On Generating Ensemble of Design Spectrum-Compatible Accelerograms

Manish Shrikhande*, Vinay K. Gupta**

SUMMARY – *A new method for the characterization of design seismic excitation through the spectrum-compatible power spectral density function (PSDF) is proposed. This method is based on assuming the excitation process to be stationary, and on explicitly accounting for the nonstationarity due to the building-up of response after the excitation is applied. The PSDF thus generated has much better consistency with the design spectra of different damping ratios. This PSDF has been used to synthesize an ensemble of accelerograms which are consistent with the specified design spectrum in a statistical sense and are also more realistic in the temporal variation of the frequency content.*

1. Introduction

Seismic design forces are usually specified by various codes and regulatory agencies in the form of smooth design (response) spectra. These spectra are convenient to use and are popular with practicing engineers for the equivalent linear analyses. For estimating nonlinear response of structures, however, it is preferred to directly integrate the equations of motion, and therefore, these spectra are used to characterize the design ground motion by generating the spectrum-compatible time histories. It is possible to generate several «different looking» time histories which are consistent with the same design spectrum in the mean sense. An ensemble of such time histories can be used to quantify the variability in structural response due to the random phasing of seismic waves. It is also possible to obtain the probabilistic response of the structures more directly by using the power spectral density function (PSDF) of the design ground motion, but such techniques can be used accurately in the case of linear systems only.

* Graduate Student.

** Associate Professor, Dept. of Civil Engineering, Indian Institute of Technology, Kanpur - 208016, India.

Several attempts have been made in the past towards the generation of synthetic accelerograms that are compatible with the specified design spectrum. Most of these studies, e.g., those by Scanlan and Sachs (1974), Vanmarcke and Gasparini (1977), Iyengar and Rao (1979), Preumont (1980, 1984), and Gupta and Joshi (1993) were based on the iterative evaluation of the Fourier amplitudes so as to match the response spectrum of the generated time-history with the target spectrum within a specified error. Except for the study by Gupta and Joshi (1993), these approaches did not account for the temporal variation of the frequency characteristics in the ground motion, and simply relied on the use of a time-dependent modulating function along with a stationary process to simulate the nonstationarity in the ground motion. Assuming time-invariant frequency content of the ground motion may not however be acceptable for the nonlinear analyses due to their sensitivity to the characteristics of individual pulses and their sequence within a ground acceleration time history (O'Connor and Ellingwood (1987)). Trifunac (1971), Wong and Trifunac (1979), Lee and Trifunac (1985, 1987, 1989), Lee (1990) and Trifunac (1990) obtained more realistic time-histories by using the group velocity curves of various waves to compute the instants of first arrival and the relative phases of these waves. Gupta and Joshi (1993) instead used the phase characteristics of real accelerograms to replicate the nonstationary characteristics in the generated time-histories. Spanos and Vargas Loli (1985) used the concept of spectrum compatible evolutionary power spectrum to generate synthetic accelerograms. However, their approach also overlooked the role of dispersion of the seismic waves in the temporal variation of the ground motion frequency content. All the previous approaches, except for those by Lee, Trifunac and Wong, have been iterative in nature such that the response spectra of the synthetic accelerograms obtained by these approaches matched «almost exactly» with the design spectrum. Hence, these approaches cannot be used to obtain an ensemble of accelerograms with realistic variations as would he produced by the random phasing of various waves and which would be essential for a statistically meaningful study of the structural response. The approaches of Lee, Trifunac and Wong give statistically more useful ensemble of accelerograms, but those require the additional input of the group velocity curves of various seismic waves.

In this paper, a simple and efficient procedure is proposed for generating an ensemble of synthetic accelerograms which are realizations of a Gaussian ground motion process and are consistent with the given design spectrum in the mean sense. This procedure is based on i) first finding the spectrum-compatible PSDF of the design ground motion assuming it to be a stationary process, ii) calculating the corresponding Fourier amplitude spectrum by using the frequency-dependent duration as in Gupta (1994a), and then on synthesizing the Fourier spectrum with a realistic phase spectrum as suggested by Gupta and Joshi (1993). The phase characteristics and frequency-dependent durations of the generated accelerograms are taken to be same as in the case of a recorded ground motion. In finding the spectrum-compatible PSDF, a new iterative procedure based on the formulation of Shrikhande and Gupta (1995) has been used. This procedure is different from the earlier approaches, e.g., those by Kaul (1978), Sundararajan (1980), and Unruh and Kana (1981), as the nonstationarity effects introduced in the system response due to the finite operating time of the excitation are filtered out of the computed PSDF. Due to this, almost similar PSDFs are obtained for the spectra of two different damping ratios. The proposed algorithm has been illustrated by generating the ensemble of accelerograms which are compatible with the USNRC design spectrum and are based on the characteristics of four example (recorded) accelerograms.

2. Generation of Spectrum-Consistent PSDF

Response spectrum ordinates are defined as the largest peak amplitudes in the responses of the single-degree-of-freedom (SDOF) oscillators of different natural frequencies and damping ratios to a given excitation. It is possible to compute these ordinates in probabilistic sense directly from the PSDF of the given excitation by using the transfer function for the SDOF system response and the peak statistics (Udwadia and Trifunac (1974), Trifunac (1995)). As an inverse problem, if the response spectrum is known, it is possible to estimate the PSDF by following an iterative procedure, e.g., as in Kaul (1978), Sundararajan (1980), Unruh and Kana (1981) etc. These PSDFs are often evaluated from a set of design (response) spectra for the aseismic design of structures e.g., those by USNRC, and are termed as the spectrum-compatible PSDFs. It is usual to assume the ground motion process and the response process to be stationary in nature for computing the spectrum-compatible PSDFs, and thus the PSDFs so computed implicitly reflect the adjustments for i) inherent nonstationarity in the ground motion and ii) additional nonstationarity introduced in the system response due to the sudden application of the excitation (Caughey and Stumpf (1961)). Since the system damping significantly influences this additional nonstationarity, the PSDFs so generated are different for the response spectra of two different damping ratios despite the use of «equivalent damping» as proposed by Rosenblueth and Elorduy (1969). Spanos and Vargas Loli (1985) considered both sources of nonstationarity explicitly by using an evolutionary process model for the PSDF of ground acceleration and time-dependent transfer function for the SDOF oscillator response, and obtained PSDFs which were consistent with the sets of response spectra for different damping ratios. However, ground motion nonstationarity is too complex a phenomenon to be accurately modelled by an evolutionary process, and it may be more convenient to characterize the design ground motion as an «equivalent stationary» process of desired duration such that the design spectra of different damping ratios are consistent in terms of the PSDF of this process. Shrikhande and Gupta (1995) have shown how the peak response of a SDOF oscillator to a suddenly applied stationary excitation can be estimated through successive application of the order statistics formulation of Gupta and Trifunac (1988) to a series of fictitious stationary processes. A brief outline of this procedure is given in the Appendix. This procedure properly accounts for the nonstationarity due to the response build-up, and thus can be employed to calculate the largest peak response in the usual iterative schemes of finding the spectrum-compatible PSDFs. A PSDF so generated will be independent of the system characteristics, and will thus be a more true description of the ground motion.

Let us assume that the PSDF of the «equivalent stationary» process varies linearly between certain control frequencies, and let the ordinates of PSDF at these frequencies be determined through iterations. To begin the iterations, the first estimate of these ordinates may be obtained by assuming the acceleration process to be ideal white-noise and peak factor equal to a constant value (irrespective of the natural frequency, damping of the oscillator, and duration of the ground motion), and by ignoring the nonstationarity in response as

$$G_{\ddot{u}_g}^{(1)}(\omega_i) = \frac{4\zeta\,\mathrm{PSA}_t^2(\omega_i,\zeta)}{\pi\omega_i\eta^2} \tag{1}$$

where, $\mathrm{PSA}_t(\omega_i, \zeta)$ is the Pseudo Spectral Acceleration ordinate of target design spectrum at the control frequency, ω_i and for the damping ratio, ζ, and η is the constant peak factor, which may be assumed equal to 3.0. The Pseudo Spectral Acceleration ordinates for the stationary excitation defined by the PSDF in Eq. (1) can now be computed at the control frequencies by using the formulation of Shrikhande and Gupta (1995). If $\mathrm{PSA}_c^{(1)}(\omega_i,\zeta)$ denotes the computed ordinate at frequency, ω_i for damping ratio, ζ in the first iteration, the PSDF ordinates for the next iterations can then be computed according to the following recursive relation

$$G_{\ddot{u}_g}^{(k+1)}(\omega_i) = G_{\ddot{u}_g}^{(k)}(\omega_i)\left[\frac{\mathrm{PSA}_t(\omega_i,\zeta)}{\mathrm{PSA}_c^{(k)}(\omega_i,\zeta)}\right]^2, \quad k = 1, 2, 3, \ldots \quad (2)$$

where, $\mathrm{PSA}_c^{(k)}(\omega_i, \zeta)$ represents the computed Pseudo Spectral Acceleration ordinate in the k^{th} iteration. This iterative procedure is continued until the match between the target and the computed spectra is found to be within acceptable error. The proposed procedure converges very fast and acceptable agreement between the target and the computed spectra with total error within 5% is achieved within 4-5 iterations. It may be observed that despite similarities, this procedure is different from that proposed by Unruh and Kana as it uses the transient transfer function instead of the steady-state transfer function along with the «equivalent damping».

To illustrate the proposed procedure, the PSDFs compatible with the response spectra for 2% and 5% damping ratios of the S00E component of the Imperial Valley earthquake of 14 May, 1940 recorded at El Centro site have been computed assuming those to be the design spectra. It has been further assumed that the duration of the «equivalent stationary» motion is same as the 90% duration (Trifunac and Brady (1975)) of the earthquake record. The actual and the computed spectra after 5 iterations for both damping ratios have been compared in Fig. 1. The PSDFs for these damping ratios are found to be in good agreement as shown in Fig. 2. This and many other examples show that the proposed procedure indeed leads to a definition of the PSDF of the excitation process which is consistent with the response spectra for different damping ratios. Figs. 3 and 4 show the similar comparison of the design spectra and PSDFs respectively in case of the USNRC design spectra for 2% and 5% damping ratios. The duration of the «equivalent stationary» process has been taken as 24.42 sec as in the case of the recorded motion during the Imperial Valley earthquake. It is seen that the two PSDFs are very similar with respect to the frequency distribution of the energy. The appar-

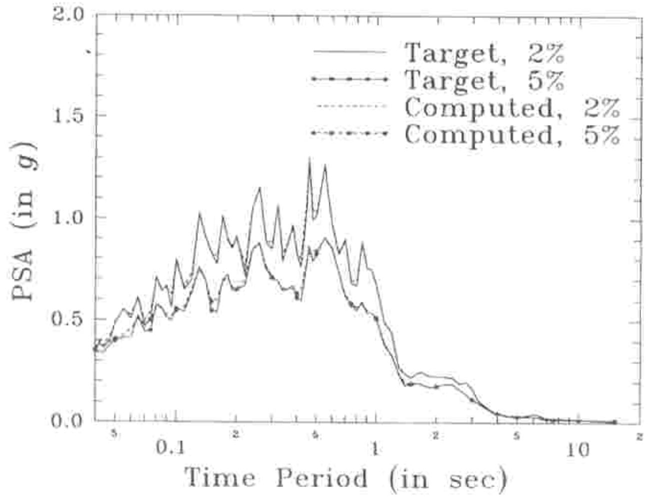

Fig. 1 – Comparison of Target and Computed Spectra for the Imperial Valley Earthquake Motion.

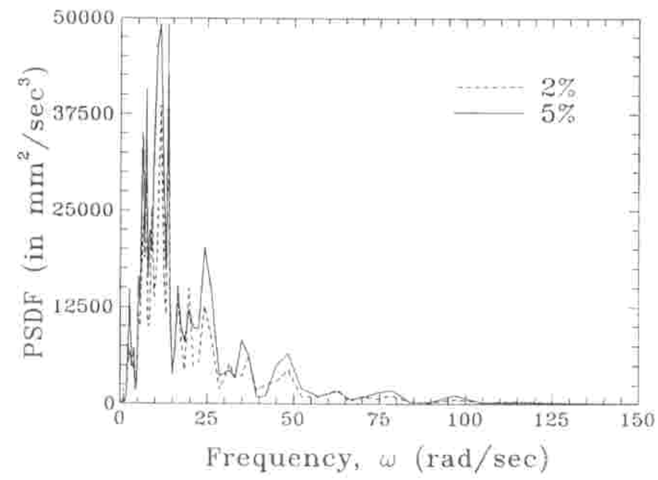

Fig. 2 – Comparison of PSDFs for 2% and 5% Damping Spectra in the Case of Imperial Valley Earthquake Motion.

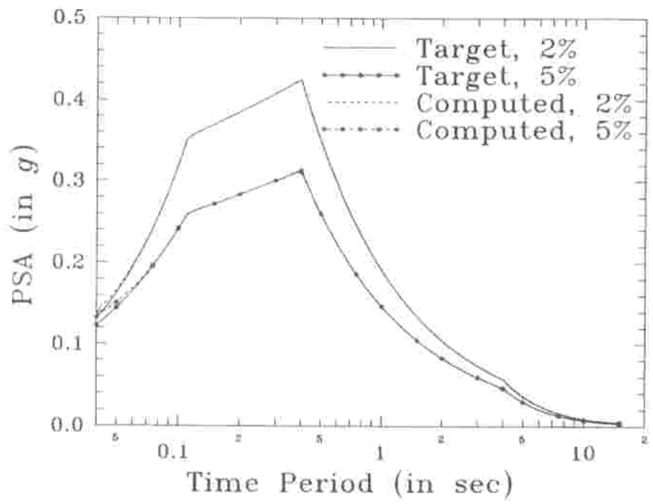

Fig. 3 – Comparison of the Target and Computed USNRC Spectra.

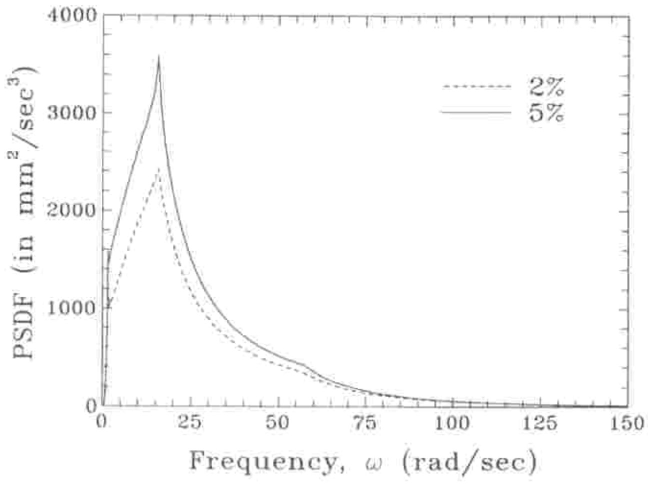

Fig. 4 – Comparison of PSDFs for the 2% and 5% Damping USNRC Spectra.

ent discrepancies in the amplitudes are possibly due to the independent smoothing and peak-broadening operations used during the construction of the design spectra for different damping ratios. These design spectra are inconsistent with each other as those do not correspond to the same ground motion process. In such cases, one can possibly use the envelope of PSDFs from various damping ratios and thus obtain a single PSDF characterizing the design ground motion for a wide variety of structural systems.

The PSDF generated by the proposed procedure may be directly used in the random vibration analyses of the structural systems. Here, it will be used for the generation of synthetic accelerograms which are consistent with a specified design spectrum in statistical sense.

3. Generation of Synthetic Accelerograms

It is common to synthesize the accelerograms by modeling the ground acceleration process, $\ddot{u}_g(t)$ as

$$\ddot{u}_g(t) = E(t)\, a(t) \tag{3}$$

where, $E(t)$ is a deterministic, slowly varying function of time employed to simulate the build-up and decay segments in the earthquake time-histories, and $a(t)$ is a stationary process given by

$$a(t) = \sum_i C_i \cos(\omega_i t + \phi_i) \tag{4}$$

In Eq. (4), ϕ_i's are the random phases uniformly distributed between 0 and 2π, and C_i's are the amplitudes related to the (one-sided) PSDF, $G(\omega)$ of $a(t)$ as

$$C_i = \sqrt{2G(\omega_i)\,\Delta\omega} \tag{5}$$

where, $\Delta\omega$ is the frequency interval centered at frequency, ω_i over which C_i is defined. To generate the synthetic time-histories, a suitable envelope function, $E(t)$ is chosen, and the PSDF, $G(\omega)$ is generally taken to be the spectrum-compatible PSDF corresponding to the given design spectrum, while assuming the ground motion and response processes to be stationary (and thus effectively assuming $E(t)$ to be identically equal to unity). The response spectrum of the time-history so generated is then forced to match with the design spectrum by iterative scaling of the coefficients, C_i's.

The time-histories generated by such a procedure are unrealistically uniform in the frequency content throughout the duration and also lack the desired ensemble variation produced by random phasing of the waves. Attempts to obtain the relative phase information using i) the group velocity curves by Lee and Trifunac (1985, 1987, 1989) and by Gupta and Joshi (1993) on the ideas of Wong and Trifunac (1979), and from ii) the phase spectrum of a recorded accelerogram as in Gupta and Joshi (1993), have led to the generation of accelerograms with far more realistic nonstationary characteristies with respect to the frequency content. Since those are based on the iterative scaling of the coefficients, C_i's for exact matching with the design spectrum, those still lack the variations due to random phasing. The approach of Gupta and Joshi (1993) based on the phase spectrum of a recorded accelerogram appears more practical with more and more recorded data, incorporating large variety of earthquake and site characteristics, becoming available to the earthquake engineering community (Lee et al. (1995)). Now, depending on the situation at hand, a suitable recorded accelerogram may be picked up and used to simulate the realistic phase characteristics for generating the synthetic data. Theoretically, it has been shown by Ohsaki (1979) and Nigam (1982) that the nature of the envelope function, $E(t)$, of a time-series can be assessed from the «group-delays» or the frequency derivative of the phase spectrum of the process. Further, in the particular case of seismic waves, the expected value of the phase derivative represents the average value of the group travel-time, thereby capturing the phenomenon of dispersion of seismic waves. In the limit of discretization, this implies that if we retain the phase differences between the waves of adjacent frequencies in the summation of Eq. (4) to be same as those obtained from a recorded accelerogram, we should, in principle, get the same nonstationary characteristics in the synthetic time-history. Hence, an ensemble of realistic accelerograms can be generated by producing different arrays of phases of the waves in the summation of Eq. (4) by keeping the phase differences between the adjacent frequency components to be same as those in the recorded accelerogram.

As mentioned above, iterative evaluation of C_i's leads to an ensemble of accelerograms with almost identical response spectra. The response spectrum ordinates of the members of an ensemble should in fact be statistically consistent with the design spectrum ordinates, instead of exactly matching with those. Hence, it is proposed to synthesize an ensemble of accelerograms by estimating Fourier amplitude spectrum and phase spectrum for one of these accelerograms, by randomly changing the phase spectrum such that the relative phases between the adjacent frequency components are preserved, and then by taking the inverse Fourier transform as

$$\ddot{u}_g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |U_g(\omega)|\, e^{i\phi(\omega)}\, e^{i\omega t}\, d\omega \tag{6}$$

where, $|U_g(\omega)|$ and $\phi(\omega)$ respectively are the Fourier amplitude and phase spectra of $\ddot{u}_g(t)$. In a nonstationary process, the amplitude and phase contents vary with time, and therefore, these two spectra contain the amplitude and phase information of the accelerogram only in a temporal «average» sense. It is proposed to take the phase spectrum, $\phi(\omega)$ same as the «average» phase spectrum of a recorded accelerogram as suggested by Gupta and Joshi (1993). The amplitude spectrum, $|U_g(\omega)|$ has to be however consistent with the specified design spectrum while being again an «average» spectrum. In case of a stationary process, $f(t)$, the PSDF may be related to the Fourier (amplitude) spectrum, $|F(\omega)|$ as

$$G_f(\omega) = \frac{\varepsilon[|F(\omega)|^2]}{\pi T} \qquad (7)$$

where, $\varepsilon[.]$ denotes the expectation of the term inside, and T is the duration of the process. For ground acceleration processes, however, the Fourier spectrum, $|U_g(\omega)|$ provides «average» information which cannot be related to the PSDF, $G_{\ddot{u}_g}(\omega)$ by taking frequency-invariant duration in the above relationship, whether $G_{\ddot{u}_g}(\omega)$ is the PSDF of the stationary segment of the process, or it is the PSDF of the «equivalent stationary» process as considered in the previous section. Several recent studies e.g., those by Trifunac and Westermo (1982), Novikova and Trifunac (1993a, 1993b, 1994), Trifunac and Novikova (1994), and Gupta (1994a), have considered the frequency-dependence of the ground motion duration. Here, it is proposed to use the frequency-dependent duration, $T_s(\omega)$ as suggested by Gupta (1994a) such that

$$G_{\ddot{u}_g}(\omega) = \frac{\varepsilon[|U_g(\omega)|]^2}{\pi T_s(\omega)}. \qquad (8)$$

We can use Eq. (8) to obtain the duration spectrum, $T_s(\omega)$ for the recorded accelerogram, the nonstationary characteristics of which are desired to be simulated in the synthetic accelerograms, by computing its PSDF through the proposed procedure in the previous section, and by evaluating and smoothing its Fourier spectrum. This duration spectrum can then be used again in Eq. (8) with $G_{\ddot{u}_g}(\omega)$ representing the PSDF compatible with the design spectrum as in the previous section, and $\varepsilon[|U_g(\omega)|^2]$ may be evaluated. This represents a reasonable estimate of the square of the «average» Fourier spectrum amplitudes of the synthetic accelerograms, and therefore its square root can be used with the «average» phase spectrum in the inverse Fourier transform. Using this logic, the estimate of $|U_g(\omega)|$ may be directly expressed as

$$|U_g(\omega)| = \sqrt{\frac{G_{\ddot{u}_g}(\omega)}{G_{\ddot{u}_{g,\text{rec}}}(\omega)}\varepsilon[|U_{g,\text{rec}}(\omega)|^2]} \qquad (9)$$

where, the sub-script, «rec» represents the quantities corresponding to the recorded accelerogram.

The proposed algorithm has been illustrated by generating four ensembles of 20 accelerograms each for the 2% damping USNRC spectrum with the recorded accelerograms being taken as (i) Imperial Valley Earthquake of 14 May, 1940, at El Centro Site, Comp: S60E, (ii) Parkfield Earthquake of 27 June, 1966 at Cholame, Shandon Site, Comp: Vertical, (iii) first aftershock of Whittier Narrows Earthquake of 1 October, 1987 at Santa Fe Springs, Comp: S60E, and (iv) Landers Earthquake of 28 June, 1992 at Santa Fe Springs, Comp: N30E. Whereas the first two accelerograms are representative of the majority of ground motion records taken at moderate epicentral distances for moderate magnitude earthquakes, the last two accelerograms respectively are very short (impulsive) and small magnitude earthquake, and very long (distant) and large magnitude earthquake records. Figs. 5-8 respectively show the comparison of the design spectrum in these cases with the response spectrum amplitudes of the generated accelerograms. These scatter plots clearly show that except in the case of the last accelerogram, the generated motions are in good agreement with the target spectrum in the mean sense, and moreover, there is sufficiently wide scatter in the response spectrum amplitudes of these motions as would be normally expected in a random process. The response spectrum amplitudes in case of the last accelerogram are consistently lower than the target spectrum for the short periods. This is so because this record is dominated by the long period waves, and hence, its duration spectrum, $T_s(\omega)$ is inconsistent with the duration spectrum one

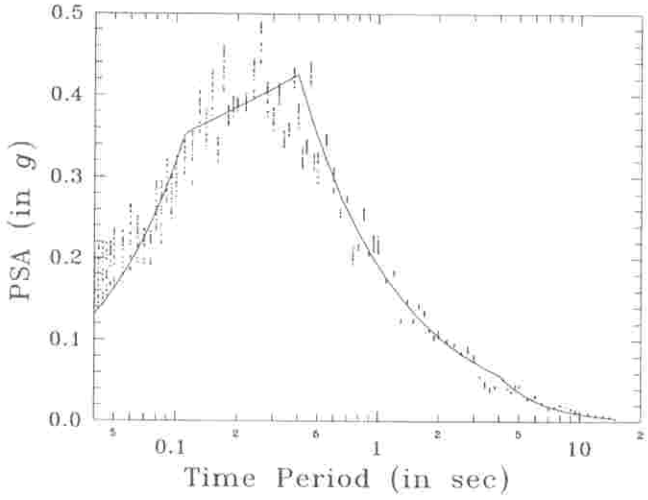

Fig. 5 – Comparison of the Target Design Spectrum with the Response Spectra of the Ensemble with Imperial Valley Motion Characteristics.

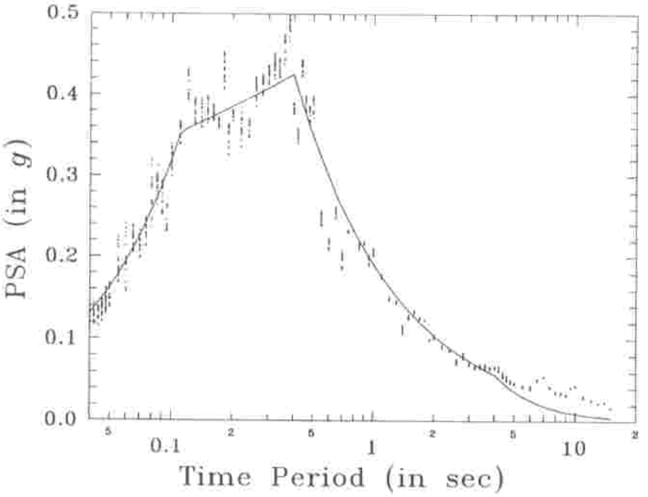

Fig. 6 – Comparison of the Target Design Spectrum with the Response Spectra of the Ensemble with Parkfield Motion Characteristics.

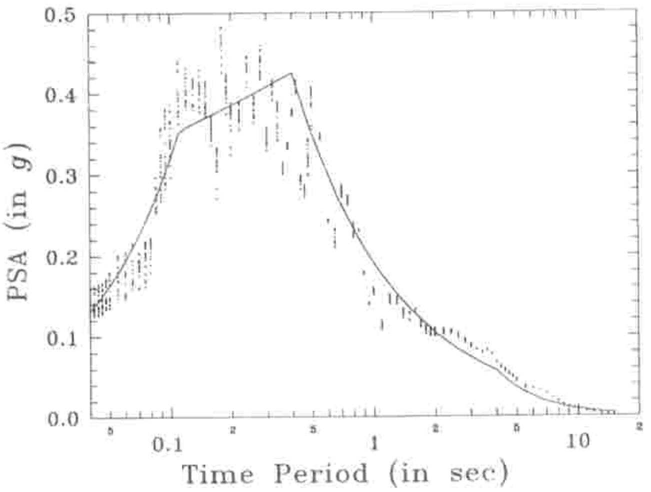

Fig. 7 – Comparison of the Target Design Spectrum with the Response Spectra of the Ensemble with Whittier Narrows Motion Characteristics.

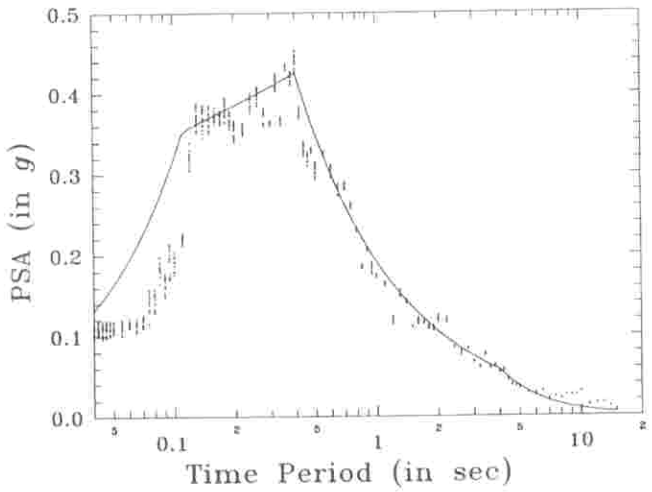

Fig. 8 – Comparison of the Target Design Spectrum with the Response Spectra of the Ensemble with Landers Motion Characteristics.

can physically obtain in case of a ground motion rich in high frequencies as well. It is therefore desirable to consider those recorded accelerograms only in the proposed algorithm for simulating nonstationary characteristics, which have «not too different» energy distributions from that in the design spectrum. Samples of the synthesized accelerograms in all the four cases are shown in Figs. 9-12. It should be noted that in the proposed approach, once the «average» Fourier amplitude and phase spectra are estimated from the design spectrum and the recorded accelerogram, the whole ensemble of accelerograms can be generated without any iterations. Thus, this approach is computationally efficient, besides being able to give a realistic ensemble of accelerograms. It is possible to generate the time histories of strains, curvatures and rotations also for a given design spectrum. For this, we can obtain the Fourier spectrum of the «equivalent stationary» process which

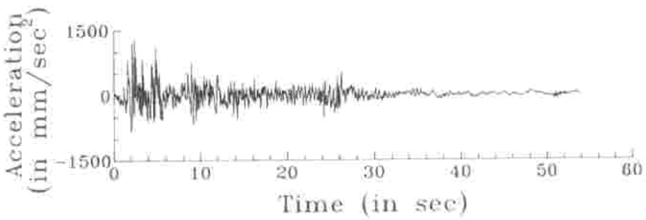

Fig. 9 – Sample Accelerogram from the Ensemble with Imperial Valley Motion Characteristics.

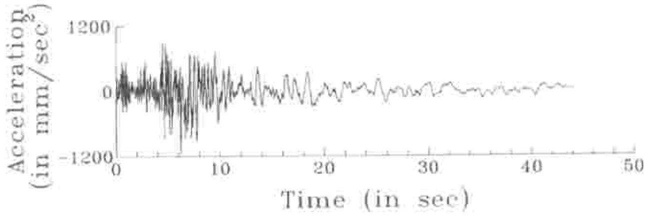

Fig. 10 – Sample Accelerogram from the Ensemble with Parkfield Motion Characteristics.

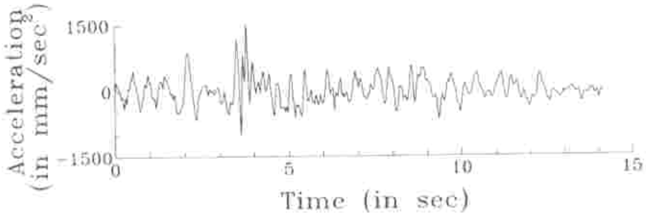

Fig. 11 – Sample Accelerogram from the Ensemble with Whittier Narrows Motion Characteristics.

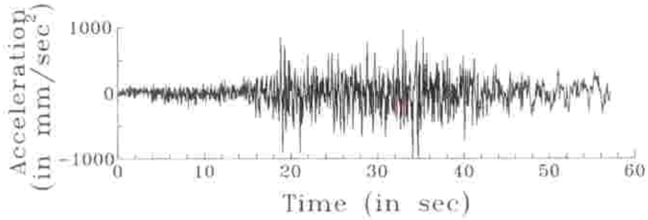

Fig. 12 – Sample Accelerogram from the Ensemble with Landers Motion Characteristics.

is compatible with the design spectrum, from its PSDF by using Eq. (7), and then use it in the methods of Lee and Trifunac (1985, 1987), Lee (1990), and Trifunac (1990).

4. Conclusions

A new method has been proposed in which the seismic excitation is characterized through the spectrum-compatible PSDF of an «equivalent stationary» motion. This is different from the earlier approaches, as it explicitly considers the response nonstationarity due to the sudden application of the excitation in computing the system response from the ground motion PSDF. The spectrum-compatible PSDF has thus acquired a new definition in which it is independent of the damping ratio of the response spectrum from which it is generated. The PSDF thus generated can be directly used in those stochastic analyses of linear systems which do not use the design spectra as the input.

The proposed spectrum-compatible PSDF has been used to synthesize an ensemble of accelerograms which are consistent with the design spectrum in a statistical sense. The phenomenon of dispersion of seismic waves has been conveniently simulated by using the phase information from the real accelerograms, and thus, the generated accelerograms are realistic in the temporal variation of the frequency content. Due to this, these accelerograms will be very useful for the stochastic Monte-Carlo studies of the nonlinear systems.

Appendix

The equation of motion of a single-degree-of-freedom (SDOF) oscillator subjected to excitation by base acceleration, $\ddot{u}_g(t)$ is given by

$$\ddot{x}(t) + 2\zeta_n\omega_n\dot{x}(t) + \omega_n^2 x(t) = -\ddot{u}_g(t) \qquad (A-1)$$

where, $x(t)$ denotes the relative displacement of the oscillator with respect to the base, and ζ_n and ω_n respectively represent the damping ratio and the natural frequency of the oscillator. Let the base acceleration be assumed as a zero mean, stationary Gaussian process. Then, the evolutionary power spectrum of the displacement response process of the oscillator is related to the power spectrum, $G_{\ddot{u}_g}(\omega)$ of the excitation process as

$$G_x(\omega, t) = |H(\omega, t)|^2 \; G_{\ddot{u}_g}(\omega) \qquad (A-2)$$

where,

$$H(\omega, t) = \frac{1}{(\omega_n^2 - \omega^2) + 2i\zeta_n(t)\omega\omega_n} \qquad (A-3)$$

represents the transient frequency response function of the oscillator in an approximate form. In Eq. (A-3), $\zeta_n(t) = 1/[1 - e^{-2\zeta_n\omega_n t}]$ is the fictitious time-dependent damping ratio of the oscillator.

The instantaneous spectral moments, bandwidth parameter, and mean rate of occurrence of peaks of the response process are obtained from $G_x(\omega, t)$ as

$$m_i(t) = \int_0^\infty \omega^i G_x(\omega, t)\,d\omega; \quad i = 0, 2, 4 \quad (A-4)$$

$$\varepsilon(t) = \left[\frac{m_0(t)m_4(t) - m_2^2(t)}{m_0(t)m_4(t)}\right]^{1/2} \qquad (A-5)$$

and

$$N(t) = \frac{1}{2\pi}(1 + \sqrt{1 - \varepsilon^2(t)}) \left[\frac{m_4(t)}{m_2(t)}\right]^{1/2}. \quad (A-6)$$

Using these parameters now, the largest peak value, x_{peak} of the response process is approximately obtained as

$$x_{peak} = \left[\frac{1}{T}\int_0^T \eta^2(t) m_0(t)\,dt\right]^{1/2} \qquad (A-7)$$

where, $\eta(t)$ represents the peak factor for the «expected» largest peak amplitude in a ficititious stationary process. This fictitious process has the same duration, T as the duration of the excitation process. Its bandwidth parameter and mean rate of occurrence of peaks are same as those of the response process at time, t, i.e. $\varepsilon(t)$ and $N(t)$. Calculations for this peak factor are carried out by using the formulations of Gupta and Trifunac (1988) and Gupta (1994b). The integral in Eq. (A-7) is easily evaluated by using any of the standard quadrature routines.

References

Caughey, T.K. and Stumpf, H.J. (1961). Transient response of a dynamic system under random excitaion, *J. Appl. Mech.*, ASME, **28E**, 563-566.

Gupta, I.D. and Trifunac, M.D. (1988). Order statistics of peaks in earthquake response, *J. Engg. Mech. Div.*, ASCE, 114, 1605-1627.

Gupta, I.D. and Joshi, R.G. (1993). On synthesizing response spectrum compatible accelerograms, *Euro. Earthq. Engg.*, (2), 25-33.

Gupta, I.D. (1994a). A probabilistic approach for estimating the response of multidegree-of-freedom structures, *Soil Dyn. Earthq. Engg.*, **13**, 79 87.

Gupta, V.K. (1994b). Higher order peaks in the seismic response of multistoried buildings, Report No. 9443, Dept. of Civil Engg., Indian Institute of Technology, Kanpur, India.

Iyengar, R.N. and Rao, P.N. (1979). Generation of spectrum compatible accelerograms, *Earthq. Engg. Str. Dyn.*, **7**, 253-263.

Kaul, M.K. (1978). Stochastic characterization of earthquakes through their response spectrum, *Earthq. Engg. Str. Dyn.*, **6**, 497-509.

Lee, V.W. (1990). Surface strains associated with strong earthquake shaking, *Strust. Engg./Earthq. Engg.*, JSCE, **7**, 187-194.

Lee, V.W. and Trifunac, M.D. (1985). Torsional accelerograms, *Soil Dyn. Earthq. Engg.*, **4**, 132-139.

Lee, V.W. and Trifunac, M.D. (1987). Rocking strong earthquake accelerograms, *Soil Dyn. Earthq. Engg.*, **6**, 75-89.

Lee, V.W. and Trifunac, M.D. (1989). A note on filtering strong motion accelerograms to produce response spectra of specified shape and amplitude, *Euro. Earthq. Engg.*, (2), 38-45.

Lee, V.W., Trifunac, M.D., Todorovska, M.I. and Novikova, E.I. (1995). Empirical equations describing attenuations of peaks of strong ground motion in terms of magnitude, distance, path effects and site conditions, Rep. No. 95-02, Dept. of Civil Engg., Univ. Southern California, Los Angeles, California.

Nigam, N.C. (1982). Phase properties of a class of random processes, *Earthq. Engg. Str. Dyn.*, **10**, 711-717.

Novikova, E.I. and Trifunac, M.D. (1993a). Modified Mercalli intensity and the geometry of the sedimentary basin as scaling parameters of the frequency dependent duration of strong ground motion, *Soil Dyn. Earthq. Engg.*, **12**, 209-225.

Novikova, E.I. and Trifunac, M.D. (1993b). Modified Mercalli intensity scaling of the frequency dependent duration of strong ground motion, *Soil Dyn Earthq. Engg.*, **12**, 309-322.

Novikova, E.I. and Trifunac, M.D (1994). Duration of strong ground motion in terms of earthquake magnitude, epicentral distance, site conditions and site geometry, *Earthq. Engg. Str. Dyn.*, **23**, 1023-1043.

O'Connor, J.M. and Ellingwood, B. (1987). Reliability of nonlinear structures with seismic loading, *J. Struet. Engg.*, ASCE, **113**, 1011-1028.

Ohsaki, Y. (1979). On the significance of phase content in earthquake ground motions, *Earthq. Engg. Str. Dyn.*, **7**, 427 439.

Preumont, A. (1980). A method for the generation of artificial earthquake accelerograms, *Nucl. Engg. Des.*, **59**, 357-368.

Preumont, A. (1984). The generation of spectrum compatible accelerograms for the design of nuclear power plants, *Earthq. Engg. Str. Dyn.*, **12**, 481-497.

Rosenblueth, E. and Elorduy, J. (1969). Response of linear systems to certain transient disturbances, *Proc. 4th World Conf. on Earthquake Engg.*, Santiago, Chile, I, 185.

Scanlan, R.H. and Sachs, K. (1974). Earthquake time-histories and response spectra, *J. Engg. Mech. Div.*, ASCE, **100**, 635-655.

Shrikhande, M. and Gupta, V.K. (1995). A generalized approach for the seismic response of structural systems, *submitted for publication*.

Spanos, P.D. and Vargas Loli, L.M. (1985). A statistical approach to generation of design spectrum compatible earthquake time-histories, *Soil Dyn. Earthq. Engg.*, **4**, 2-8.

Sundararajan, C. (1980). An iterative method for the generation of seismic power spectral density functions, *Nucl. Engg. Des.*, **61**, 13-23.

Trifunac, M.D. (1971). A method for synthesizing realistic strong ground motion, *Bull. Seism. Soc. Amer.*, **61**, 1739-1753.

Trifunac, M.D. (1990). Curvograss of strong ground motions, *J. Engg. Mech. Div.*, ASCE, **116**, 1426-1432.

Trifunac, M.D. (1995). Pseudo relative velocity spectra of earthquake ground motions at high frequencies, *Earthq. Engg. Str. Dyn.*, **24**, 1113-1130.

Trifunac, M.D. and Brady, A.G. (1975). A study on the duration of strong earthquake ground motion, *Bull. Seism. Soc. Amer.*, **65**, 581-626.

Trifunac, M.D. and Westermo, B.D. (1982). Duration of strong earthquake shaking, *Int. J. Soil Dyn. Earthq. Engg.*, **2**, 117-121.

Trifunac, M.D. and Novikova, E.I. (1994). State of the art review on strong motion duration, *Proc. 10th European Conf. on Earthquake Engg.*, Vienna, Austria, **1**, 131-140.

Udwadia, F.E. and Trifunac, M.D. (1974). Characterization of response spectra through the statistics of oscillator response, *Bull. Seism. Soc. Amer.*, **64**, 205-219.

Unruh, J.F. and Kana, D.D. (1981). An iterative procedure for the generation of consistent power/response spectrum, *Nucl. Engg. Des.*, **66**, 427-435.

Vanmarcke, E.H. and Gasparini, D.A. (1977). Simulated earthquake ground motions, *Trans. 4th Intnl. Conf. on SMiRT*, Paper K1/9, San Francisco, U.S.A., 1-12.

Wong, H.L. and Trifunac, M.D. (1979). Generation of artificial strong motion accelerograms, *Earthq. Engg. Str. Dyn.*, **7**, 509-527.