A Generalized Approach for the Seismic Response of Structural Systems

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SUMMARY – Most approaches presently used for the seismic analysis of linear structural systems are response spectrum-based due to the inherent simplicity in the characterization of design ground motions through response spectra and in the description of the peak structural response in terms of these ordinates. For complex systems where such a description is not so obvious, one may have to use much more complicated nonstationary random vibration analyses employing appropriate envelope functions for the ground motions. The purpose of this paper is to present an alternative stochastic approach which is simpler than these nonstationary analyses, and is more generalized than the response spectrum based procedures. For this purpose, the computation of peak factors using the theory of order statistics has been generalized to the nonstationary processes, and the earthquake ground motion has been modeled as an 'equivalent stationary' excitation process. The design ground motion can thus be characterized in terms of a probabilistic estimate of the peak ground acceleration and the Fourier spectrum in the 'time-averaged' sense. The proposed approach has been illustrated in case of simple SDOF and MDOF systems through three example excitations.

1. Introduction

The response spectrum based procedures are widely used for the seismic analysis of civil engineering structures, e.g., multistoried buildings, nuclear power plants, bridges etc. Many studies in the past have significantly contributed to the development of these procedures. These include those by Singh and Chu (1976), Der Kiureghian (1981), Wilson et al. (1981), Singh and Mehta (1983), Amini and Trifunac (1985), Gupta and Trifunac (1990b), Der Kiureghian and Nakamura (1993), Gupta (1994b) for the fixed-base buildings, those by Gupta and Trifunac (1990a), Gupta and Trifunac (1991) for the flexible base buildings, and those by Yamamura and Tanaka (1990), Berrah and Kausel (1992), Der Kiureghian and Neuenhofer (1992), Heredia-Zavoni and Vanmarcke (1994) for the multi-support systems. While some of these studies have utilized the white noise or filtered white noise models of excitation, others required the use of either the Fourier spectra of the response spectrum compatible power spectral density functions (PSDFs) to characterize the earthquake ground motion. Essentially all of these studies have attempted to model the peak seismic responses of a multi-degree-of-freedom (MDOF) system in terms of the peak seismic responses of several single-degree-of-freedom (SDOF) systems. Though such an approach may be convenient from the practising engineer's point of view, this may not be generalized to all practical situations. For example, the response spectrum studies accounting for the soil-structure interaction effects invariably ignore the kinematic interaction [e.g., see Gupta and Trifunac (1991)]. It has been shown by Betti et al. (1993) that in case of structures with massive, embedded foundations, e.g., suspension bridges, the effects of kinematic interaction can be quite significant. In fact, a mixed type of boundary value problem has to be solved in frequency domain in such cases to compute the actual foundation input motion which may be quite different from the free field ground motion. Similarly, response spectrum based techniques account for the incoherence between the support motions of a multi-support system only in an approximate way. Such problems can be tackled more accurately by working in the frequency domain and by directly accounting for the nonstationarity in excitation and response, e.g., as in Shinozuka and Yang (1972), Deb Chaudhary and

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Gasparini (1980), Madsen and Krenk (1983); Igusa (1987), Bucher (1988), Hou (1990) etc. These non-response spectrum based studies are however cumbersome for practical applications and have also lacked generality due to modeling of nonstationarity in the ground motion through particular modulating functions.

The purpose of this study is to develop a simpler and yet reasonably accurate approach to estimate the stochastic response of linear structural systems subjected to earthquake excitations. For this purpose, the excitation has been modeled as a finite duration segment of a stationary random process such that this is equivalent to the given nonstationary process in terms of the largest peak value, i.e. the peak ground acceleration. The nonstationarity in response due to finite operating time has been accounted for exactly by considering the transient transfer function, and the peak factors have been obtained by generalizing the order statistics formulation of Gupta and Trifunac (1988) to the nonstationary processes. The proposed approach has been illustrated by computing the response spectra and the seismic response of an example multistoried building for a set of example earthquake ground motions.

2. Formulation of the Proposed Approach

(i) Brief Review

Let us first consider the response of a linear dynamical system to a suddenly applied stationary excitation. It has been shown by Corotis et al. (1972) that such a response process is evolutionary in nature with the response gradually building up with time in the beginning. The evolutionary power spectral density, $G_x(\omega, t)$ of the systems response, $X(t)$ is then given by [Corotis and Vanmarcke (1975)]

$$G_x(\omega, t) = |H(\omega, t)|^2 \, G_f(\omega) \tag{1}$$

where, $G_f(\omega)$ denotes the PSDF of the excitations, $F(t)$, and $H(\omega, t)$ represents the transient frequency response function of the system. This relates the system response to the input excitation at time, t, and is given by the truncated Fourier transform of the unit impulse response function of the system. As t goes to infinity, $H(\omega, t)$ approaches the steady-state frequency response function, $H(\omega)$, and the response, $X(t)$ tends to become a stationary process with spectral density, $G_x(\omega)$. Peak amplitudes in stationary processes can be estimated by using the moments of the spectral density of the process about the origin. There have been two popular approaches for estimating the largest peak in a stationary process. One is based on the solution of first passage problem as in Vanmarcke (1975), and the other is based on the theory of order statistics as in Gupta and Trifunac (1988). The latter formulation is more generalized since it can be used to compute not only the largest peak in a process, but also the higher order peaks. The salient point of this approach are given in the Appendix. Since this approach is formulated for the stationary processes, this requires all the local maxima in a random process to have an identical probability distribution. This may work reasonably well also when the processes are nonstationary for small fractions of their total durations in the beginning and at the end, e.g., as in the case of the response of stiff structures to the long duration excitations. In the case of flexible and/or lightly damped systems, the transient phase, in which the response process gradually builds up, may be appreciably long, thereby leading to a very small of nonexistent phase of stationarity. In such processes, the local maxima occurring at different instants of time would correspond to different probability distributions, and thus, it will not be possible to use the formulation of Gupta and Trifunac (1988) even for the approximate extimations.

(ii) Peak Factors for Nonstationary Processes

In the case of an evolutionary response process, $X(t)$, the instantaneous bandwidth parameter at time, t, can be defined in analogy with the stationary case as [Cartwright and Longuet-Higgins (1956)]

$$\varepsilon(t) = \left[\frac{m_0(t)m_4(t) - m_2^2(t)}{m_0(t)m_4(t)}\right]^{1/2} \tag{2}$$

where, $m_0(t)$, $m_2(t)$, and $m_4(t)$ are the instantaneous spectral moments given by

$$m_i(t) = \int_0^\infty \omega^i G_x(\omega, t)\mathrm{d}\omega; \quad i = 0, 2, 4. \tag{3}$$

The corresponding probability density function of the maxima of $X(t)$ at time, t, may be expressed as

$$p(\eta) = \frac{1}{\sqrt{2\pi}}\left[\varepsilon(t)e^{-\eta^2/2\varepsilon^2(t)} + \right.$$
$$\left. +(1-\varepsilon^2(t))^{1/2}\eta e^{-\eta^2/2}\int_{-\infty}^{\eta(1-\varepsilon^2(t))^{1/2}/\varepsilon(t)} e^{-x^2/2}\mathrm{d}x\right] \tag{4}$$

where, η is the normalized amplitude of maxima with respect to the root-mean-square (r.m.s.) value, $x_{\mathrm{rms}}(t)$ $(= \sqrt{m_0(t)})$.

For the purpose of design, however, the peak value should be computed for the random process, $|X(t)|$ instead of $X(t)$. Following the rationale of the formulation by Gupta (1994a) for the stationary processes, the instantaneous probability density in the process, $|X(t)|$ may be given by

$$\tilde{p}(\eta) = \frac{1}{\sqrt{2\pi}}\left[2\varepsilon(t)e^{-\eta^2/2\varepsilon^2(t)} + \right.$$
$$\left. +(1-\varepsilon^2(t))^{1/2}\eta e^{-\eta^2/2}\int_{-\eta(1-\varepsilon^2(t))^{1/2}/\varepsilon(t)}^{\eta(1-\varepsilon^2(t))^{1/2}/\varepsilon(t)} e^{-x^2/2}\mathrm{d}x\right]; \tag{5}$$

In a similar way, the instantaneous mean of occurrence of peaks in the process, $|X(t)|$ may be defined as [Cartwright and Longuet-Higgins (1956), Gupta (1994a)]

$$N(t) = \frac{1}{2\pi}(1 + \sqrt{1 - \varepsilon^2(t)})\left[\frac{m_4(t)}{m_2(t)}\right]^{1/2}. \qquad (6)$$

To estimate the largest peak in an evolutionary random process, $X(t)$, with time-dependent bandwidth parameter and mean rate of peak occurrence, let us consider the temporal mean-square value of one of its realizations as given by

$$\bar{x}^2_{\rm rms} = \frac{1}{T}\int_0^T x^2(t)\mathrm{d}t \qquad (7)$$

where, T is the total duration of the process.

Assuming this realization to be composed of finitely many, say M, stationary segments of duration, ΔT, the above expression can be written as

$$\bar{x}^2_{\rm rms} = \frac{1}{MT}\sum_{m=1}^{M} M \int_{(m-1)\Delta T}^{m\Delta T} x^2(t)\mathrm{d}t \qquad (8)$$

Taking expectation of both the sides in the above equation, we obtain

$$E[\bar{x}^2_{\rm rms}] = \frac{1}{MT}\sum_{m=1}^{M} \left(M\int_{(m-1)\Delta T}^{m\Delta T} E[x^2(t)]\mathrm{d}t \right) \qquad (9)$$

Inside the parentheses above, there are M integrals of same magnitude involving the process, $X(t)$, between $t = (m-1)\Delta T$ and $t = m\Delta T$. We can alternatively consider a fictitious stationary process, say $X_m(t)$, of duration, T, which is composed of M realizations of the process, $X(t)$, in this interval. Thus, Eq. (9) may be expressed as

$$E[\bar{x}^2_{\rm rms}] = \frac{1}{MT}\sum_{m=1}^{M} \int_0^T E[x^2_m(t)]\mathrm{d}t \qquad (10)$$

For very small ΔT, $E[x^2_m(t)]$ may be considered as time-invariant over the interval of the integral, and may be represented by $x^2_{m,\,\rm rms}$, the mean square value of $X_m(t)$ at the mid-point of the interval, $t = (m-1)\Delta T$ to $t = m\Delta T$. Eq. (10) thus becomes

$$E[\bar{x}^2_{\rm rms}] = \frac{1}{M}\sum_{m=1}^{M} x^2_{m,\,\rm rms} \qquad (11)$$

Since $E[\bar{x}^2_{\rm rms}]$ is a measure of 'average' mean square value of the nonstationary process, $X(t)$, it can be related to the larges peak value, $x_{\rm peak}$, in the process through a peak factor, say η, and thus,

$$x^2_{\rm peak} = \eta^2 E[\bar{x}^2_{\rm rms}] = \frac{\eta^2}{M}\sum_{m=1}^{M} x^2_{m,\,\rm rms} \qquad (12)$$

The mean square value, $x^2_{m,\,\rm rms}$ can also be related to the largest peak, $x_{m,\,\rm peak}$ through the peak factor, η_m where, η_m is obtained by using the formulation of Gupta and Trifunac (1988) for the stationary process, $X_m(t)$. Eq. (12) thus becomes

$$x^2_{peak} = \frac{1}{M}\sum_{m=1}^{M} \frac{\eta^2}{\eta^2_m} x^2_{m,\,\rm peak} \qquad (13)$$

It may be noted that the peak factors, η_m as well as ω are computed for the same process duration, T, and that the total number of peaks for the M different (fictitious) stationary processes will change only slightly from a higher value for $X_1(t)$ to a lower value for $X_2(t)$, $X_3(t)$, ... as the system response slowly evolves from a broad-band process to a narrow-band one. Since the peak factor for a process depends little on its bandwidth parameter, except when the process is highly broad-banded, it may be expected that the ratio, η/η_m will be close to unity for all the segments. It will be smaller than unity for some segments, and larger than unity for other. Thus, assuming the ratio uniformly equal to unity for all m may lead to only small deviations from the exact value. Thus, the peak value, $x_{\rm peak}$, of the process, $X(t)$, may be estimated from

$$x_{peak} = \left(\frac{1}{M}\sum_{m=1}^{M} x^2_{m,\,\rm peak}\right)^{1/2} \qquad (14)$$

Letting $\Delta T \to 0$, Eq. (14) becomes

$$x_{\rm peak} = \left(\frac{1}{T}\int_0^T x^2_{\rm peak}(t)\mathrm{d}t\right)^{1/2} \qquad (15)$$

where, $x_{\rm peak}(t)$ is the peak value of a fictitious stationary process of duration, T and with the mean square value same as the instantaneous mean square value of the nonstationary process, $X(t)$ at time, t. Similarly, by considering the peak values of the 'absolute' fictitious processes, desired peak amplitudes in the process, $|X(t)|$ may be estimated. Eq. (15) may be seen as a generalization of the formulation by Gupta and Trifunac (1988) and Gupta (1994a) in an approximate form. For stationary processes, $x_{\rm peak}(t)$ is invariant of time and thus, the right hand side becomes identically equal to the left hand side. Further, since the evolutionary spectrum of the system response is a smooth function of time, standard quadrature routines can be used to evaluate the integral in Eq. (15).

(iii) Equivalent Stationary Excitation

The procedure described in the proceding sub-sections may be used for estimating the peak responses of a linear system subjected to a suddenly applied stationary excitation. Seismic ground acceleration records, however, are nonstationary as those exhibit a building-up segment, a segment of sustained intense shaking, and a decaying tail portion. Different earthquake records exhibit widely different patterns of these three segments, and thus, it is not possible to specify a unique modulating function which would be representative of the nonstationary characteristics of all possible or a majority of earthquake records. Employing a modulating function to describe nonstationarity as has been done in several past studies, also makes the analysis very complicated, and yet fails to account for the changing frequency composition even in an approximate form. To propose a simple, approximate, yet more generalized procedure which does not depend on the form of nonstationarity in the excitation, it is sought to replace the given nonstationary motion by an equivalent stationary motion. It is proposed to have equivalence in terms of the 'average' energy distribution, strong motion duration, and intensity of shaking of the given motion.

Let $\ddot{u}_g(t)$ denote the ground acceleration process with the strong motion duration, T, defined as the 90% duration of Trifunac and Brady (1975). An estimate of the 'average' power spectrum of this acceleration process may be obtained from

$$\tilde{G}_{\ddot{u}_g}(\omega) = \frac{E[|\ddot{U}_g(\omega)|^2]}{\pi T} \tag{16}$$

where, $\ddot{U}_g(\omega)$ is the Fourier transform of $\ddot{u}_g(t)$. We can consider the equivalent stationary motion to have duration, T, and the same energy distribution as in $\tilde{G}_{\ddot{u}_g}(\omega)$. To make it compatible with the nonstationary process as regards the intensity of shaking, $\tilde{G}_{\ddot{u}_g}(\omega)$ may be uniformly scaled so as to lead to the peak ground acceleration, a_{max}, of desired level of confidence on taking the spectral moments and multiplying the r.m.s. value with the peak factor. Thus, if $\bar{a}_{max}$ is the largest peak of the stationary process with the PSDF given by Eq. (16) and same level of confidence as of a_{max},

$$\eta_g = \frac{a_{max}}{\bar{a}_{max}} \tag{17}$$

represents the required scaling factor, and

$$G_{\ddot{u}_g}(\omega) = \frac{E[|\eta_g \ddot{U}_g(\omega)|^2]}{\pi T} \tag{18}$$

represents the PSDF of the equivalent excitation. It may be emphasized that the energy distribution represented by this PSDF is only an 'average' energy distribution in the ground motion process. This may in fact be inaccurate for those frequencies which are present only in the small segments of the ground motion. The resulting inaccurracies in the response calculations may be substantial, depending on the system frequencies, for a ground motion with short stationary phase. A more accurate approach would involve taking the Fourier transform of the truncated ground motion as proposed by Udwadia and Trifunac (1974) or considering frequency-dependent duration of the ground motion (Gupta (1994)]. However, for simplicity in the present study, it is assumed that the 'average' distribution is adequate to describe the 'equivalent stationary' motion. A similar procedure based on the equivalence of mean square energy has also been suggested by Lin and Tyan (1986) for the equivalent stationary excitations and this has been found to be adequate for the response of most linear systems.

3. Application to Response Spectra Calculations

Let us consider a SDOF oscillator subjected to ground acceleration, $\ddot{u}_g(t)$. The equation of motion for the relative displacement response of the oscillator can be written as

$$\ddot{x}(t) + 2\zeta_n\omega_n\dot{x}(t) + \omega_n^2 x(t) = -\ddot{u}_g(t) \tag{19}$$

where, ω_n and ζ_n are the natural frequency and critical damping ratio of the oscillator, $x(t)$ represents the relative displacement of the oscillator mass, and an overdot denotes the differentiation with respect to time. Now, with reference to Eq. (1), the transient response function relating the response, $x(t)$ to the input acceleration, $\ddot{u}_g(t)$ may be expressed as [Vanmarcke (1977)]

$$H(\omega, t) = \frac{1}{(\omega_n^2 - \omega^2) + 2i\zeta_n(t)\omega\omega_n} \tag{20}$$

with $\zeta_n(t) = \zeta_n/[1 - e^{-2\omega_n\zeta_n t}]$ being the fictitious time-dependent damping ratio of the oscillator. This form of the frequency response function is an approximate form similar to that of the steady-state transfer function with the fictitious damping replacing the actual oscillator damping. The peak oscillator response can now be computed by using this frequency response function and the PSDF of equivalent stationary motion [as in Eq. (18)] in Eq. (1) to obtain the evolutionary PSDF of the response and then by considering the peak values for fictitious stationary process through Eq. (15).

To illustrate the proposed procedure, response spectrum ordinates for Pseudo Spectral Acceleration (PSA) response have been computed for a set of three accelerograms: (*i*) recorded S00E component of Imperial Valley earthquake of May 18, 1940 at El Centro, having the total duration of 53.74 sec and the strong motion duration of 24.42 sec, (*ii*) synthetic accelerogram for the horizontal component of Michoacan earthquake, 1985 at Mexico City, having the total duration of 80.96 sec and the strong motion duration of 46.44 sec [as in

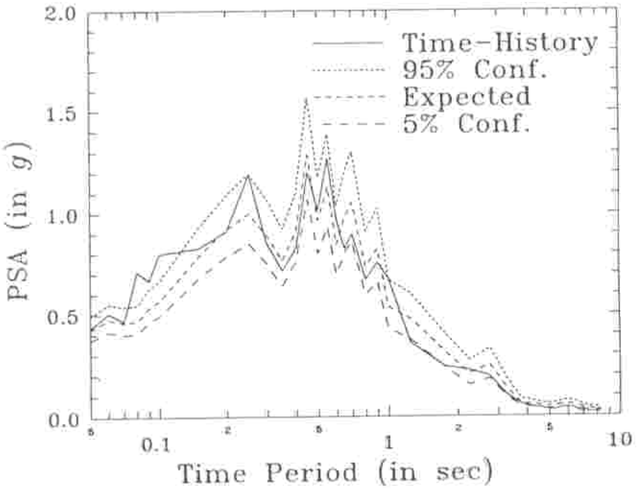

Fig. 1 – Comparison of the Response Spectra for the Imperial Valley Earthquake Motion.

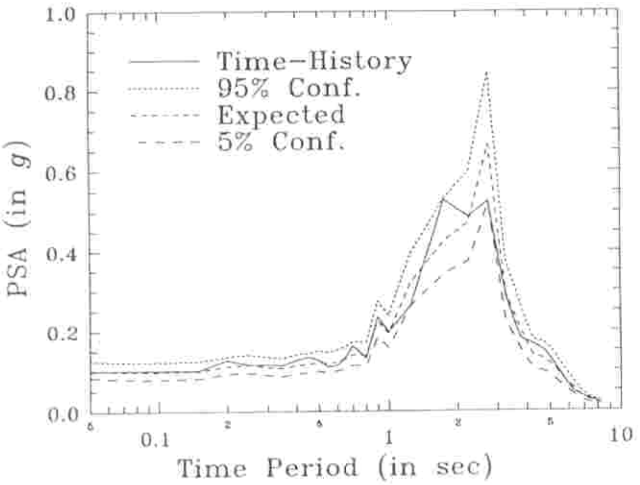

Fig. 2 – Comparison of the Response Spectra for the Michoacan Earthquake Motion.

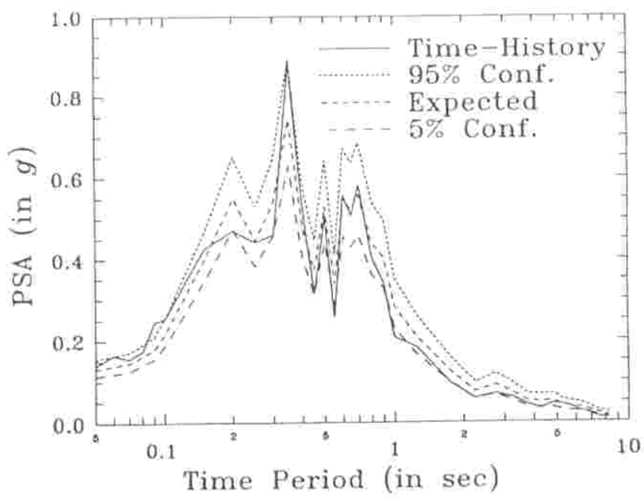

Fig. 3 – Comparison of the Response Spectra for the Kern County Earthquake Motion.

Gupta and Trifunac (1989)], and (*iii*) recorded N21E component of Kern County earthquake of July 21, 1952 at Lincoln Taft School tunnel, having total duration of 54.36 sec and the strong motion duration of 30.54 sec. The strong motions corresponding to the Imperial Valley and Kern County earthquakes are broad-banded whereas the motion corresponding to the Michoacan earthquake is a narrow-banded motion. For the results in this study, it is assumed that the Fourier spectra based on the single records represent the ensemble behavior and that the observed peak ground accelerations in these records are the expected values of the largest peaks in the considered ground acceleration processes. Further, the oscillator damping has been assumed to be 2% of the critical damping in each case. The ordinates of the probabilistic spectra have been computed for the 5% and 95% confidence levels in addition to the expected values. These spectra have been compared with those obtained from the time-history analyses as shown in Figures 1-3. It is seen that the 'expected' spectra computed by the proposed procedure are in good agreement with the spectra obtained from the time-history analyses. The time history results are also well bounded by the 5% and 95% estimates on both sides, except for a few narrow bands of natural periods in the case of Imperial Valley and Kern County earthquakes. These deviations may be due to the 'time averaging' of the energy distributions in obtaining the 'equivalent stationary' motions. In the case of Michoacan earthquake, this approximation works better, perhaps due to the fact that the input motion is narrow banded and therefore, there is little variation in the frequency-dependent duration over the entire band of significant energy.

The probabilistic estimates in Figures 1-3 have been obtained by assuming 'equivalent stationary' motions to be of the 90% energy durations as suggested by Trifunac and Brady (1975). However, it is seen that these estimates undergo little changes, only at large periods, if different durations are considered. Thus, it may be sufficient to have just an approximate estimation of the stationary duration in using the proposed approach. Further, it may be observed that the application of the proposed approach in obtaining the response spectrum ordinates is, in essence, similar to the method proposed by Udwadia and Trifunac (1974). The proposed formulation however considers the effects of response nonstationarity more accurately, and is based on more rational estimation of the peak factors.

4. Application to MDOF Systems

Let us now consider a n-degree-of-freedom, classically damped system subjected to ground acceleration, $\ddot{u}_g(t)$, at its base. Using the normal mode approach, any response quantity of interest, say $z(t)$, in this system can be expressed as [Clough and Penzien (1993)]

$$z(t) = \sum_{j=1}^{n} \alpha_j \eta_j(t) \qquad (21)$$

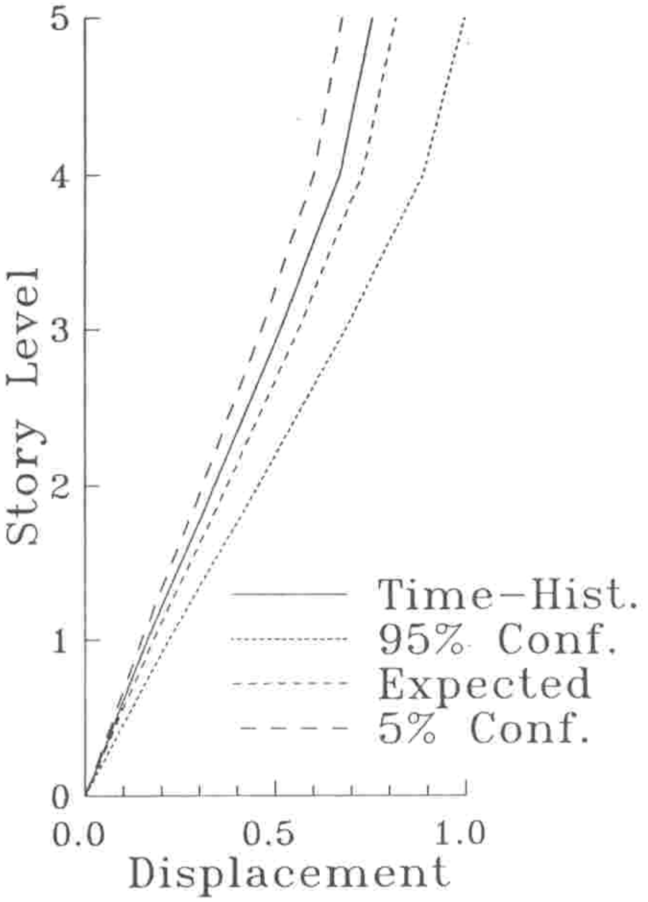

Fig. 4 – Comparison of the Displacement Response Envelopes for the Imperial Valley Earthquake Motion.

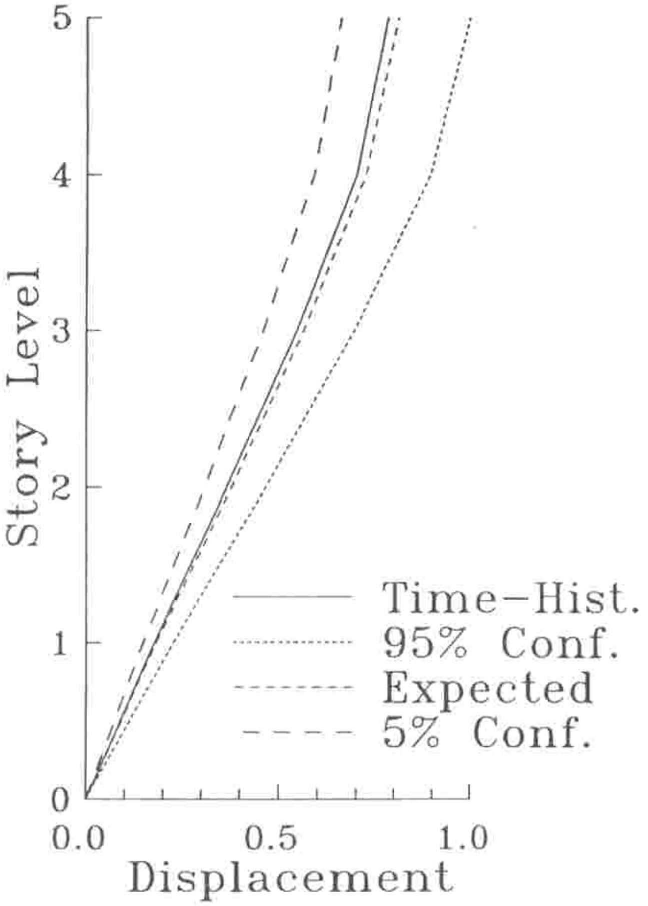

Fig. 5 – Comparison of the Displacement Response Envelopes for the Michoacan Earthquake Motion.

where, $\eta_j(t)$ is the system response in the j^{th} mode given by the response of a SDOF oscillator with natural frequency, ω_j, and critical damping ratio, ζ_j to the base acceleration, $\ddot{u}_g(t)$ [as in Eq. (19)]. Further, α_j is the effective mode participation factor in the j^{th} mode for the desired response quantity. For example, $\alpha_j = \phi_i^{(j)}\Gamma_j$ represents the effective modal participation factor for the displacement response at the i^{th} degree of freedom where $\phi_i^{(j)}$ is the i^{th} element of the j^{th} orthonormal mode shape vector, $\{\phi\}^{(j)}$, and $\Gamma_j(= -\{\phi\}^{(j)T}[M]\{r\})$ is the participation factor for the j^{th} mode, with $\{r\}$ representing the vector of rigid body influence coefficients.

The transient frequency response function for the response, $z(t)$ can be easily obtained by linearly combining the transient response functions for the modal responses, $\eta_1(t)$, $\eta_2(t)$, ..., $\eta_n(t)$, as in Eq. (21). Using this in Eq. (1), the time-dependent PSDF of the response process, $z(t)$ may thus be given by [Vanmarcke (1977)]

$$G_z(\omega, t) = G_{\ddot{u}_g}(\omega)\left[\sum_{j=1}^{n}\alpha_j^2\left|H_j(\omega, t)\right|^2 + \right.$$
$$\left. +2\,\mathrm{Re}\left\{\sum_{j=1}^{n-1}\sum_{k=j+1}^{n}\alpha_j\alpha_k H_j(\omega, t)H_k^*(\omega, t)\right\}\right] \quad (22)$$

where, $G_{\ddot{u}_g}(\omega)$ is the PSDF of the equivalent stationary ground motion as defined in Eq. (18), and $H_j(\omega, t)$ is the transient frequency response function for $\eta_j(t)$ defined as

$$H_j(\omega, t) = \frac{1}{(\omega_j^2 - \omega^2) + 2i\zeta_j(t)\omega\omega_j} \quad (23)$$

In Eq. (23), $\zeta_j(t) = \zeta_j/[1 - e^{-2\omega_j\zeta_j t}]$ represents the time-dependent damping in the j^{th} mode. Using the spectral density as in Eq. (22) now, the peak amplitudes in $z(t)$ response can be computed for any level of confidence by considering the peaks in fictitious stationary processes through Eq. (15). It may be noted that the computational effort in these calculations can be substantially reduced by carrying out the summation in Eq. (22) over a reduced number of modes.

The proposed approach has been illustrated by analyzing a symmetric, 5-storied, shear building for the same set of accelerograms as used in illustrating the response spectra calculations. The floor masses in this building are varying linearly from 3000 tonnes for the first floor to 1000 tonnes for the top floor. The story stiffnesses are respectively 3.795×10^6, 3.105×10^6, 2.415×10^6, 1.725×10^6, and 1.380×10^6 kN/m for

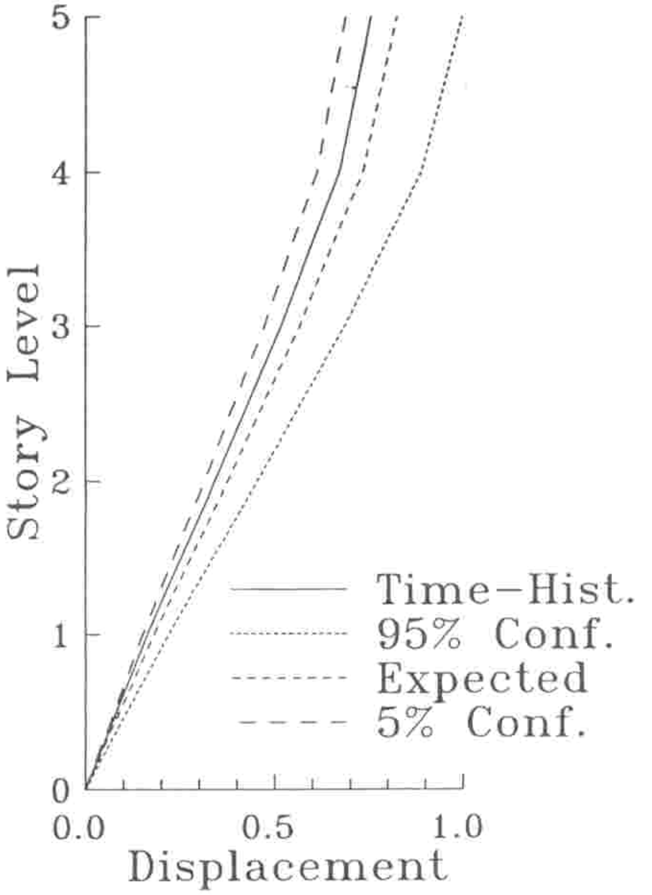

Fig. 6 – Comparison of the Displacement Response Envelopes for the Kern County Earthquake Motion.

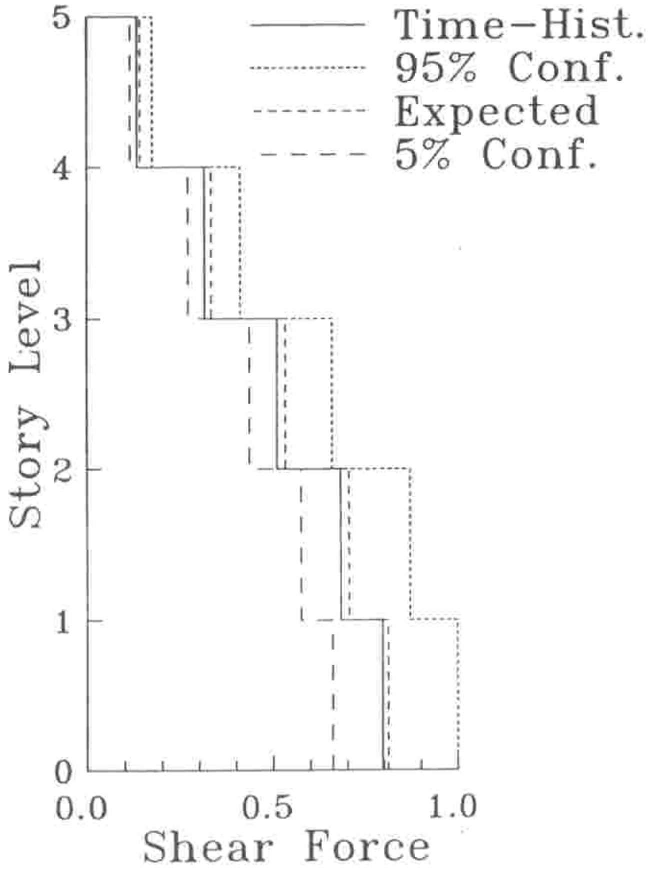

Fig. 8 – Comparison of the Shear Force Response Envelopes for the Michoacan Earthquake Motion.

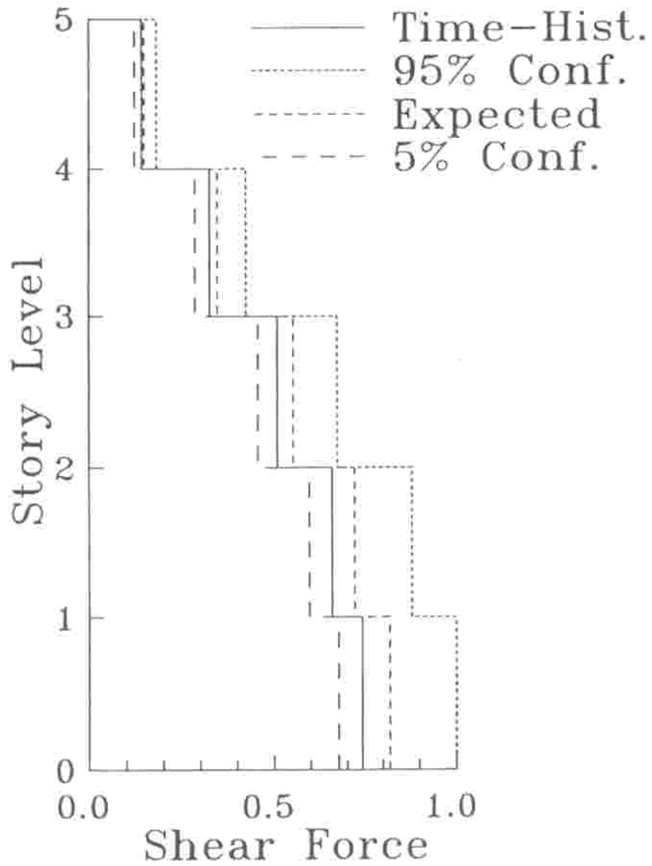

Fig. 7 – Comparison of the Shear Force Response Envelopes for the Imperial Valley Earthquake Motion.

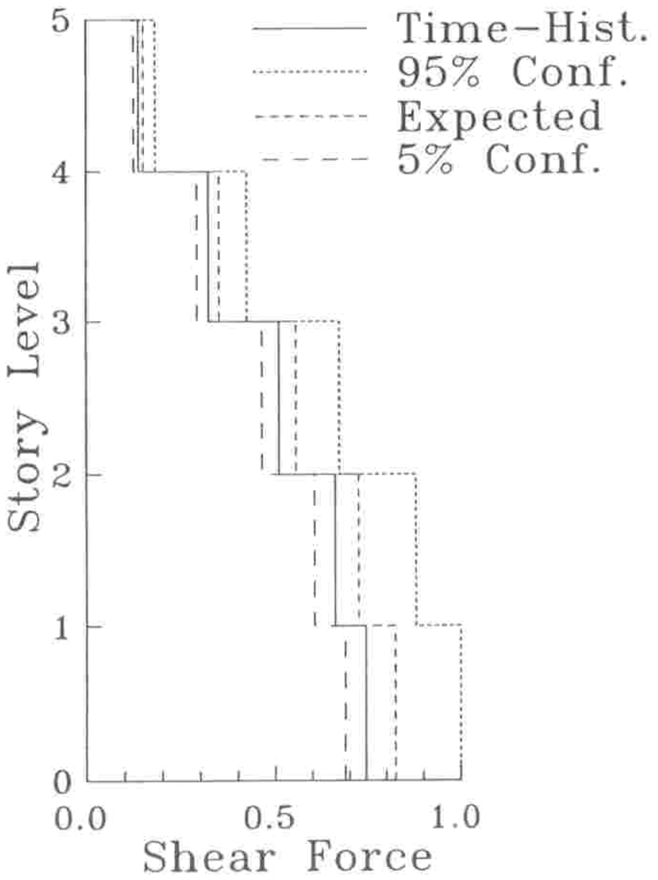

Fig. 9 – Comparison of the Shear Force Response Envelopes for the Kern County Erathquake Motion.

the bottom, second, third, fourth and top stories. The natural frequencies are found to be 12.29, 28.61, 44.30, 55.96, 62.96 rad/sec. The damping ratio is assumed to be 2% uniformly for all the modes of the example building. Expected envelopes of floor displacement and story shear responses have been obtained by using the proposed approach, and compared with the time-history results respectively in Figures 4-6 and 7-9. To illustrate the 90% confidence interval for these envelopes, the 5% and 95% confidence level estimates have also been obtained by using the proposed approach, and shown in these figures. In each figure, all the response values have been normalized with respect to the overall maximum value. It is seen that there is a good agreement of the expected values with the time-history results, and that the time-history results are well bounded by the 5% and 95% confidence estimates on both sides.

5. Conclusions

A new stochastic approach has been formulated in frequency domain for the seismic analysis of linear structural systems. This is based on the decoupling of the nonstationarity in response arising due to sudden application of excitation from the inherent nonstationarity in the excitation process. Th order statistics approach has been extended to the nonstationary processed for the approximate peak factor computations. Thus, this approach can also be used to estimate the higher order peak amplitudes, not just the largest peak amplitudes. The proposed approach is simple yet more generalized than the existing approaches. Simplicity in the approach has been introduced by modeling the input excitation as an 'equivalent stationary' process, and by characterizing it in terms of *i*) the peak ground acceleration with certain level of confidence, and *ii*) the Fourier spectrum representing 'average' energy distribution. Though this approach has been illustrated in case of simple SDOF and MDOF systems only, this can be easily extended to more complicated systems involving soil-structure interaction and spatial correlation in support motions.

The proposed approach has been shown to work well in the example cases as considered in this study. However, it can be improved further by considering more realistic energy distributions of the 'equivalent stationary' ground motion (as against the 'time-averaged' energy distribution). To this end, it may be useful to consider the recorded motions in segments rather than for the entire durations. Further, due to the independent treatment of the nonstationarity in the response process, the PSDF of the equivalent excitation may also be generated iteratively so as to be compatible with the response spectra for different damping levels. Separate studies are required, however, to explore these possibilities in detail.

Appendix

Let us consider a stationary, zero mean and Gaussian process, $X(t)$ with the (one-sided) PSDF denoted by $G_x(\omega)$. Extending th work of Rice (1944, 1945), Cartwright and Longuet-Higgins (1956) derived the probability density function (p.d.f.) for the distribution of maxima of $X(t)$, in terms of the r.m.s. value of $x(t)$, x_{rms}, and a parameter ε, which is a measure of th spread of the PSDF, $G_x(\omega)$. These parameters are defined in terms of the moments of the PSDF as follows

$$x_{rms} = m_0^{1/2} \quad \text{(A.1)}$$

and

$$\varepsilon = \left[\frac{m_0 m_4 - m_2^2}{m_0 m_4}\right]^{1/2} \quad \text{(A.2)}$$

where, in general, the i^{th} moment, m_i of the PSDF is defined by

$$m_i = \int_0^\infty \omega^i G_x(\omega)\mathrm{d}\omega \quad (n = 0, 1, 2, \ldots). \quad \text{(A.3)}$$

The probability density function of the maxima of $X(t)$ as normalized with respect to x_{rms}, is given as

$$p(\eta) = \frac{1}{\sqrt{2\pi}}\Big[\varepsilon e^{-\eta^2/2\varepsilon^2} + $$

$$+ (1-\varepsilon^2)^{1/2}\,\eta e^{-\eta^2/2}\int_{-\infty}^{\eta(1-\varepsilon^2)^{1/2}/\varepsilon} e^{-x^2/2}\mathrm{d}x\Big] \quad \text{(A.4)}$$

For $\varepsilon = 0$, this becomes a Rayleigh distribution, and for $\varepsilon = 1$, it become a Gaussian distribution.

Let the random process, $X(t)$ under our consideration have a total number of N peaks or maxima in its entire duration. These peaks are distributed as per Eq. (A.4), and may be assumed to be statistically independent for the first few orders [Basu and Gupta (1994)]. For that case, Gupta and Trifunac (1988) have obtained the probability density function of the i^{th} order peak as

$$p_{(i)}(\eta) = \frac{N!}{(N-i)!(i-1)!}[P(\eta)]^{i-1}[1-P(\eta)]^{N-i}p(\eta), \quad \text{(A.5)}$$

where, the total number of maxima, N in process duration, T is given in terms of the moments of $G_x(\omega)$ as

$$N = \frac{T}{2\pi}\left[\frac{m_4}{m_2}\right]^{1/2}, \quad \text{(A.6)}$$

and $P(\eta)$ is the cumulative probability given by $P(\eta) = \int_\eta^\infty p(u)\mathrm{d}u$.

For a given confidence level, Eq. (A.5) can be iteratively used to find the peak factor, η which, on being multiplied with x_{rms}, gives *the* i^{th} order peak amplitude. For the 'expected' peak amplitude, the peak factor, $\eta = \overline{\eta}$ may be computed as

$$\overline{\eta} = \int_{-\infty}^{\infty} u p_{(i)}(u) \mathrm{d}u. \tag{A.7}$$

The integral in this equation may be obtained by an approximate approach given by David and Johnson (1954), and also used later by Gupta and Trifunac (1988).

References

Amini, A. and M.D. Trifunac (1985). Statistical extension of response spectrum superposition, *Soil Dyn Earthq. Eng.*, **4(2)**, 54-63.

Basu, B. and V.K. Gupta (1994). Statistics of orderd peaks in a random response, *Proc. 10th Symp. Earthq. Eng., Roorkee*, 907-915.

Berrah, M. and E. Kausel (1992). Response spectrum analysis of structures subjected to spatially varying motions, *Earthq. Eng. Struct. Dyn.*, **21**, 461-470.

Betti, R., A.M. Abdel-Ghaffar, and A.S. Niazy (1993). Kinematic soil-structure interaction for long-span cable-supported bridges, *Earthq. Eng. Struct. Dyn.*, **22**, 415-430.

Bucher, C.G. (1988). Approximate nonstationary random vibration analysis for MDOF systems, *J. Appl. Mech. (ASME)*, **55**, 197-200.

Cartwright, D.E. and M.S. Longuet-Higgins (1956). The statistical distribution of maxima of a random function, *Proc. Roy. Soc. London*, **A 237**, 212-232.

Clough, R.W. and J. Penzien (1993). Dynamics of Structures, *McGraw-Hill, Singapore*.

Corotis, R.B., E.H. Vanmarcke, and C.A. Cornell (1972). First passage of nonstationary random processed, *J. Eng. Mech. Div., Proc. ASCE*, **98(EM2)**, 401-414.

Corotis, R.B. and E.H. Vanmarcke (1975). Time-dependent spectral content of system response, *J. End. Mech. Div., Proc. ASCE*, **101(EM5)**, 623-637.

David, F.N. and N.L. Johnson (1954). Statistical treatment of censored data, Part I: Fundamental formulae, *Biometrika*, **41**, 228-240.

Deb Chaudhury, A. and D.A. Gasparini (1980). Dynamic response to nonstationary nonwhite excitation, *J. Eng. Mech. Div., Proc. ASCE*, **106(EM6)**, 1233-1248.

Der Kiureghian, A. (1981). A response spectrum method for random vibration analysis of MDF systems, *Earthq. Eng. Struct. Dyn.*, **9**, 419-435.

Der Kiureghian, A. and Y. Nakamura (1993). CQC modal combination rule for high-frequency modes, *Earthq. Eng. Struct. Dyn.*, **22**, 943-956.

Der Kiureghian, A. and A. Neuenhofer (1992). Response spectrum method for the multi-support seismic excitation, *Earthq. Eng. Struct. Dyn.*, **21**, 713-740.

Gupta, I.D. (1994). A probabilistic approach for estimating th response of MDOF structures, *Soil Dyn. Earthq. Eng.*, **13**, 79-87.

Gupta, I.D. and M.D. Trifunac (1988) Order statistics of peaks in earthquake response, *J. Eng. Mech.* (ASCE), **114(10)**, 1605-1627.

Gupta, I.D. and M.D. Trifunac (1990a). Probabilistic spectrum superposition for response analysis including the effects of soil-structure interaction, *J. Prob. Eng. Mech.*, **5(1)**, 9-18.

Gupta, V.K. (1994a). Stochastic approach to seismic floor spectra in nuclear power plants, *Report 94-02, Dep. of Civil Eng., I.I.T. Kanpur, Kanpur, India.*

Gupta, V.K. (1994b) Higher order peaks in the seismic response of multistoried buildings, *Report 94-03, Dep. of Civil Eng., I.I.T. Kanpur, Kanpur, India.*

Gupta, V.K. and M.D. Trifunac (1989). Investigation of building response to translational and rotational earthquake excitations, *Report CE 89-02, University of Southern California, Los Angeles, California.*

Gupta, V.K. and M.D. Trifunac (1990b). Response of multistoried buildings to ground translation and rocking during earthquakes, *J. Prob. Eng. Mech.*, **5(3)**, 138-145.

Gupta, V.K. and M.D. Trifunac (1991). Seismic response of multistoried buildings including the effects of soil-structure interaction, *Soil Dyn. Earthq. Eng.*, **10(8)**, 414-422.

Heredia-Zavoni, E. and E.H. Vanmarcke (1994). Seismic random vibration analysis of multisupport structural systems, *J. Eng. Mech.* (ASCE), **120(5)**, 1107-1128.

Hou, Z. (1990). Nonstationary response of structures and its application to earthquake engineering, *Report EERL 90-01, California Inst. of Tech., Pasadena, California.*

Igusa, T. (1987). Characteristics of nonstationary response, *Report 87-10/TI-02, Dep. Civil Eng., Northwestern Univ., Evanston, Illinois.*

Lin, J.S. and J.Y. Tyan (1986). Equivalent stationary motion and average response spectra, *Earthq. Eng. Struct. Dyn.*, **14**, 267-279.

Madsen, P.H. and S. Krenk (1983). Stationary and transient response statistics, *J. Eng. Mech. Div.* ASCE, **108(EM4)**, 622-635.

Rice, S.O. (1944). Mathematical analysis of random noise, *Bell Syst. Tech. J.*, **23**, 282-332.

Rice, S.O. (1945). Mathematical analysis of random noise, *Bell Syst. Tech. J.*, **24**, 46-156.

Shinozuka, M. and J.N. Yang (1972). Peak structural response to nonstationary random excitations, *J. Sound Vib.*, **14**(4), 505-517.

Singh, M.P. and S.L. Chu (1976). Stochastic considerations in seismic analysis of structures, *Earthq. Eng. Struct. Dyn.*, **4**, 295-307.

Singh, M.P. and K.B. Mehta (1983). Seismic design response by an alternative SRSS rule, *Earthq. Eng. Struct. Dyn.*, **11**, 771-783.

Trifunac, M.D. and A.G. Brady (1975). A study on the duration of strong earthquake ground notion, *Bull. Seism. Soc. Amer.*, **65(3)**, 581-626.

Udwadia, F.E. and M.D. Trifunac (1974). Characterization of response spectra through the statistics of oscillator response, *Bull. Seism. Soc. Amer.*, **64**, 205-219.

Vanmarcke, E.H. (1975). On the distribution of the first-passage time for normal stationary random processes, *J. Appl. Mech. (ASME)*, **42(E)**, 215-220.

Vanmarcke, E.H. (1977). Structural response to earthquakes, in *Seismic Risk and Engineering Decisions (Eds. C. Lomnitz and E. Rosenblueth), Elsevier, Amsterdam*, 287-337.

Wilson, E.L., A. Der Kiurenghian, and E.P. Bayo (1981). A replacement for the SRSS method in seismic analysis, *Earthq. Eng. Struct. Dyn.*, **9**, 187-194.

Yamamura, N. and H. Tanaka (1990). Response analysis of flexible MDF systems for multiple support seismic excitations, *Earthq. Eng. Struct. Dyn.*, **19**, 345-357.