A note on IS: 875 - 1987 approach to along-wind building response

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A review of the gust factor method for the alongwind response of buildings has been presented and the provisions of Indian Standard Code, IS: 875 (Part 3)- 1987 based on this method have been critically examined. It is shown that the codal recommendations, based on the assumption of wind energy spectra being independent of height above the ground, may be too conservative. Also, the assumption of perfect correlation between the wind velocities on the windward and leeward faces of the building is seen to give conservative estimates.

Wind loading on structures is usually idealized as an equivalent static loading up to a specified height. Although this practice of disregarding the fluctuations in the wind velocity leads to simple calculations, it is far from being realistic for the analysis of tall structures. The development of modern materials and construction techniques has resulted in the emergence of new generation of tall structures that are very flexible, light in weight and are having low damping. The natural frequencies of these structures may lie in the same range as the average frequency of the powerful gusts and therefore, these structures are likely to experience large resonant motions under severe wind loads. This has led to the development of several methods for calculating the dynamic response of structures due to the wind gusts in the past three decades. These include i) the gust factor approach by Davenport1,2, Vellozzi and Cohen3, Vickery4, Simiu5,6, and Solari7, ii) the response spectrum techniques by Torkamani and Pramono8, and Solari9,10,11 and iii) the solutions in time domain by Vaiatis et al12, etc. Among these, the gust factor approach has been most popular due to its simplicity. It has been originally proposed by Davenport1 who did the pioneering work in estimating the alongwind response of single-degree-of-freedom (SDOF) systems while accounting for the random nature of wind and later, extended these concepts to the continuous systems2. His model was further modified by Vickery4 who introduced more flexibility in the spectrum of velocity fluctuations with respect to the choice of meteorological parameters like scale of turbulence etc. Vellozzi and Cohen3 introduced a reduction factor to account for the imperfect correlation between the pressures on the windward and leeward faces of the structure. Simiu5,6 proposed a different method to account for this alongwind correlation. He considered the reduction of energy in turbulent fluctuations with the increasing elevation and proposed a wind velocity spectrum which accounted for the height dependence in calculation of the spectrum energy in any frequency band. Using the same formulation as in Simiu6, Solari7 developed a closed form solution for the alongwind response by approximating the results of the numerical integrations by simple functions.

The Indian Standard code13 recommends the gust factor method for calculation of the dynamic wind loads on buildings. This method calculates the peak response of a structure to turbulent, gusty wind, and is essentially based on the works of Davenport1 and Vickery4. The recommendations of the code do not include the more recent developments in the calculation of gust factors5,7 regarding alongwind correlation and spectra of velocity fluctuations. The purpose of this study is to compare the gust factors for
tall buildings based on these developments with those specified by the code and then to critically examine the codal recommendations. A brief theoretical formulation of the gust factor approach has been included for completeness in the paper. It has been shown through a few example cases that the gust factor provisions in the code may lead to too conservative designs for the tall buildings.

FORMULATION OF THE GUST FACTOR METHOD

The gust factor method uses the statistical concepts of a stationary time series to calculate the response of a structure to a gusty wind with the following assumptions.

1. **The response of the structure is primarily in the fundamental mode.**

2. **The wind pressure due to skin friction drag on faces transverse to the wind direction is negligible.**

3. **The fundamental mode shape, \( \mu(z) \) at any height, \( z \), can be approximated by a straight line, \( \mu(z) = z/H \) where, \( H \) represents the height of the structure.**

![FIG.1 SCHEMATIC VIEW OF A BUILDING](image)

Consider a structure as shown in Fig.1 subjected to the action of a gusty wind. The wind pressure, \( P(x,z,t) \), at any point, \( (x,z) \) on the windward and leeward faces of the structure at any instant of time, \( t \) is given by:

\[
P(x, z, t) = \frac{1}{2} \rho c_p(x, z) U^2(x, z, t).
\]

(1)

where, \( \rho \) is the density of air, \( c_p(x,z) \) is the pressure coefficient, and \( U(x,z,t) \) is the wind velocity at time, \( t \). The velocity, \( U(x,z,t) \) can be expressed as the sum of a mean component, \( \bar{u}(x,z) \), invariant of time, and a fluctuating component, \( u(x,z,t) \).

\[
U(x, z, t) = \bar{u}(x, z) + u(x, z, t)
\]

(2)

It has been established that \( \bar{u}(x, z) \) is mainly a function of height, \( z \), and thus \( \bar{u}(x,z) = \bar{u}(z) \). Further it has been observed that the fluctuating component, \( u(x,z,t) \) is a random function of time and space. Hence, in order to determine the structure's response, the statistical properties of \( u(x,z,t) \) have to be determined. Now, using Eq. (2) in Eq. (1),

\[
P(x, z, t) = \frac{1}{2} \rho c_p(x, z) [\bar{u}^2(z) + 2u(x, z, t) \bar{u}(z) + u^2(x, z, t)]
\]

(3)

It has been found that the value of \( u(x,z,t) \) is small when compared with \( \bar{u}(z) \), particularly in the case of usually encountered high mean wind speeds. Hence in Eq. (3), the third term, \( u^2(x,z,t) \) can be neglected. Using this approximation, \( P(x,z,t) \) can be expressed as:

\[
P(x, z, t) = \bar{p}(x, z) + p(x, z, t)
\]

(4)

where, \( \bar{p}(x, z) = \frac{1}{2} \rho c_p(x, z) \bar{u}^2(z) \).

(5)

and

\[
p(x, z, t) = p c_p(x, z) \bar{u}(z) u(x, z, t)
\]

(6)

These are respectively called as the mean and the fluctuating pressures. From Eq. (6), it can be seen that \( p(x,z,t) \) is a linear function of \( u(x,z,t) \). Further, for a linear structure, the relationship between force and the response is also linear. Hence, the probability distributions of \( u(x,z,t) \), \( P(x,z,t) \), and the response will be similar. Further, it has been established that \( u(x,z,t) \) is a zero mean, stationary Gaussian process. For the heights very close to the ground level, there may be some departure from the Gaussian distribution. Thus, by knowing the variance of \( u(x,z,t) \), its probability distribution can be completely determined.

Now, the total wind load acting on the structure can be obtained by integrating the wind pressures acting on the windward and leeward faces of the structure. Further, it is assumed that the coefficient of pressure, \( c_p(x,z) \) assumes a constant value, \( c_w \) or \( c_l \) depending on windward or leeward side. Thus, the generalized mean wind load on the structure due to \( \bar{p}(x,z) \) is:

\[
\bar{F} = \int_{A_w} \bar{p}(x, z) \mu(z) dA + \int_{A_l} \bar{p}(x, z) \mu(z) dA
\]

(7)

where, \( A_w \) and \( A_l \) are the areas of the windward and leeward faces of the structure. The mean displacement response, \( \bar{y}(z) \) due to \( \bar{F} \) will be given by:

\[
\bar{y}(z) = \frac{\bar{F}}{(2\pi n_o)^2 M} \mu(z)
\]

(8)

where \( n_o \) is the fundamental frequency of the structure and \( M = \int_0^H m(z) \mu^2(z) dz \) is the modal mass in the funda-
mental mode, \( m(z) \) denoting the distribution of the system mass per unit length. For an approximate shape of \( \mu(z) \), e.g., a linear shape, \( M \) denotes the generalized mass. For a linear elastic structure, the fluctuating part of the response, \( y(z) \) is described by the spectral density function, \( S_y(z,n) \) where

\[
S_y(z,n) = \frac{\mu^2(z) H(n) H_1^2}{M^2} \int_{A} \mu(z_1) \mu(z_2) S_p(M_1, M_2; n) dA_1 dA_2
\]

(9)

In Eq. (9), \( A \) is the total windward and leeward area of the structure and \( S_p(M_1, M_2; n) \) is the cross-spectral density of the pressure fluctuations acting at points, \( M_1 \) and \( M_2 \) with elemental areas, \( dA_1 \) and \( dA_2 \), and with ordinates, \( x_1, z_1 \) and \( x_2, z_2 \) respectively. \( H(n) \) is the complex frequency response function defined by:

\[
H(n) = \frac{1}{4\pi^2 \left( n_0^2 - n^2 + 2i\zeta n_0 n \right)}
\]

(10)

\( \zeta \) being the critical damping ratio in the fundamental mode.

The function \( S_p(M_1, M_2; n) \) is, in general, a complex function. It has been found from the experiments\(^2\), however, that the imaginary part of this function is negligible. The real part of \( S_p(M_1, M_2; n) \) is given by the expression:

\[
S_p(M_1, M_2; n) = \rho^2 c_u c_w \mu(z_2) \mu(z_1) S_u^2(z_2, n) R(x_1, x_2, z_1, z_2; n)
\]

(11)

where,

\[
c = c_w^2 \text{ if points, } M_1 \text{ and } M_2 \text{ are on the windward side}
\]

\[
= c_l^2 \text{ if points, } M_1 \text{ and } M_2 \text{ are on the leeward side}
\]

\[= \tau(n) c_w c_l \text{ if points, } M_1 \text{ and } M_2 \text{ are on the opposite sides,}\]

\( \tau(n) \) is called the alongwind correlation factor, and \( S_u(z,n) \) is the spectrum of wind velocity fluctuations at \( z \). The function, \( R(x_1, x_2, z_1, z_2; n) \) is the square root of the coherence function of velocities at points, \( M_1 \) and \( M_2 \). Its value, as proposed by Davenport\(^2\), is:

\[
R(x_1, x_2, z_1, z_2; n) = \left[ \frac{-2(c_u (c_w - c_l)^2 + (c_u + c_l)^2 (x_1 - x_2)^2)}{\mu(z_1) + \mu(z_2)} \right]^{1/2}
\]

(12)

where \( c_u \) and \( c_l \) are the coefficients representing the decay of velocity correlations between two points on the surface of the building in the horizontal and vertical directions respectively.

Integrating on the windward and leeward faces, Eq. (9) leads to

\[
S_y(z,n) = \frac{\mu^2(z) H(n) H_1^2}{M^2} \left[ \int_{A} \mu(z_1) \mu(z_2) S_p(M_1', M_2'; n) dA_1 dA_2 + 2 \int_{A} \mu(z_1) \mu(z_2) S_p(M_1', M_2^2; n) dA_1 dA_2 + \int_{A} \mu(z_1) \mu(z_2) S_p(M_1^2, M_2'; n) dA_1 dA_2 \right]
\]

(13)

Denoting the first, second and third integrals by \( S_{ww}(n), S_{lw}(n), \) and \( S_{lf}(n) \), the above equation can be rewritten as:

\[
S_y(z,n) = \frac{\mu^2(z) H(n) H_1^2}{M^2} \left[ S_{ww}(n) + 2S_{lw}(n) + S_{lf}(n) \right]
\]

(14)

In the above derivation, it has been implicitly assumed that the velocity distribution at any time on the leeward side is same as that on the windward side. Results of the full scale measurements suggest, however, that the velocity fluctuations on leeward side are smaller than those on the windward side\(^5\). Thus, the use of Eq. (11) is conservative. Also, the results have shown that the local pressure distributions on the leeward side are strongly non-Gaussian in behaviour\(^6\). For simplicity in this study, however, the assumption of Gaussian nature of these pressure fluctuations will be retained. Now the variance of the response is given by \( \sigma_y^2(z) = \int_0^\infty S_y(z,n) dn \). Further, it has been shown\(^7\) that the peak fluctuating displacement response, \( y_p(z) \) is given by:

\[
y_p(z) = \overline{y}(z) \left[ 1 + g(vT) \frac{\sigma_y(z)}{\overline{y}(z)} \right]
\]

(15)

with \( g(vT) \), called as the peak factor, being given by:

\[
g(vT) = \left[ \frac{\sqrt{2lnT}}{\sqrt{2lnT}} + \frac{0.5772}{\sqrt{2lnT}} \right]
\]

(16)

Here, \( T \) is the total time over which the velocity has been averaged, and \( v \) is the average frequency of positive crossings.

\[
\nu = \left[ \int_0^\infty n^2 S_y(n) dn \right]^{1/2}
\]

(17)

The term, \( 1 + g(vT) \sigma_y(z)/\overline{y}(z) \) is called as the gust factor, and it measures the amplification in the mean system response due to the wind velocity fluctuations. Thus, taking the gust factor equal to unity is equivalent to treating the wind loading, as a static loading. Now, substituting the relevant values from Eqs. (8) and (14) in Eq. (15), the final expression for the gust factor becomes
\[ G(z) = 1 + g(vT)(2nw) \left\{ \frac{1}{2} \left[ \int H(n) \left( S_{\omega_1} (n) + 2S_{\omega_2} (n) + S_f (n) \right) \, dn \right] + \int \int \rho \nu \mu (x,z) \, dx \right\} \]

By adopting suitable expressions for the velocity spectra, alongwind correlation factor, mean wind profile and fundamental mode shape of the structure, and by using Eq. (18), the value of the gust factor, \( G(z) \) can be determined.

**Review of Codal Provisions for Gust Factors**

The Indian Standard code on wind loading\(^{13}\) prescribes the gust factor approach for the estimation of wind loads on the flexible structures. This is based on the following assumptions.

1. The value of alongwind correlation coefficient \( N(n) \) (see the definition of \( c \) in Eq. (11)) is equal to unity.
2. The spectrum of velocity fluctuations, \( S_u(z,n) \) (as used in Eq. (11)) is independent of height, \( z \).
3. The variation of mean wind speed with height follows the logarithmic law.\(^{19}\)

Here, the validity of the first two assumptions has been critically examined in the light of recent developments. For this, gust factors for four example buildings have been calculated as per the code, and compared with the values obtained by using improved expressions for the longitudinal wind velocity spectra and alongwind correlation. The dimensions of these example buildings are as follows:

<table>
<thead>
<tr>
<th>Example Building #</th>
<th>Breadth in m</th>
<th>Height in m</th>
<th>Depth in m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>120</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>150</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>365</td>
<td>60</td>
</tr>
</tbody>
</table>

Example buildings # 1 and 4 are the same as considered by Vickery\(^4\). The fact that these buildings are square in plan, is not likely to limit the applicability of the observations to follow in qualitative terms. These buildings are assumed to be located in a densely populated urban area, since this type of terrain is associated with more pronounced fluctuations in the wind velocity due to the increased surface roughness. This leads to greater contribution of the dynamic component in the structural response. The basic wind speed, as defined in the code, has been assumed to be 50 m/sec. Further, for the calculation of gust factors using the height-dependent velocity spectra, the value of friction velocity, \( u_* \), is needed. For urban areas, this value has been specified to be around 2.26 m/sec. and has been adopted in this study.

The expression for gust factor, \( G \) is conventionally taken as in Eq. (18). It has been shown by Vickery\(^4\) and Simiu\(^6\) that the value of peak factor, \( g(vT) \) varies in a narrow range of 3.4–3.7. It is the ratio of the standard deviation of the fluctuating part of the response to the mean response, i.e., \( \sigma_r(z) / \bar{y}(z) \), hereafter called as the gust ratio, which mainly causes variations in the gust factors.\(^5,20\) It depends on the different velocity spectra and the alongwind correlation factors. For a constant value of peak factor, \( g(vT) \), it can be seen that the variations in gust factors will be truly reflected by the variations in gust ratios. The variations in gust factors have thus been studied here by considering the variations in gust ratios only.

**Alongwind Correlation**

The I.S. code assumes that the wind pressures on windward and leeward faces of the structure are perfectly correlated. In other words, the value of \( N(n) \) in Eq. (11) is assumed to be unity. Experiments and full scale tests have indicated that the value on \( N(n) \) is less than unity\(^{19}\). For this study, the expression proposed by Vellozi and Cohen\(^3\) is adopted where,

\[ N(n) = \frac{1}{\xi} - \frac{1}{2\xi^2} \left( 1 - e^{-2\xi} \right) \quad (19) \]

with

\[ \xi = \frac{15.4n\Delta x}{U} \quad (20) \]

Here, \( U \) is the mean wind velocity at \( z = 2H/3 \), and \( \Delta x \) is the smallest of the dimensions \( B, H, \) and \( D, B \) being the breadth, \( H, \) the height, and \( D, \) the depth of the structure. It has been found that the values obtained from this expression agree fairly well with the experimental results.\(^{15}\)

For the purposes of comparison, the gust ratios have been computed for example buildings\(^# 3 \) and \( 4 \). The fundamental frequencies of the buildings have been considered to lie in the range from 0.05 to 0.5 Hz, and the damping ratio, \( \zeta \), has been taken as 0.01. Two cases have been considered, i) for correlation as per Eq. (19), and ii) for perfect correlation as in the code. Fig. 2 shows the results of gust ratios for these two cases. It is seen that the imperfect correlation (as in Eq. (19)) may cause a departure in the gust ratios from as low as 4 per cent to as high as 32 per cent, the values for perfect correlations being on the conservative side.

To avoid the overconservatism associated with the perfect correlation assumption while retaining the simplicity in calculations, Simiu\(^{16}\) has suggested the alongwind correlation factor to be approximated by a constant value of 0.2. However, for more accuracy, further calculations of gust ratios in this study will be based on the more complicated expression of alongwind correlation as in Eq. (19).
Velocity Spectra

The spectra, $S_u(z, n)$ of velocity fluctuations as considered in the code is given by:

$$S_u(z, n) = 4\kappa \frac{u_1^2 x^2}{n (1 + x^2)^{\frac{5}{3}}} \tag{21}$$

where $x = \frac{nL(H)}{\bar{u}(H)} \tag{22}$

and $\kappa$ is a drag coefficient which depends on the surface roughness and is corresponding to the mean wind velocity at $10 \text{ m}$ height, $\bar{u}$, $L(\cdot)$ is a turbulence scale which depends on the terrain and height, $H$ of the structure; and $\bar{u}(H)$ is the mean wind velocity at the top of the structure. This expression is independent of $z$ and it was empirically suggested by Davenport\textsuperscript{21} on the basis of several wind measurements over terrains of different roughness and at different heights. Davenport\textsuperscript{21} showed that different velocity spectra will be obtained for different values of $z$ and the spectra at lower heights are likely to have greater amplitudes for a given frequency, $n$. However, he neglected these variations in the spectral amplitudes with height assuming those to be of the same order as that of the errors in the estimation of the spectra, and obtained the above expression by averaging the spectra at various heights. Simiu\textsuperscript{5} has shown that this simplification is associated with significantly large errors in the calculations of structural response, and has proposed the following height-dependent spectra:

$$S_u(z, n) = \frac{200f}{(1 + 50f)^{\frac{5}{3}}} \frac{u_1^2}{n} \tag{23}$$

where $f = \frac{0.5z}{\bar{u}(z)} \tag{24}$

$u_1$ is the friction velocity and $\bar{u}(z)$ is the mean wind velocity at height, $z$ above the ground. This characterisation of spectra has, on an average, been shown to agree very well with the experimental data\textsuperscript{5} in the high frequency range with $f > 0.2$. In the low frequency range, however, the velocity spectra cannot be described by a single expression as above. Since, most of the buildings have their natural frequencies in the range $0.1 < n < 1.0$, the spectra as in Eq. (23) can be used for the entire frequency range, including the low frequencies, without causing appreciable errors.

The gust ratios have been calculated for all the four example buildings with varying natural frequencies for two cases: i) the spectra is height-independent as in Eq. (21), and ii) the spectra is height-dependent as in Eq. (23). Two damping ratios, $\zeta = 0.01$ for steel structures and $\zeta = 0.016$ for concrete structures have been considered. These results have been shown in the form of curves showing variation of gust ratios with the fundamental frequency of the building (Figs. 3-6).

It is seen from the figures that the dependence of gust ratios on the natural frequency is almost identical in both the approaches. However, the gust ratios as estimated by using the height-independent spectrum are much higher than those based on the Eq. (23), especially for the long period buildings. As shown in Fig. 7, this is due to the fact that the height-independent spectrum overestimates the energy in velocity fluctuations in the relevant frequency range. For very stiff structures, the observed differences become much smaller. Further, the gust ratios approaches constant values beyond certain frequencies thus indicating the response to be pseudo-static. The frequencies beyond which the pseudo-static response is obtained, are governed by the band of energy distribution in the force spectrum, which shifts towards the lower frequencies for the buildings.
with larger dimensions, for a given velocity spectrum. Larger is the building, smaller will be the value of this frequency. This happens due to the decreased velocity correlation between any two points at higher frequencies. For any given building, for the same reason, convergence to the constant value is obtained at the higher values in the case of height-independent spectrum (Fig. 3). From the above, it follows that the effects of gusts are likely to be important in most of the cases when the natural frequency of the structure does not exceed 0.6 Hz. In this range, the over-conservatism resulting from the use of height-independent spectra may be as much as 60 per cent for typical tall buildings with time periods close to 10 seconds (Fig. 6). This, however, gets reduced for the taller buildings as can be seen from the comparison of gust ratios for different buildings with the same fundamental frequency. Further, as expected, all of these figures show reduced gust ratios for the higher damping ratios. The code provisions seem to properly account for these effects.

CONCLUSIONS

The dynamic alongwind response of buildings using the gust factor method has been studied. A review of this method as adopted by the Indian Standard code\textsuperscript{13} has shown that the recommended gust factors are too conservative due to i) the use of height-independent velocity spectra, and ii) assuming perfect correlation between the pressures on windward and leeward faces of the building. It has been observed that the gust factors approaches constant values for stiffer buildings, thus indicating the response to be pseudo-static. The frequency beyond which the pseudo-static response is obtained, is governed by the band of energy distribution in the force spectrum. Larger the building, smaller is the value of this frequency.

REFERENCES


