STRUCTURAL ENGINEERING CONVENTION 1997
FEB. 12 - 14 1997

ON EVOLUTIONARY SEISMIC RESPONSE AND PEAK FACTORS

MANISH SHRIKHANDE **VINAY K. GUPTA**
Graduate Student *Associate Professor*
Department of Civil Engineering, I.I.T. Kanpur, Kanpur 208 016

SYNOPSIS: A new formulation is presented for estimating the largest and higher order peak responses of a single-degree-of-freedom oscillator subjected to a stationary excitation of finite duration. This formulation explicitly accounts for the non-stationarity in response arising due to the finite operating time of excitation. For this purpose, the order statistics formulation for stationary peak factors has been extended to apply in case of 'evolutionary' response processes. Numerical simulations have been carried out to check the validity of this formulation on the 'non-stationary' peak factors in such processes. The proposed formulation may be particularly useful for the computation of more realistic response spectrum-compatible power spectral density functions as required in the aseismic design of structures.

INTRODUCTION

The earthquake ground motion processes are inherently non-stationary in nature. Further, those are of short durations, and thus, cause further non-stationarity in the responses of dynamical systems initially at rest. A rigorous probabilistic assessment of seismic response of a dynamical system would thus require an appropriate modelling of the ground motion and consideration of additional non-stationarity due to the excitation not acting long enough. The transient nature of the system response to a finite duration stationary excitation had been first considered by Caughey and Stumpf (1961), and since then various analytical formulations have been proposed within the framework of random vibrations for estimating the seismic response. The ground motion process has been modelled in these formulations through suitable deterministic modulating functions, and thus these formulations have lacked generality in application, besides being mathematically cumbersome. In another class of simpler methods, response process of a dynamical system is assumed to be a stationary process, thus assuming the earthquake ground motion to be a stationary process and ignoring the effects of finite duration operation of excitation. Resulting errors in the estimation of peak responses are corrected with the help of response spectrum characterization of the ground motions. Despite simplicity, these methods permit the estimation of higher orders of response peaks [see, for example, Gupta and Trifunac (1987, 1990, 1991) and Gupta (1994a)] which may be useful in the damage-based probabilistic design of structures [Basu and Gupta (1995a,b, 1996a,b)]. These methods are however suitable for application when the ground motion lasts long enough to permit many cycles of system response.

This study aims to consider an alternative random vibration approach for estimating the seismic response of single-degree-of-freedom (SDOF) systems wherein the ground motion is idealized as an 'equivalent stationary process' and the non-stationarity in the structural response due to the finite duration of the excitation is accurately accounted for. Thus, it is aimed to combine the simplicity of the response spectrum-based techniques with the rigour of the non-stationary formulations. A new formulation based on this approach has recently been suggested by Shrikhande and Gupta (1996a). This considers i) explicit consideration of the response non-stationarity by the use of time-dependent transfer function, and ii) extension of the peak factor formulation of Gupta and Trifunac (1988) and Gupta (1994b) for stationary processes to determine the ordered peak amplitudes in a non-stationary SDOF system response. A brief review of this formulation has been presented in this paper and it has been shown through a numerical study that the 'expected' peak factors for the SDOF system response process as computed by this formulation are in good agreement with those directly obtained from time-history simulations.

REVIEW OF THE NEW FORMULATION

Let us consider a SDOF oscillator which is subjected to a stationary base excitation, $\ddot{u}(t)$, for a finite duration, T. The (evolutionary) power spectral density function (PSDF), $G_x(\omega, t)$, of the displacement response process, $x(t)$, of the oscillator may be expressed as [Corotis and Vanmarcke (1975)]

$$G_x(\omega, t) = |H(\omega, t)|^2 G_{\ddot{u}}(\omega) \tag{1}$$

where, $G_{\ddot{u}}(\omega)$ is the PSDF of the input excitation, and $H(\omega, t)$ denotes the transient frequency response function of the relative displacement response. For an oscillator with damping, ζ and natural frequency, ω_n, $H(\omega, t)$ is given by

$$H(\omega, t) = \frac{1}{(\omega_n^2 - \omega^2) + 2i\zeta(t)\omega\omega_n} \tag{2}$$

with $\zeta(t) = \zeta/[1-e^{-2\zeta\omega_n t}]$ being the fictitious time-dependent damping [Vanmarcke (1976)].

The amplitudes of the ordered peaks in a stationary random process for a given level of confidence may be estimated in terms of the moments of the process PSDF about origin and by using the theory of order statistics [Gupta and Trifunac (1988), Gupta (1994b)]. Extending this for application in case of evolutionary displacement response process, the instantaneous probability density function of peaks may be written as [Shrikhande and Gupta (1996a)]

$$p(\eta,t) = \frac{1}{\sqrt{2\pi}}\left[2\varepsilon(t)e^{-\eta^2/2\varepsilon^2(t)} + (1-\varepsilon^2(t))^{1/2}\eta e^{\eta^2/2}\int_{-\eta(1-\varepsilon^2(t))^{1/2}/\varepsilon(t)}^{\eta(1-\varepsilon^2(t))^{1/2}/\varepsilon(t)} e^{-x^2/2}\mathrm{d}x\right];\ \eta \geq 0 \tag{3}$$

where,

$$\varepsilon(t) = \left[\frac{m_0(t)m_4(t) - m_2^2(t)}{m_0(t)m_4(t)}\right]^{1/2} \tag{4}$$

is the instantaneous band-width parameter, and

$$m_i(t) = \int_0^\infty \omega^i G_x(\omega,t)\mathrm{d}\omega\,;\quad i = 0,2,4 \tag{5}$$

represents the ith instantaneous spectral moment (about origin) of the displacement process. Further, η is the normalized amplitude of maxima with respect to the instantaneous root-mean-square (r.m.s.) value, $x_{\mathrm{rms}}(t)$ $(= \sqrt{m_0(t)})$. Let us now consider a fictitious stationary process with the same duration, T as the duration of the excitation process, and the ensemble statistics same as the instantaneous statistics of the process, $x(t)$, at a time instant, t. Let $\eta^t_{(j)}$ represent the peak factor for the jth largest peak amplitude corresponding to a specified probability of confidence in this fictitious process. This may be determined by the knowledge of the probability density function, $p^t_{(j)}(\eta)$, of the jth largest peak as

$$p^t_{(j)}(\eta) = j\binom{N(t)}{j}[P(\eta,t)]^{j-1}[1-P(\eta,t)]^{N(t)-j}p(\eta,t) \tag{6}$$

where, $P(\eta,t)$ $(= \int_\eta^\infty p(u,t)\mathrm{d}u)$ is the distribution function of peaks corresponding to $p(\eta,t)$, and

$$N(t) = \frac{1}{2\pi}(1+\sqrt{1-\varepsilon^2(t)})\left[\frac{m_4(t)}{m_2(t)}\right]^{1/2} \tag{7}$$

is the instantaneous mean rate of occurrence of peaks. It has been shown by Shrikhande and Gupta (1996a) heuristically that the jth largest peak in the evolutionary displacement process may be expressed as

$$x_{\text{peak}}^{(j)} = \left[\frac{1}{T}\int_0^T (\eta_{(j)}^t)^2 m_0(t)\mathrm{d}t\right]^{1/2} \tag{8}$$

Similarly, the r.m.s. value of the displacement process may be written as

$$x_{\text{rms}} = \left[\frac{1}{T}\int_0^T m_0(t)\mathrm{d}t\right]^{1/2} \tag{9}$$

Since the transient frequency response function of the oscillator is a smooth function of time, the integrals in Eqs. 8 and 9 may be evaluated efficiently by means of the standard quadrature routines. It may be noted that the ratio, $x_{\text{peak}}^{(j)}/x_{\text{rms}}$ represents the 'non-stationary' peak factor, $\eta_{(j)}$, for the jth order peak which when multiplied with the (temporal) r.m.s. value of the response process, gives the jth largest peak estimate.

ILLUSTRATION OF 'NON-STATIONARY' PEAK FACTORS

For a numerical study on illustrating the 'non-stationary' peak factors, input ground motion process has been assumed to be characterized by the modified Kanai-Tajimi filtered white-noise spectrum [Clough and Penzien (1982)]. This is expressed as

$$G_{\ddot{u}}(\omega) = G_0 \frac{\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \cdot \frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + 4\zeta_f^2\omega_f^2\omega^2} \tag{10}$$

where, G_0 represents the intensity of the excitation, and ω_g, ω_f, ζ_g, and ζ_f are the site dependent frequency and damping parameters. Let these parameters be taken as 1.5 rad/s, 15.0 rad/s, 0.6, and 0.6 respectively as in case of the the rocky sites. An ensemble of 30 stationary time-histories corresponding to the assumed PSDF and of 20 sec duration has been generated for the simulation results. A set of SDOF oscillators with natural periods varying from 0.01 sec to 3.0 sec and with damping ratios of 2%, 5%, and 10% have been subjected to these excitation time-histories and the (relative displacement) response time-histories have been computed by the numerical evaluation of Duhamel's integral. The temporal r.m.s. value and the amplitudes of the first, third and sixth order peaks have been determined for each response time-history and then averaged over the ensemble. The 'non-stationary' peak factors, $\eta_{(j)}$s, for $j = 1$, 3 and 6 have been then computed by taking the ratio of 'averaged' peak amplitudes to the 'averaged' r.m.s. value.

The 'expected' peak factors corresponding to the first, third and sixth order peaks have also been computed by using the formulation of the previous section and compared with those obtained from the simulations as shown in Figs. 1–3. It may be seen that the two sets of peak factors are in excellent agreement for the largest as well as higher orders of peaks. Further, this agreement is improved with an increase in either the damping ratio or the natural frequency of the SDOF oscillators. This is as expected because the response process approaches a stationary process with the increase in these two parameters [Caughey and Stumpf (1961)]. In such a situation, the 'non-stationary' peak factors approach the 'stationary' peak factors as proposed by Gupta and Trifunac (1988) and Gupta (1994b).

CONCLUSIONS

A new formulation has been presented for the non-stationary seismic analysis of a SDOF system subjected to stationary base excitation. This is simple and explicitly accounts for the effect of response non-stationarity on the response peak amplitudes. Through numerical simulation, it has been shown that the peak factors of the non-stationary response process are predicted with good accuracy by this formulation for a large variety of SDOF oscillators. This formulation can be easily extended to the multi-degree-of-freedom systems as shown by Shrikhande and Gupta (1996a). This may also be suitable for more realistic computation of PSDF from a given design spectrum [see Shrikhande and Gupta (1996b) for details].

REFERENCES

Basu, B. and Gupta, V.K. (1995a), "A probabilistic assessment of seismic damage in ductile structures," Earthq. Engg. Struct. Dyn., V. 24, pp. 1333–1342.

Basu, B. and Gupta, V.K. (1995b), "A note on ductility for probabilistic seismic design of structures," Euro. Earthq. Engg., V. IX, pp. 34–43.

Basu, B. and Gupta, V.K. (1996a), "A note on damage-based inelastic spectra," Earthq. Engg. Struct. Dyn., V. 25, pp. 421–433.

Basu, B. and Gupta, V.K. (1996b), "Expected seismic damage during narrow-band structural responses," Struct. Engg./Earthq. Engg., JSCE, V. 13, pp. 119–123.

Caughey, T.K. and Stumpf, H.J. (1961), "Transient response of a dynamic system under random excitation," Jl. Appl. Mech., ASME, V. 28:E, pp. 563–566.

Clough, R.W. and Penzien, J. (1982), "Dynamics of Structures," McGraw-Hill Book Co., Singapore.

Corotis, R.B. and Vanmarcke, E.H. (1975), "Time-dependent spectral content of systems response," Jl. Engg. Mech. Div., ASCE, V. 101, pp. 623–637.

Gupta, I.D. and Trifunac, M.D. (1987), "Order statistics of peaks in earthquake response of multi-degree-of-freedom systems," Earthq. Engg. Engg. Vib., V. 7, pp. 15–50.

Gupta, I.D. and Trifunac, M.D. (1988), "Order statistics of peaks in earthquake response," Jl. Engg. Mech., ASCE, V. 114, pp. 1605–1627.

Gupta, V.K. (1994a), "Higher order peaks in the seismic response of multistoried buildings," Report 94-03, Dept. of Civil Engg., I.I.T. Kanpur, Kanpur.

Gupta, V.K. (1994b), "Stochastic approach to seismic floor spectra in nuclear power plants," Report 94-02, Dept. of Civil Engg., I.I.T. Kanpur, Kanpur.

Gupta, V.K. and Trifunac, M.D. (1990), "Response of multistoried buildings to ground translation and rocking during earthquakes," Jl. Prob. Engg. Mech., V. 5, pp. 138–145.

Gupta, V.K. and Trifunac, M.D. (1991), "Seismic response of multistoried buildings including effects of soil-structure interaction," Soil Dyn. Earthq. Engg., V. 10, pp. 414–422.

Shrikhande, M. and Gupta, V.K. (1996a), "A generalized approach for the seismic response of structural systems," Euro. Earthq. Engg., (in press).

Shrikhande, M. and Gupta, V.K. (1996b), "On generating ensembles of design spectrum compatible accelerograms," Euro. Earthq. Engg., (in press).

Vanmarcke, E.H. (1976), "Structural response to earthquakes," in 'Seismic Risk and Engineering Decisions' (Eds. C. Lomnitz and E. Rosenblueth), Elsevier, Amsterdam, pp. 287–337.

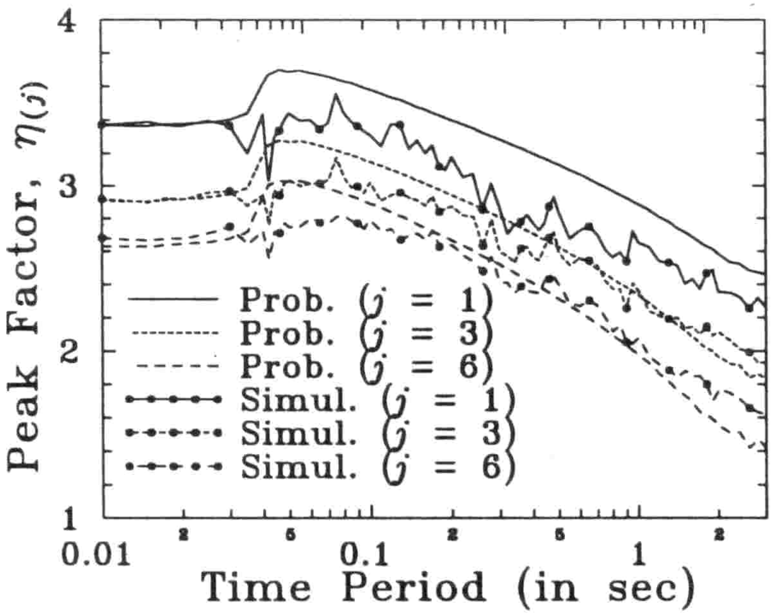

Fig. 1 Comparison of 'probabilistic' and 'simulation' peak factors for 2% damping oscillators

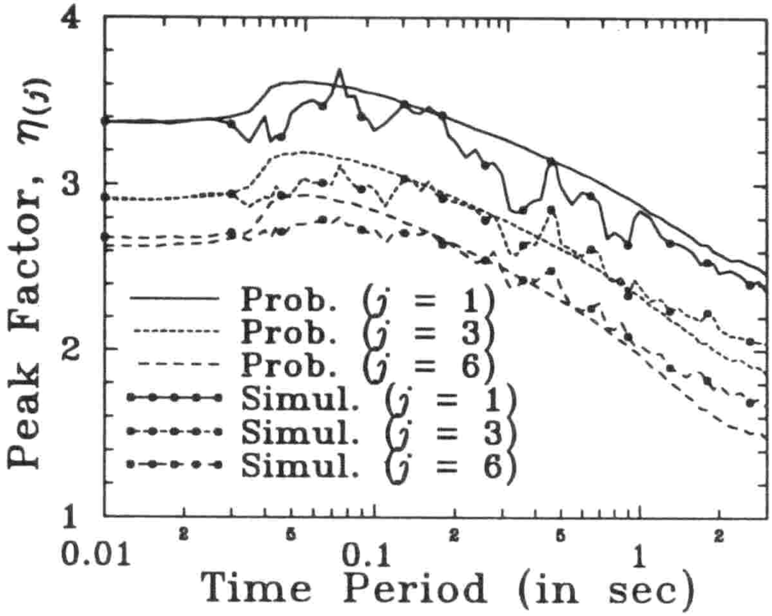

Fig. 2 Comparison of 'probabilistic' and 'simulation' peak factors for 5% damping oscillators

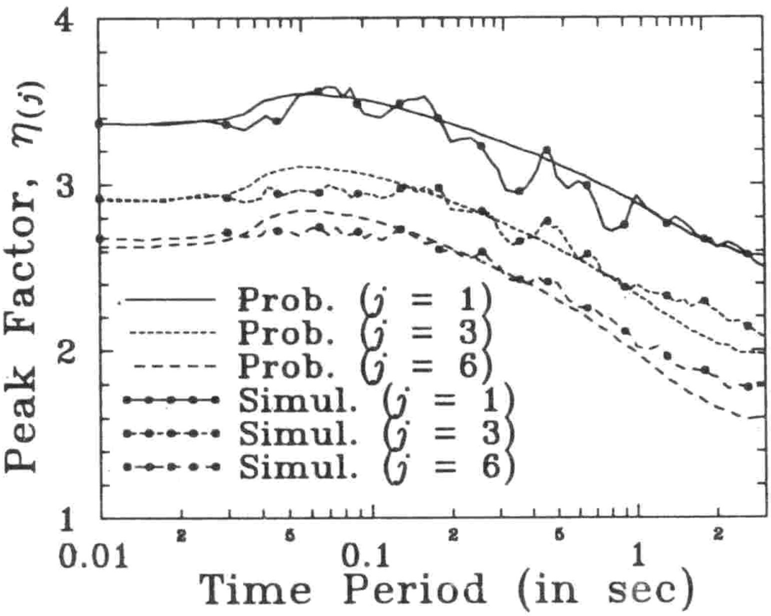

Fig. 3 Comparison of 'probabilistic' and 'simulation' peak factors for 10% damping oscillators