on the locations of instrumented floors. However, these results do not indicate which set of instrumented floors would provide the most accurate prediction of motions, because the recorded motions for the selected buildings are available only at a limited number of floors. For this purpose, data is needed from the buildings with motions available at all floors. Unfortunately, the mid- or high-rise buildings, such as those considered in this investigation, are almost never instrumented at each floor. In the absence of the data from buildings instrumented at each floor, data simulated from the response-history analysis (RHA) of carefully calibrated computer models of buildings provides the best-available option. It is useful to note that this approach has been used previously by Taghavi and Miranda (2005) to evaluate an interpolation scheme developed to estimate the floor acceleration demands in multistory buildings.

In this investigation, the motions at each floor were simulated by the RHA of building models due to the motions that were recorded at the base of the building. For this purpose, three-dimensional computer models of the two selected buildings were developed using the structural analysis software "OpenSees" (McKenna and Fenves, 2001). The beams, columns, and shear walls were modeled with the "nonlinearBeamColumn" element with fiber section in "OpenSees". The damping was defined as the classical damping with damping ratios in the first and second modes to be 5%. The base input motions were taken as those recorded at the base of these buildings during the 1994 Northridge earthquake, and soil-structure interaction effects were ignored. Further details of the modeling of the selected buildings are available elsewhere (Goel and Chadwell, 2007).

The RHA motions at a limited number of floors were then used to interpolate motions at the remaining floors. The sets of floors used to interpolate were selected to be the same as those in the preceding section. The accuracy of the interpolation procedure is evaluated by comparing the motions simulated from RHA with those from the PWCPI procedure.

While accuracy of an interpolation procedure may be evaluated by considering numerous combinations of instrumented floors, only a few practical combinations are considered in this investigation. These combinations include the locations of significant stiffness discontinuities and nearly evenly distributed locations over the remaining height of the building. Combinations with one of the intervals being much larger than the others, such as those obtained by dropping the 8th floor in the Sherman Oaks building, lead to unacceptable results and are not considered in this investigation.

The height-wise variations of the peak values of floor displacements, story drifts, and floor accelerations from RHA are compared with those from the PWCPI procedure in Figure 8 for the Sherman Oaks building and in Figure 9 for the North Hollywood building. The PWCPI procedure uses the same three sets of floors for interpolation as those considered in the preceding section. The error functions are computed for the floor displacements, story drifts, and floor accelerations at each of the floors not used in the interpolation procedure. The means of these errors are also reported in Figures 8 and 9.

The presented results for the Sherman Oaks building indicate that one of the three interpolations—the case in which the motions at 2S, G, 2nd floor, 8th floor, and roof are used in the PWCPI procedure—provides reasonable predictions of the floor displacements (see Figure 8(a)) and floor accelerations (see Figure 8(c)) when compared with the RHA results. Ignoring one of the floors leads to further deterioration of the accuracy of the predictions from the PWCPI procedure when compared to the results from the RHA. The mean error function is the lowest for floor displacements and accelerations for the interpolation based on the motions at 2S, G, 2nd floor, 8th floor, and roof, but becomes larger when one of these floors is ignored in the interpolation procedure. The story drifts from the PWCPI procedure differ significantly from the RHA values (see Figure 8(b)): the PWCPI procedure considering the motions of aforementioned five floors leads to an under-prediction in the mid-portion of the building (i.e., for the stories 6 to 10) and to an over-prediction in the lower- and upper-portions (i.e., for the stories 3 and 13) of the building. The mean error function for story drifts is also much larger compared to the values for floor displacements and accelerations. Ignoring one of the five floors in the interpolation procedure tends to provide slightly better correlation of story drifts with the values from RHA but the mean error still remains quite high.

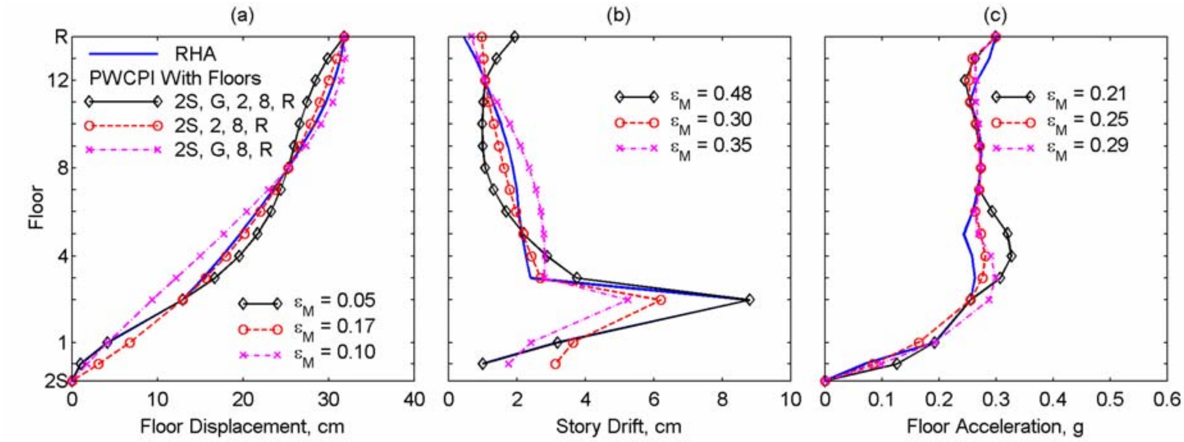

Fig. 8 Height-wise distribution of the peak response from the PWCPI procedure using simulated motions of the Sherman Oaks 13-Story Commercial Building for three different sets of monitored floors

The results for the North Hollywood building (see Figure 9) indicate that the case in which the motions at basement, 3rd floor, 9th Floor, 16th floor, and roof are used, the PWCPI procedure generally provides very good predictions of the floor displacements throughout the building height (see Figure 9(a)). The predictions for the story drifts and floor accelerations, however, may be poor in some portions of the building (see Figures 9(b) and 9(c)). Furthermore, the accuracy of the predictions deteriorates significantly if one of the floors is ignored in the interpolation procedure as indicated by the significant increase in the mean error.

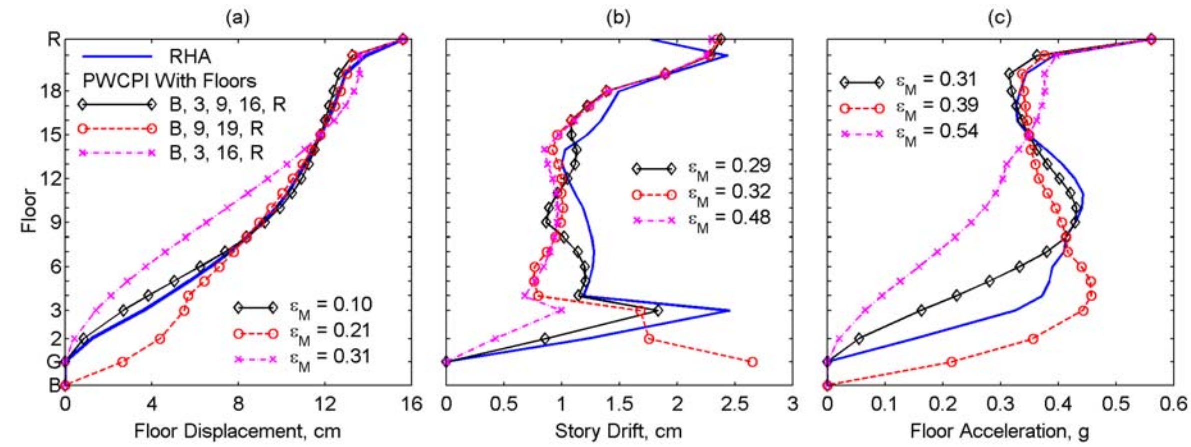

Fig. 9 Height-wise distribution of the peak response from the PWCPI procedure using simulated motions of the North Hollywood 20-Story Hotel for three different sets of monitored floors

The results presented in this section indicate that the PWCPI procedure may provide reasonable predictions of the floor displacements at non-instrumented floors if (1) the building is instrumented at a regular interval over its height, and (2) additional instruments are located in the building where stiffness changes significantly either due to the very-stiff condition resulting from the basement shear walls or due to the soft-story condition resulting from a taller story compared to the other stories. This becomes apparent from the displacements in Figures 8(a) and 9(a) where ignoring one of the five floors—a case which results in non-uniform locations of the instrumented floors—or ignoring the 1st floor in the Sherman Oaks building and ignoring the 3rd floor in the North Hollywood building—a case which leads to ignoring the floor near the stiff-story condition due to the basement shear wall—or ignoring the 2nd floor in the Sherman Oaks building—a case which leads to ignoring the floor near the soft-story condition—results in significant discrepancies between the results from the PWCPI procedure and the RHA. The story drifts and floor accelerations, however, may not still be accurately predicted by the PWCPI procedure. It may be noted that the mean errors for story drifts and floor accelerations tend to be much larger compared to those for the floor displacements.

RELATIVE ACCURACY OF PWCPI AND MBI PROCEDURES

The interpolated seismic demands from the two interpolation procedures—PWCPI and MBI—are compared with those from the RHA procedure (or simulated seismic demands) in Figure 10 for the Sherman Oaks building and in Figure 11 for the North Hollywood building. The "known" motions at the selected floors in both the interpolation procedures are taken same as the simulated motions from the RHA. The motions at 2S, G, 2nd, 8th, and roof of the Sherman Oaks building and the motions at B, 3rd, 9th, 16th, and roof of the North Hollywood building are used in the interpolation procedures. The estimates from the MBI procedure are based on the first four modes of the two selected buildings.

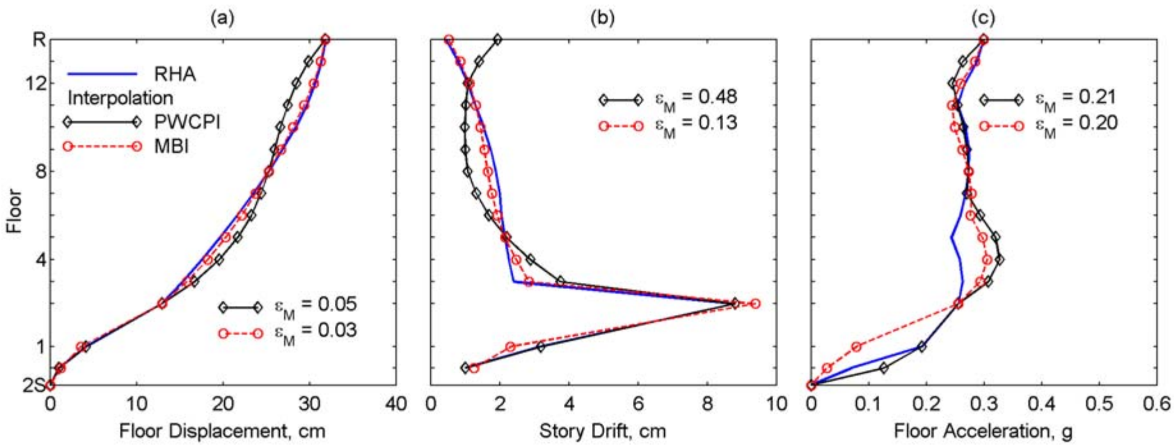

Fig. 10 Comparison of the height-wise distributions of peak response from two interpolation procedures—PWCPI and MBI—with those from RHA for the Sherman Oaks 13-Story Commercial Building

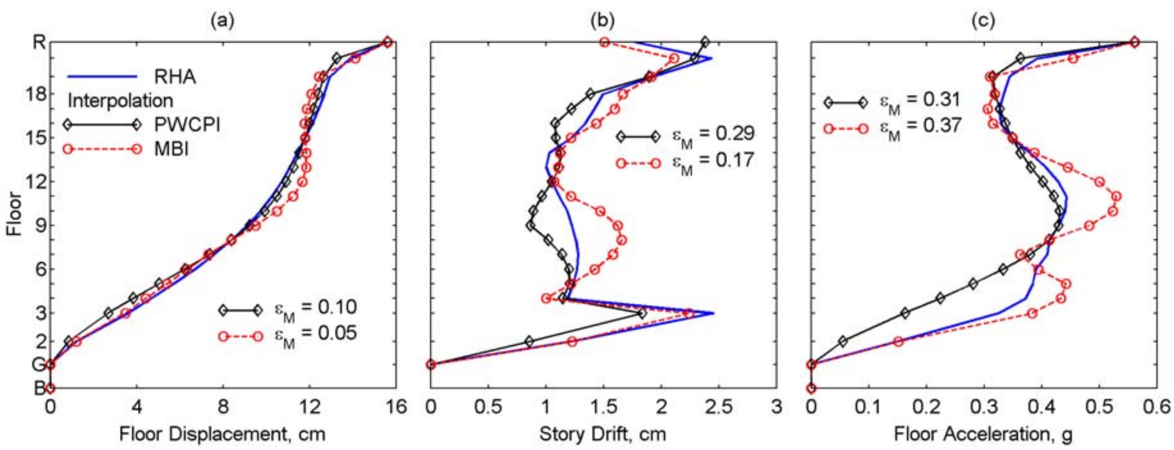

Fig. 11 Comparison of the height-wise distributions of peak response from two interpolation procedures—PWCPI and MBI—with those from RHA for the North Hollywood 20-Story Hotel

The height-wise distributions of floor displacements indicate that both the PWCPI and MBI procedures provide reasonable estimates of the floor displacements over building height (see Figures 10(a) and 11(a)). However, the MBI procedure provides slightly better estimates of the floor displacements, as indicated by a lower mean error for the MBI procedure in comparison with the PWCPI procedure: the mean error from the MBI procedure is 0.03 and 0.05 for the Sherman Oaks and North Hollywood buildings, respectively, compared to the values of 0.05 and 0.1 from the PWCPI procedure (see Figures 10(a) and 11(a)).

The estimates of story drifts from the MBI procedure are much better compared to those from the PWCPI procedure. In particular, the PWCPI procedure under-predicts story drifts in the middle stories and over-predicts story drifts in the upper stories of the Sherman Oaks building, whereas the MBI procedure closely tracks the story drifts from the RHA (see Figure 10(b)). With one exception, similar trends are apparent for the story drifts of the North Hollywood building (see Figure 11(b)); the exception occurs for the top-story drift where the MBI may slightly under-predict the drift compared to the RHA procedure. The mean error in story drifts from the MBI procedure is significantly less than that from the PWCPI procedure: the mean error in story drifts from the MBI procedure is 0.13 and 0.17 for the Sherman Oaks and North Hollywood buildings, respectively, compared to the values of 0.48 and 0.29 from the PWCPI procedure (see Figures 10(b) and 11(b)).

The accuracy of both the MBI and PWCPI procedures for estimating floor accelerations appears to be similar as apparent from the similar mean errors (see Figures 10(c) and 11(c)). However, the discrepancies in floor accelerations from the two procedures may occur at different locations when compared with the results from the RHA.

The preceding results indicate that for the same selection of instrumented floors the MBI procedure provides improved estimates of the floor displacements and story drifts compared to the PWCPI procedure. The two procedures, however, provide similar estimates of the floor accelerations.

SENSITIVITY OF MBI PROCEDURE

Sensitivity of the MBI procedure, i.e., the dependence of results from the MBI procedure to the sensor distribution—number and locations of sensors—is investigated next by comparing the results from RHA with those from the MBI procedure using two sets of floor motions for the two selected buildings. For the Sherman Oaks building, the first set consist of the motions at all floors—G, 2nd, 8th, and roof—whereas the second set consists of motions only at two floors—G, 8th and roof. Obviously, first four modes were included in the MBI procedure using the first set of motions, and first three modes were included in the MBI procedure using the second set. For the North Hollywood building, the first set consists of motions at four floors—3rd, 9th, 16th, and roof—permitting the use of first four modes in the MBI procedure, and the second set includes the motions at three floors—9th, 16th, and roof—allowing the use of first three modes.

The results for the Sherman Oaks building indicate that both sets of motions lead to very good estimates of floor displacements, story drifts, and floor accelerations when compared with the RHA results (see Figure 12): the mean errors from the two sets are generally very similar. More importantly, the dependence of results from the MBI procedure on the locations and number of instrumented floors is much less, as apparent from the much smaller variation in mean error with selection of floors in the interpolation procedure, compared to that of the results from the PWCPI procedure (compare Figure 12 with Figure 8). Similar to the Sherman Oaks building, both sets of motions provide very good estimates of the floor displacements for the North Hollywood building (see Figure 13(a)), and the dependence of results from the MBI procedure on the locations and number of instrumented floors is much less than that of the results from the PWCPI procedure (compare Figure 13(a) with Figure 9(a)). While the story drifts and floor accelerations from the MBI procedure using the second set of motions are much better than those from the PWCPI procedure using the same set (compare Figure 13(b) with Figure 9(b), and Figure 13(c) with Figure 9(c)), noticeable differences occur in the results from the MBI procedure when compared with the RHA results at few locations in the building. Such is the case because higher modes contribute much more to the response quantities like story drifts and floor accelerations of the flexible North Hollywood building, and dropping one floor from the set of motions leads to the inclusion of one less mode in the MBI procedure, which in turn leads to a loss of accuracy.

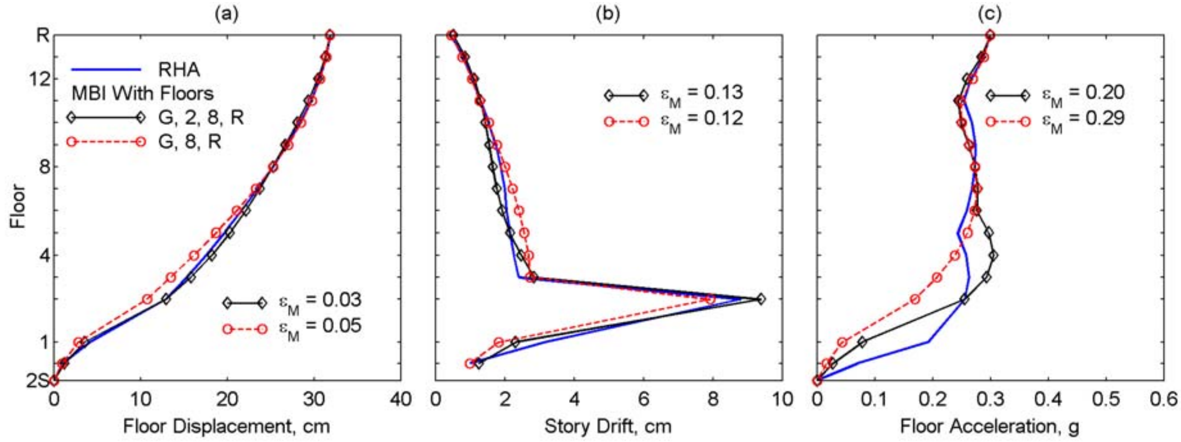

Fig. 12 Height-wise distribution of peak response from the MBI procedure, for the motions from RHA of the Sherman Oaks 13-Story Commercial Building, for two different sets of monitored floors

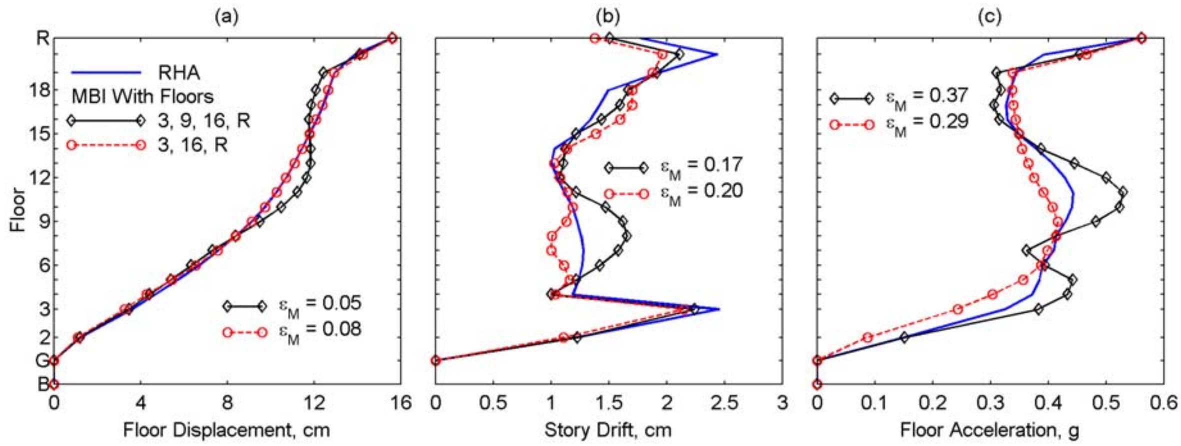

Fig. 13 Height-wise distribution of peak response from the MBI procedure, for the motions from RHA of the North Hollywood 20-Story Hotel, for two different sets of monitored floors

SUMMARY AND CONCLUSIONS

Investigated in this paper is the accuracy of the piece-wise cubic polynomial interpolation (PWCPI) procedure to estimate the motions—floor displacements, story drifts, and floor accelerations—at non-instrumented floors in the buildings with significant stiffness discontinuities. Based on the interpolation of recorded motions of two buildings, it has been shown that the results from the PWCPI procedure may depend noticeably on the locations of instrumented floors. Since recorded motions are typically available only at a few floors, this did not clearly indicate which locations in the PWCPI procedure would provide the most accurate estimates of the motions at non-instrumented floors. Therefore, the motions at each floor of the selected buildings were simulated from the response history analysis (RHA) of the computer models of these buildings. The RHA motions at a limited number of floors were then used to investigate the accuracy of the PWCPI procedure. It has been shown that the PWCPI procedure may provide good estimates of the displacements at non-instrumented floors if (1) the building is instrumented at a regular interval over its height, and (2) additional instruments are located in the building where stiffness changes significantly. The story drifts and floor accelerations, however, may not be accurately predicted by the PWCPI procedure.

The mode-based interpolation (MBI) procedure presented in this paper provides much-improved estimates of the motions—floor displacements, story drifts, and floor accelerations—at non-instrumented floors compared to those from the PWCPI procedure. Furthermore, the dependence of results from the

MBI procedure on the number and locations of the instrumented floors is much less compared to that from the PWCPI procedure.

It is useful to emphasize that the observations and conclusions in this paper are based on a limited set of data: two buildings and only a few combinations of the number of sensors and their locations. Clearly, a much larger dataset is needed to develop further confidence in these conclusions. Furthermore, the MBI procedure in this investigation has been applied to interpolate only the translational motions. Since the recorded motions of even essentially symmetric buildings have been known to include some level of torsional motions, it would be useful to extend the MBI procedure to estimate torsional motions as well. A comprehensive investigation to address this issue is currently underway, and the results would be reported upon completion.

ACKNOWLEDGEMENTS

This investigation is supported by the California Department of Conservation, California Geological Survey, Strong Motion Instrumentation Program under Contract No. 1005-832. This support is gratefully acknowledged. Also acknowledged is the useful insight provided by Dr. Praveen Malhotra on various interpolation procedures.

APPENDIX I: PIECE-WISE CUBIC POLYNOMIAL INTERPOLATION PROCEDURE

Let us consider a building with sensors at *J* locations and sub-divided into $J-1$ sub-intervals. Also, let r_j be the recorded response from the *j*th sensor located at height h_j; response r_j is available for $j = 1, 2, ..., J$. Using the piece-wise cubic polynomial interpolation, the response r at height h located in the *j*th sub-interval may be expressed as

$$r(h) = a_j \overline{w}_j + b_j w_j + c_j \frac{\left(\Delta h_j\right)^2}{6}\left(\overline{w}_j^3 - \overline{w}_j\right) + d_j \frac{\left(\Delta h_j\right)^2}{6}\left(w_j^3 - w_j\right) \tag{A.1}$$

in which $w_j = \left(h - h_j\right) / \left(h_{j+1} - h_j\right)$; $\overline{w}_j = \left(h_{j+1} - h\right) / \left(h_{j+1} - h_j\right)$; $\Delta h_j = h_{j+1} - h_j$; and a_j, b_j, c_j, and d_j are the cubic-polynomial constants for the *j*th sub-interval.

The polynomial constants for each of the $J-1$ sub-intervals are $a_j = r_j$, $b_j = r_{j+1}$, $c_j = \sigma_j$, and $d_j = \sigma_{j+1}$. The *J* values of σ_j for the piece-wise cubic polynomial interpolation with “not-a-knot” end conditions are computed from the following set of linear equations (Beatson, 1986):

$$\begin{gathered} -\overline{\theta}_1 \sigma_1 + \sigma_2 - \theta_1 \sigma_3 = 0 \\ \theta_{j-1}\sigma_{j-1} + 2\sigma_j + \overline{\theta}_{j-1}\sigma_{j+1} = \frac{6}{h_{j+1} - h_{j-1}}\left[\left(\frac{r_{j+1} - r_j}{h_{j+1} - h_j}\right) - \left(\frac{r_j - r_{j-1}}{h_j - h_{j-1}}\right)\right]; \ j = 2, 3, ..., J-1 \\ -\overline{\theta}_{J-2}\sigma_{J-2} + \sigma_{J-1} - \theta_{J-2}\sigma_J = 0 \end{gathered} \tag{A.2}$$

in which

$$\begin{gathered} \theta_j = \frac{h_{j+1} - h_j}{h_{j+2} - h_j}; \ j = 1, 2, ..., J-2 \\ \overline{\theta}_j = \frac{h_{j+2} - h_{j+1}}{h_{j+2} - h_j}; \ j = 1, 2, ..., J-2 \end{gathered} \tag{A.3}$$

In the matrix form, the solution for *J* values of σ_j involves solving the following system:

$$[A]\{\sigma\} = \{b\} \tag{A.4}$$

in which $\{\sigma\} = \{\sigma_1 \ \ \sigma_2 \dots \sigma_J\}^T$,

$$[A]=\begin{bmatrix} -\bar{\theta}_1 & 1 & -\theta_1 & 0 & & & & \\ \theta_1 & 2 & \bar{\theta}_1 & 0 & & & & \\ 0 & \theta_2 & 2 & \bar{\theta}_2 & & & & \\ & & & \ddots & & & & \\ & & & \theta_{J-3} & 2 & \bar{\theta}_{J-3} & 0 \\ & & & 0 & \theta_{J-2} & 2 & \bar{\theta}_{J-2} \\ & & & 0 & -\bar{\theta}_{J-2} & 1 & -\theta_{J-2} \end{bmatrix} \tag{A.5}$$

and

$$\{b\}=\begin{Bmatrix} 0 \\ \dfrac{6}{h_3-h_1}\left(\dfrac{r_3-r_2}{h_3-h_2}-\dfrac{r_2-r_1}{h_2-h_1}\right) \\ \dfrac{6}{h_4-h_2}\left(\dfrac{r_4-r_3}{h_4-h_3}-\dfrac{r_3-r_2}{h_3-h_2}\right) \\ \vdots \\ \dfrac{6}{h_{J-1}-h_{J-3}}\left(\dfrac{r_{J-1}-r_{J-2}}{h_{J-1}-h_{J-2}}-\dfrac{r_{J-2}-r_{J-3}}{h_{J-2}-h_{J-3}}\right) \\ \dfrac{6}{h_J-h_{J-2}}\left(\dfrac{r_J-r_{J-1}}{h_J-h_{J-1}}-\dfrac{r_{J-1}-r_{J-2}}{h_{J-1}-h_{J-2}}\right) \\ 0 \end{Bmatrix} \tag{A.6}$$

Once σ_js have been computed from Equation (A.4), constants c_j and d_j can be determined, and the motion at any non-instrumented floor can be estimated from Equation (A.1). It may be noted that the formulation presented here differs slightly from Equation (2). However, this formulation is presented as it permits a much more convenient computation of the polynomial constants. The formulation presented here has been verified against the "spline" function in MATLAB (MathWorks, 2006) with "not-a-knot" end conditions.

REFERENCES

1. Beatson, R.K. (1986). "On the Convergence of Some Cubic Spline Interpolation Schemes", SIAM Journal on Numerical Analysis, Vol. 23, No. 4, pp. 903–912.
2. Chopra, A.K. (2007). "Dynamics of Structures: Theory and Applications to Earthquake Engineering", Prentice-Hall, Upper Saddle River, U.S.A.
3. De la Llera, J.C. and Chopra, A.K. (1997). "Evaluation of Seismic Code Provisions Using Strong-Motion Building Records from the 1994 Northridge Earthquake", Report UCB/EERC-97/16, University of California, Berkeley, U.S.A.
4. FEMA (2000). "Prestandard and Commentary for the Seismic Rehabilitation of Buildings", Report FEMA 356, Federal Emergency Management Agency, Washington, DC, U.S.A.
5. Goel, R.K. (2005). "Evaluation of Modal and FEMA Pushover Procedures Using Strong-Motion Records of Buildings", Earthquake Spectra, Vol. 21, No. 3, pp. 653–684.
6. Goel, R.K. (2007). "Analysis of Strong Motion Records from Reinforced-Concrete Buildings", Proceedings of the 2007 Structures Congress, Long Beach, U.S.A. (on CD).
7. Goel, R.K. and Chadwell, C. (2007). "Evaluation of Current Nonlinear Static Procedures for Concrete Buildings Using Recorded Strong-Motion Data", Data Utilization Report, California Strong Motion Instrumentation Program, California Geological Survey, California Department of Conservation, Sacramento, U.S.A.

8. Limongelli, M.P. (2003). "Optimal Location of Sensors for Reconstruction of Seismic Responses through Spline Function Interpolation", Earthquake Engineering & Structural Dynamics, Vol. 32, No. 7, pp. 1055–1074.
9. Limongelli, M.P. (2005). "Performance Evaluation of Instrumented Buildings", ISET Journal of Earthquake Technology, Vol. 42, No. 2-3, pp. 47–61.
10. MathWorks (2006). "MATLAB—The Language of Technical Computing", The MathWorks, Natick, U.S.A.
11. Mau, S.T. and Aruna, V. (1994). "Story-Drift, Shear, and OTM Estimation from Building Seismic Records", Journal of Structural Engineering, ASCE, Vol. 120, No. 11, pp. 3366–3385.
12. McKenna, F. and Fenves, G.L. (2001). "OpenSees Command Language Manual: Version 1.2", Pacific Earthquake Engineering Center, University of California, Berkeley, U.S.A.
13. Miranda, E. and Akkar, S.D. (2006). "Generalized Interstory Drift Spectrum", Journal of Structural Engineering, ASCE, Vol. 132, No. 6, pp. 840–852.
14. Miranda, E. and Reyes, C.J. (2002). "Approximate Lateral Drift Demands in Multistory Buildings with Nonuniform Stiffness", Journal of Structural Engineering, ASCE, Vol. 128, No. 7, pp. 840–849.
15. Miranda, E. and Taghavi, S. (2005). "Approximate Floor Acceleration Demands in Multistory Buildings. I: Formulation", Journal of Structural Engineering, ASCE, Vol. 131, No. 2, pp. 203–211.
16. Naeim, F. (1997). "Performance of Extensively Instrumented Buildings during the January 17, 1994 Northridge Earthquake: An Interactive Information System", JAMA Report 97-7530.68, John A. Martin & Associates, Los Angeles, U.S.A.
17. Naeim, F., Lee, H., Bhatia, H., Hagie, S. and Skliros, K. (2004). "CSMIP Instrumented Building Response Analysis and 3-D Visualization System (CSMIP-3DV)", Proceedings of the SMIP04 Seminar on Utilization of Strong-Motion Data, Sacramento, U.S.A., pp. 83–102.
18. Shakal, A., Huang, M. and Graizer, V. (2003). "Strong-Motion Data Processing" in "International Handbook of Earthquake & Engineering Seismology, Part B (edited by W.H.K. Lee, H. Kanamori, P.C. Jennings and C. Kisslinger)", Academic Press, New York, U.S.A.
19. Taghavi, S. and Miranda, E. (2005). "Approximate Floor Acceleration Demands in Multistory Buildings. II: Applications", Journal of Structural Engineering, ASCE, Vol. 131, No. 2, pp. 212–220.