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APPLICATION OF LEAST SQUARES SUPPORT VECTOR MACHINE IN SEISMIC ATTENUATION PREDICTION

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ABSTRACT

The potential of least squares support vector machine (LSSVM) in the prediction of seismic attenuation based on rock properties is investigated in this paper. LSSVM is firmly based on the theory of statistical learning. Here, LSSVM is used as a regression technique. In LSSVM, Vapnik's ε-insensitive loss function is replaced by a cost function, which corresponds to a form of ridge regression. LSSVM involves equality instead of inequality constraints and works with a least-squares cost function. LSSVM is also used to compute error bars. A sensitivity analysis is performed to investigate the importance of each of the input parameters. The results show that the LSSVM approach has the potential to be a practical tool for the determination of seismic attenuation.

KEYWORDS: Seismic Attenuation, Least Squares Support Vector Machine, Predictions, Artificial Neural Network

INTRODUCTION

The determination of attenuation of seismic waves is an imperative task in geological engineering. The prediction of seismic attenuation has the two-fold aim of characterizing the propagation properties of seismic waves in order to focus on the seismic risk studied, and of understanding the physics underlying the observed phenomenology. It has long been believed that attenuation is important for the characterization of rock and fluid properties, e.g., saturation, porosity, permeability, and viscosity, because attenuation is more sensitive than velocity to some of these properties (e.g., see Best et al., 1994). Attenuation has been used also to locate hydrocarbons. The knowledge of seismic attenuation is crucial in the studies on seismic source, as the spectral shape of the seismic pulse is modified by attenuation. Seismic attenuation s mainly depends on different rock properties, namely, porosity n, clay content c, grain size d, and permeability k. The knowledge of seismic wave attenuation mechanisms and causative rock properties is very useful for the evaluation and interpretation of field and laboratory seismic data.

There are different attenuation mechanisms available in the literature (Biot, 1956a, 1956b; Walsh, 1966, 1969; Stoll and Bryan, 1970; Solomon, 1973; Kuster and Toksoz, 1974; Mavko and Nur, 1975, 1979). In the context of the prediction methodology of seismic attenuation, a regression model was developed by Klimentos and Mccann (1990). Brzostowski and McMechan (1992), and Leggett et al. (1993) used the change in seismic amplitude as observed data for the prediction of seismic attenuation. However, amplitudes are easily contaminated by many factors, such as scattering, geometric spreading, source and receiver coupling, radiation patterns, and transmission/reflection effects. Therefore, this method is not so reliable. When a seismic pulse spreads in a medium, the shape of the pulse broadens because of the dispersion caused by attenuation. The rise time associated with this broadening effect has been used to predict seismic attenuation (Kjartansson, 1979; Zucca et al., 1994). However, it is very difficult to measure this rise time for the field data. Recently, artificial neural network (ANN) has been successfully used for the prediction of seismic attenuation (Boadu, 1997). However, there are some limitations in using ANN, such as less generalizing performance, arriving at local minimum, and over-fitting (Haykin, 1999; Park and Rilett, 1999; Kecman, 2001). Quan and Harris (1997) used the frequency-shift method to predict seismic attenuation. However, this method is sensitive to small frequency changes. Roth et al. (2000) gave an empirical equation to predict seismic attenuation. As a result, alternative methods are needed, which can predict seismic attenuation more accurately.

In this study, least squares support vector machine (LSSVM) is used to predict the seismic attenuation based on rock properties. This study uses the database of Klimentos and McCann (1990) as reproduced in Table 1. LSSVM is a statistical learning theory, which adopts a least-squares linear system as a loss function instead of the quadratic program in the original support vector machine (SVM) (Suykens et al., 1999). It is closely related to the Gaussian processes and regularization networks. It requires solving a set of linear equations (i.e., linear programming), which is much easier and computationally very simple.

This paper has the following aims:

1. to investigate the feasibility of a LSSVM model for predicting seismic attenuation based on rock properties,
2. to compute the error bars of predicted data,
3. to compare the performance of developed LSSVM model with the other available models for the prediction of seismic attenuation, and
4. to explore the relative importance of the factors affecting seismic attenuation prediction by carrying out a sensitivity analysis.

Table 1: Dataset Used in This Study

Wet-Dry Porosity (%)	Nitrogen Permeability (md)	Average Grain Size (μm)	Clay Content (%)	P-Wave Attenuation at 40 MPa (dB/cm)
29.81	17.17	74	12	4.51
35.98	21.16	76	16	4.54
20.55	0.05	80	15	3.15
3.489	0	78	0	0.01
2.9	0	70	0	0.08
32.397	9.3	78	23	8.92
35.129	73.26	82	30	8.48
33.1	10.05	83	20	4.83
31.02	5.47	84	18	4.5
23.42	11.42	85	15	2.4
24.7	7.1	91	22	3.47
29.943	9.59	84	17	5.02
30.69	3.5	102	17	4.59
27.57	0.45	76	25	8.61
24.84	1.13	91	25	7.68
11.93	0.01	87	7	1.79
15.11	0.06	74	14	4.92
19.05	0.13	72	15	6.83
20.48	0.44	79	8	1.57
28.83	10.27	82	20	3.33
33.249	2.25	80	15	5.26
27.74	5.78	87	20	4.19
29.412	7.03	91	23	4.93
28.543	33.67	139	15	4.18
18.645	2.21	145	12	2.68
17.794	0.37	140	12	2.36
14.783	220.9	242	0.2	0.08
15.065	150.7	229	1	0.7
15.431	255.9	272	1	0.29
14.518	160.4	260	0.7	0.14
15.64	87.65	235	0.5	0.09
16.514	41.74	377	15	3.63
17.06	50.51	312	15	3.3
16.03	52.42	330	15	3.38
12.045	3.67	226	7	2.1
16.48	87.55	226	5	0.47

8.21	0.13	140	6	2.46
26.56	305.8	187	5	2.73
14.125	11.06	271	4	1.65
11.98	0.46	153	6	1.6
5.45	0	97	3	0.19
9.94	0.16	97	9	4.63

LEAST SQUARES SUPPORT VECTOR MACHINE (LSSVM) MODEL

LSSVM models are based on an alternate formulation of SVM regression (Vapnik and Lerner, 1963) proposed by Suykens et al. (2002). Consider a given training set of N data points $\{x_k, y_k\}_{k=1}^{N}$ with input data $x_k \in \mathbb{R}^N$ and output data $y_k \in r$ where $\mathbb{R}^N$ is the N-dimensional vector space and r is the one-dimensional vector space. The four input variables used for an LSSVM model in this study are porosity n, clay content c, grain size d, and permeability k. The output of the LSSVM model is seismic attenuation s. Hence, in this study, we take $x = [n, c, k, d]$ and $y = s$. In the feature space a LSSVM model thus takes the form

$$y(x) = w^T \phi(x) + b \tag{1}$$

where the nonlinear mapping $\phi(.)$ maps the input data into a higher dimensional feature space, $w \in \mathbb{R}^N$, $b \in r$, w = an adjustable weight vector, and b = the scalar threshold. In LSSVM, for function estimation, the following optimization problem is formulated:

$$\text{minimize } \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^{N} e_k^2$$

$$\text{subject to } y(x) = w^T \phi(x_k) + b + e_k ;\ k = 1, \ldots, N \tag{2}$$

The following equation for the prediction of s is obtained by solving the above optimization problem (Vapnik, 1995; Smola and Schölkopf, 1998):

$$s = y(x) = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b \tag{3}$$

The Gaussian kernel is used in this analysis. The Gaussian kernel is given by

$$K(x_k, x_l) = \exp\left\{ -\frac{\|x_k - x_l\|}{2\sigma^2} \right\};\ k, l = 1, \ldots, N \tag{4}$$

where σ is the width of the Gaussian kernel.

DETAILS OF LSSVM

The main scope of this work is to implement the above model in the problem of seismic attenuation prediction. The dataset consists of 42 experimental results shown in Table 1. The input parameters that are selected are related to rock properties. More specifically, the input parameters are n, k, c, and d. The output of the model is s. In carrying out the formulation, the data is divided into two sub-sets such as

(a) A training dataset: this is required to construct the model. In this study, 34 out of the 42 data points are considered for the training dataset.
(b) A testing dataset: this is required to estimate the model performance. In this study, the set of remaining 8 data points is considered as the testing dataset.

In order to train the LSSVM model, Gaussian kernel function is used. The data is scaled between 0 and 1 before being presented to the model. In the training process, the values of γ and σ are chosen by using the trial-and-error approach.

In this study, a sensitivity analysis is carried out to extract the cause and effect relationship between the inputs and outputs of the LSSVM model. The basic idea is that each input of the model is perturbed slightly and the corresponding change in the output is reported. The procedure used is same as that in Liong et al. (2000). Accordingly, the percent sensitivity S of each input parameter is calculated by the following formula:

$$S = \frac{1}{N} \sum_{j=1}^{N} \left(\frac{\text{percent change in output}}{\text{percent change in input}} \right)_j \times 100 \tag{5}$$

where N is the number of data points. The sensitivity analysis is carried out on the trained model by varying each of the input parameters, one at a time, at a uniform rate of 20%. Further, the training, testing and sensitivity analysis of LSSVM are carried out by using MATLAB® (MathWorks, 1998).

RESULTS AND DISCUSSION

In this study, the coefficient of correlation R is considered to be the main criterion to evaluate the performance of the developed LSSVM model. Different combinations of γ and σ values are tried to yield the best performance on the training dataset for the LSSVM model. The optimum values of γ and σ are obtained as 600 and 5 respectively. Figure 1 illustrates the performance of the training dataset on using the Gaussian kernel and the value of R (= 0.973) is found to be close to unity.

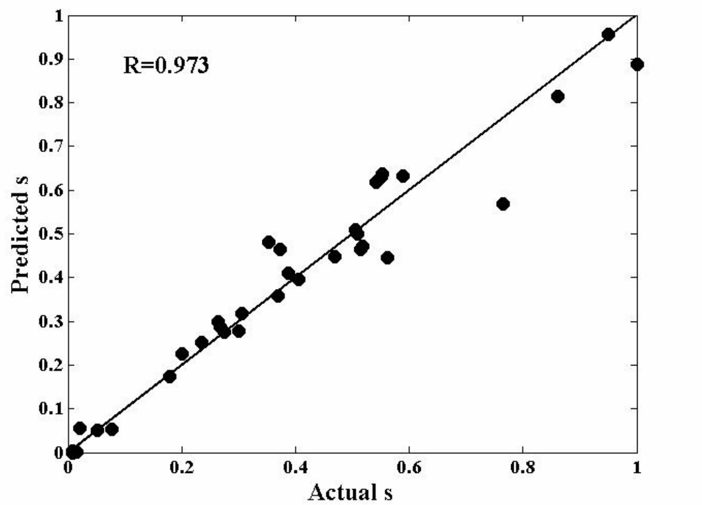

Fig. 1 Performance of LSSVM model for training dataset on using Gaussian kernel

In order to evaluate the capabilities of the LSSVM model, the model is validated with new data that is not part of the training dataset. Figure 2 shows the performance of the LSSVM model (with R = 0.963) for the testing datset.

The loss of performance with respect to the testing dataset addresses the generalization capability and LSSVM's susceptibility to overtraining. There is a marginal reduction in the performance on the testing dataset (i.e., there is a difference between the performance on the training and testing datasets) for the LSSVM model. Thus, LSSVMs have the ability to avoid overtraining, and in turn, those have good generalization capability for the prediction of seismic attenuation. From the results, it is clear that the LSSVM model predicts the actual value s very well and that it can be used as a practical tool for the determination of s. Figures 3 and 4 show the 95% error bars for the training and testing datasets respectively. An error bar is used to indicate the range of one standard deviation on a prediction. This can also be used to determine whether the differences are statistically significant. For the prediction of seismic attenuation, the determination of error bars at different points is important in order to estimate the corresponding risk. In ANN, error bars are obtained using a local quadratic approximation to the nonconvex cost function. But no approximation has to be made for LSSVM, since a quadratic cost function is used. The following equation can be developed from the LSSVM model for the determination of s:

$$s=\sum_{i=1}^{34}\alpha_i \exp\left[\frac{-\left\{(n,c,k,d)_i-(n,c,k,d)\right\}\times\left\{(n,c,k,d)_i-(n,c,k,d)\right\}^T}{50}\right]+0.246 \quad (6)$$

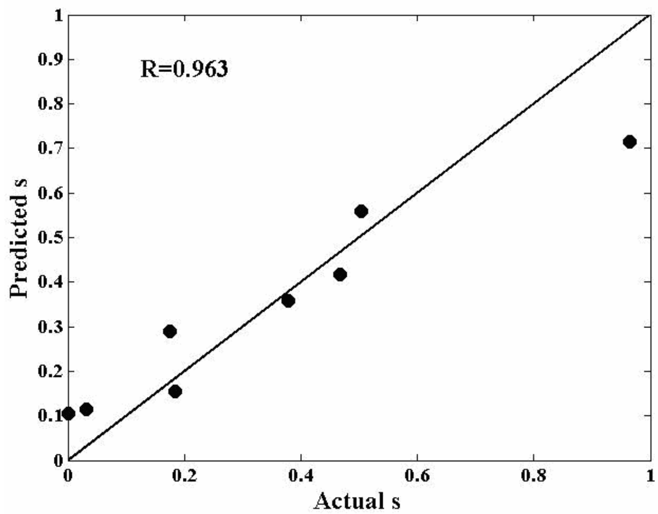

Fig. 2 Performance of LSSVM model for testing dataset on using Gaussian kernel

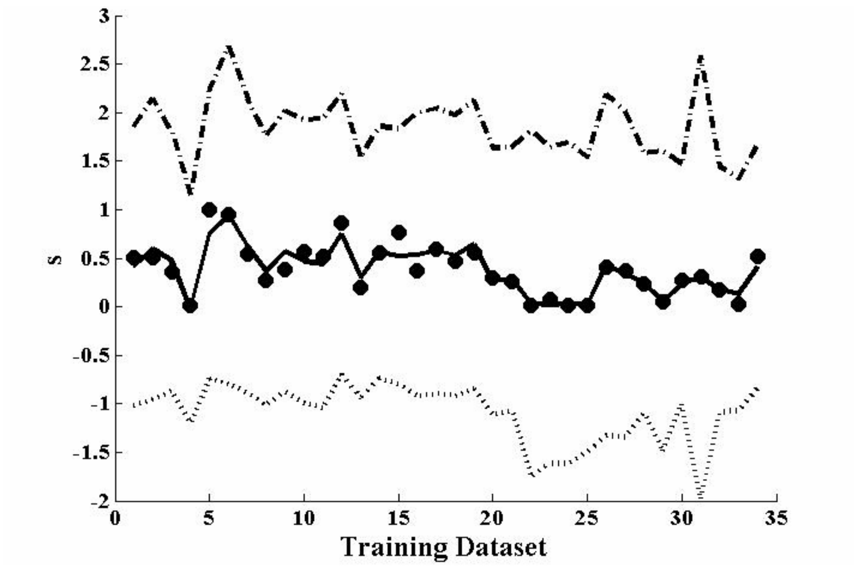

Fig. 3 95% error bars for training dataset

The values of α_i are given in Table 2. A comparative study is done between different models in Figure 5. From Figure 5, it is clear that the developed LSSVM model outperforms the ANN model proposed by Das and Samui (2008). The performance of the LSSVM model is almost same as those of the SVM model proposed by Das and Samui (2008) and the relevance vector machine (RVM) model proposed by Samui and Sitharam (2007). The RVM model has some limitations, such as highly nonlinear optimization process and difficulties in finding the optimum solution for a large data set. The use of the structural risk minimization (SRM) principle in defining the cost function provides more generalization capacity with the LSSVM model compared to the ANN model, which uses the empirical risk minimization principle. In ANN, there are a larger number of controlling parameters, including the number of hidden layers, number of hidden nodes, learning rate, momentum term, number of training epochs, transfer functions, and weight initialization methods. Obtaining an optimal combination of these parameters is a difficult task as well. On the other hand, the LSSVM model uses only two parameters (i.e., γ and σ).

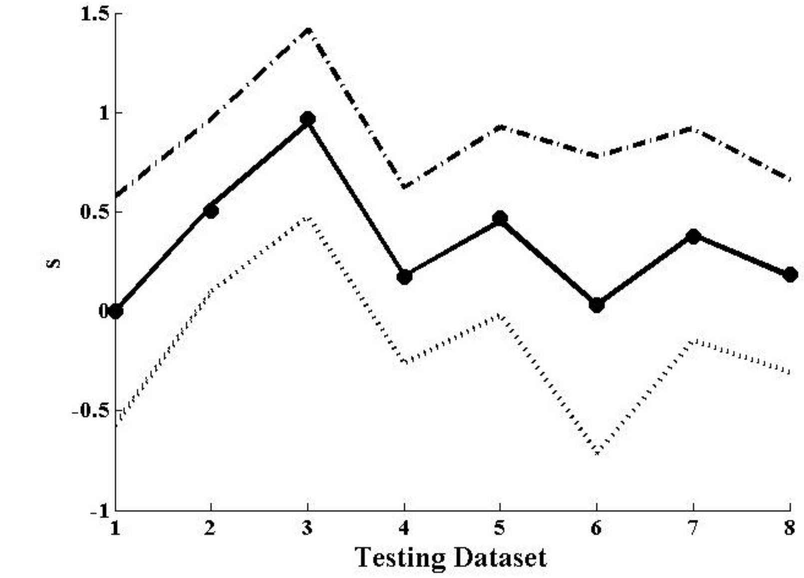

Fig. 4 95% error bars for testing dataset

Table 2: Values of Coefficient α_i

Wet-Dry Porosity (%)	Nitrogen Permeability (md)	Average Grain Size (μm)	Clay Content (%)	α_i
29.81	17.17	74	12	–7.0881
35.98	21.16	76	16	10.8523
20.55	0.05	80	15	–298.78
2.9	0	70	0	31.0665
32.397	9.3	78	23	256.0591
35.129	73.26	82	30	–3.7381
33.1	10.05	83	20	–186.013
23.42	11.42	85	15	–48.0217
24.7	7.1	91	22	–49.4881
29.943	9.59	84	17	263.2917
30.69	3.5	102	17	124.6512
24.84	1.13	91	25	97.489
11.93	0.01	87	7	–52.0951
15.11	0.06	74	14	–179.607
19.05	0.13	72	15	441.7206
28.83	10.27	82	20	–206.282
33.249	2.25	80	15	–101.135
27.74	5.78	87	20	43.5591
29.412	7.03	91	23	–197.669
18.645	2.21	145	12	67.2107
17.794	0.37	140	12	–68.8887
14.783	220.9	242	0.2	–5.6382
15.065	150.7	229	1	15.1032
14.518	160.4	260	0.7	–7.0643
15.64	87.65	235	0.5	2.9346
16.514	41.74	377	15	–5.1255
17.06	50.51	312	15	8.7566
12.045	3.67	226	7	–14.4941
16.48	87.55	226	5	–11.8257
8.21	0.13	140	6	13.6501
26.56	305.8	187	5	–0.609

11.98	0.46	153	6	19.1652
5.45	0	97	3	–75.3699
9.94	0.16	97	9	123.3906

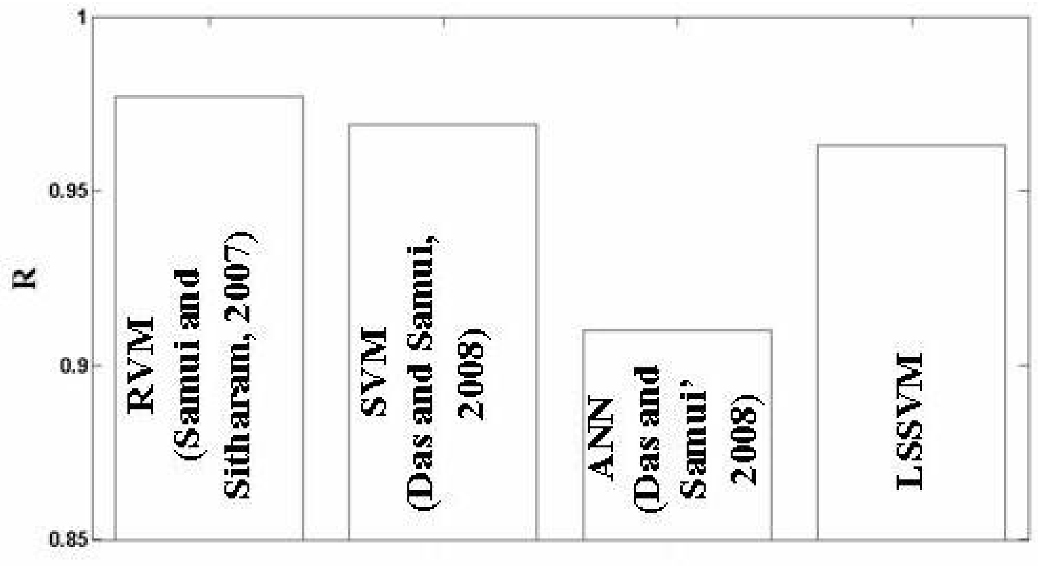

Fig. 5 Comparison of different models

The SVM model proposed by Das and Samui (2008) requires three parameters: capacity factor C, error insensitive zone ε, and kernel parameter. This model is solved using the quadratic programming methods. However, those methods are often time consuming and are difficult to implement adaptively. LSSVM has some properties that are related to the implementation and the computational method. For example, training requires solving a set of linear equations instead of solving the quadratic programming problem involved in the original SVM formulation of Vapnik (1995). The original SVM formulation is modified by considering equality constraints within a form of ridge regression rather than by considering the inequality constraints.

The sensitivity analysis of the proposed model is carried out by using the radial basis function. The results of the sensitivity analysis are shown in Figure 6. It is seen that c has the most significant effect on the predicted s, followed by n, k and d. However, the fact that the developed LSSVM model accurately provides a relationship between a limited amount of data, does not necessarily imply that the sensitivity of the model to all parameters is captured correctly.

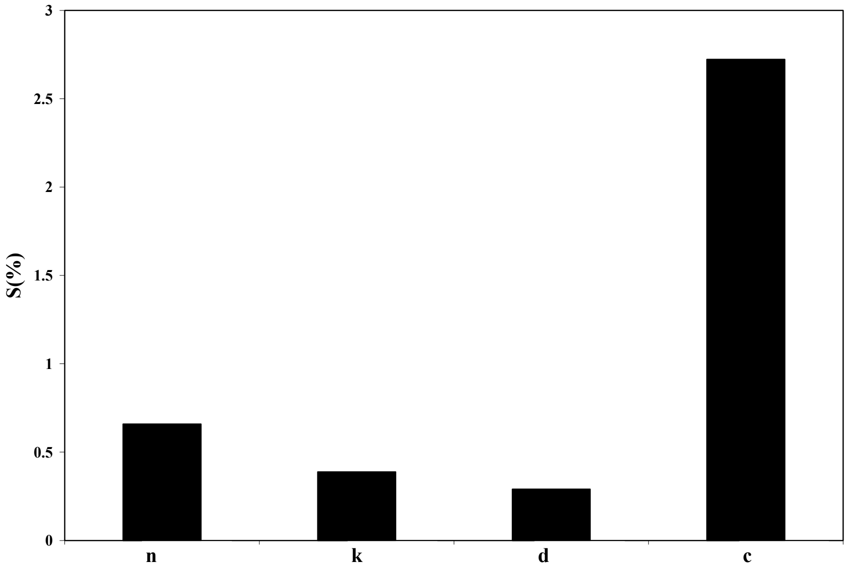

Fig. 6 Sensitivity analysis of input parameters

CONCLUSIONS

A method using LSSVM to predict seismic attenuation has been proposed. A database containing 42 case records of actual seismic attenuation measurements has been used to develop and verify the model. It is clear from the results that, with Gaussian kernel and appropriate γ and σ values, an accuracy corresponding to R = 0.963 can be achieved from the proposed LSSVM model. LSSVM has the advantage that once the model is developed, it can be used as an accurate and quick tool for predicting the seismic attenuation. LSSVM has the additional advantage of error bars that yield confidence intervals. From the sensitivity analysis, it is clear that c has the most significant effect on the predicted s and that d has the least effect on s. The results of this study indicate that LSSVM is a promising tool for the prediction of seismic attenuation.

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