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Technical Note

Higher modes in along-wind building response

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Among several methods available for calculating the dynamic response of structures due to the wind gusts, the gust factor approach[1-7] has been very popular due to the simplicity in its application. In this approach, considerable simplification is achieved by assuming the along-wind response of the structure to be in the fundamental mode. This partly follows from the fact that the wind excitation contains little energy at high frequencies, and is thus considered to be acceptable to consider the foundamental mode only, provided the ratios of natural frequencies in the higher modes to the fundamental frequency are sufficiently large[8]. In case of the tall buildings, however, the modes may often be very closely spaced. For such cases, it becomes necessary to examine whether this assumption can still be made in estimating gust factors for the along wind building response. This has been examined in this study by comparing the power spectral density functions of modal force for the first three modes in case of a 365 m high shear building. For completeness, a brief review of the gust factor approach has also been included along with the numerical results.

GUST FACTOR METHOD FOR ALONG-WIND RESPONSE

The gust factor method uses the statistical concepts of a stationary time series to calculate the response of a structure to gusty wind. It consists of first calculating the static response of structure to the mean wind load and then amplifying it by the gust factor to account for the fluctuations in the wind velocity. Thus, taking the gust factor equal to unity is equivalent to treating the wind loading as a static loading. If $\sigma_y^2(z)$ represents the variance of the structural response at height z due to the pressure fluctuations, and $\bar{y}(z)$ is the mean system response at the same height, $[1 + g(\nu T)\,\sigma_y(z)/\bar{y}(z)]$ is the gust factor, where $g(\nu T)$ is the peak factor expressed in terms of total number of positive crossings in the response fluctuations during the gusty wind[9]. Customarily, the response parameters, $\sigma_y(z)$ and $\bar{y}(z)$ are obtained by assuming that the structure deflects in the fundamental mode under the effect of wind. In general, the linear dynamic response of a structure can be expressed as a combination of several modal responses, and therefore, this assumption that the response can be estimated from a single mode, may not always be valid.

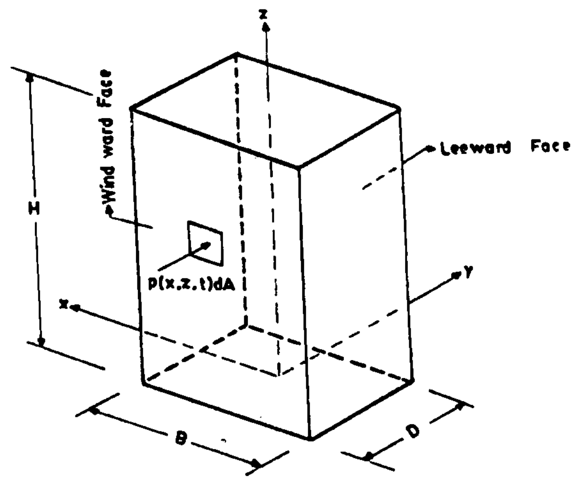

FIG.1 SCHEMATIC VIEW OF A BUILDING

Let us consider a building idealized as continuous system (Fig.1). Under the effect of gusty wind, at time, t, it is subjected to the wind pressure, $p(x, z, t)$ at points with ordinates, x, z on the windward and leeward faces. If the velocity fluctuations, $u'(x, z, t)$ are considered to be small compared to the mean wind speed, $\bar{u}(z)$ and wind pressure at any point can be expressed as

$$p(x, z, t) = \bar{p}(x, z) + p'(x, z, t) \tag{1}$$

where, $$\bar{p}(x, z) = \frac{1}{2}\,\rho c_p(x, z)\,\bar{u}^2(z) \tag{2}$$

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and $\quad p'(x,z,t) = \rho c_p(x,z)\,\bar{u}(z)\,u'(x,z,t) \quad (3)$

are respectively the mean and fluctuating pressures. Here, ρ is the density of air, and $c_p(x, z)$ is the pressure coefficient, generally assumed to be a constant (c_w or c_l), depending on whether the point under consideration is on the windward or the leeward side. Thus, $\bar{p}(x,z)$ can be assumed to be independent of x and equal to $\bar{p}_w(z)$ or $\bar{p}_l(z)$. The mean displacement response $\bar{y}(z)$ due to $\bar{p}(x,z)$ can thus be obtained as[8]:

$$\bar{y}(z) = \sum_r \mu_r(z) \frac{\int_{A_w} \bar{p}_w(z)\,\mu_r(z)\,dA + \int_{A_l} \bar{p}_l(z)\,\mu_r(z)\,dA}{(2\pi n_r)^2\,M_r} \quad (4)$$

where, $\mu_r(z)$ and n_r respectively are the r^{th} mode shape and natural frequency; A_w and A_l are the areas of the windward and leeward faces of the building; and M_r $(=\int_0^H m(z)\,\mu_r^2(z)\,dz)$ is the r^{th} modal mass, $m(z)$ denotes the system mass per unit height. Further, for a linear elastic structure, the standard deviation, $\sigma_y(z)$ of the fluctuating part of the response i.e. $y'(z)$, can be obtained on neglecting the effects of the modal cross-correlations on response as[8]:

$$\sigma_y(z) = \left[\sum_r \frac{\mu_r^2(z)}{M_r^2} \int_0^\infty |H_r(n)|^2\, S_{F_r}(n)\,dn\right]^{1/2} \quad (5)$$

Here, $H_r(n)$ is the complex frequency response function defined by:

$$H_r(n) = \frac{1}{4\pi^2\,(n_r^2 - n^2 + 2i\zeta_r\,n_r\,n)} \quad (6)$$

ζ_r being the critical damping ratio in the r^{th} mode. Further, $S_{F_r}(n)$ is the mean-square spectral density of modal force in the r^{th} mode given by:

$$\begin{aligned} S_{F_r}(n) = &\int_{A_w}\int_{A_w} \mu_r(z_1)\,\mu_r(z_2)\,S_p(P_1^w, P_2^w; n)\,dA_1\,dA_2 \\ &+ 2\int_{A_w}\int_{A_l} \mu_r(z_1)\,\mu_r(z_2)\,S_p(P_1^w, P_2^l; n)\,dA_1\,dA_2 \\ &+ \int_{A_l}\int_{A_l} \mu_r(z_1)\,\mu_r(z_2)\,S_p(P_1^l, P_2^l; n)\,dA_1\,dA_2 \end{aligned} \quad (7)$$

Here, $S_p(\,.\,)$ represents the cross-spectral density of the pressure fluctuations acting at the specified points on the windward or leeward sides, P_1^w on the windward side with ordinates, x_1, z_1 and P_2^l on the leeward side with ordinates, x_2, z_2. This complex function is generally simplified to the following real form[8].

$$S_p(P_1, P_2; n) = \rho^2\,c\,\bar{u}(z_1)\bar{u}(z_2)\,S_u^{1/2}(z_1, n)\,S_u^{1/2}(z_2, n)\,R(x_1, x_2, z_1, z_2; n) \quad (8)$$

where, c is a measure of correlation between the wind velocity fluctuations on the two points, P_1 and P_2 ($c = c_w^2$ or c_l^2 for both the points being on the windward or the leeward side; $c = N(n)c_w\,c_l$ for the two points on the opposite sides, $N(n)$ being the alongwind correlation factor), $S_u(z,n)$ is the spectrum of velocity fluctuations at z height (assumed to be same on the leeward and windward sides), and $R(x_1, x_2, z_1, z_2; n)$ is the square root of the coherence function of velocities at the points, P_1 and P_2. It may be observed that the Eqs.(4) and (5) for $\bar{y}(z)$ and $\sigma_y(z)$ have also the contributions from the higher modes with $r = 2, 3, 4, ...$

CONTRIBUTIONS OF HIGHER MODES: AN EXAMPLE

A 365 m high building with 60 x 60 m base dimensions has been considered. With the mode shapes described by the sinusoidal curves, $\mu_r(z) = \sin(2r-1)\pi\, z/2H$, $r = 1, 2, 3,...$ as in the case of shear beams. The building is assumed to be located in a densely populated urban area for greater contribution of the dynamic component in the structural response. The fluctuations in the wind velocity are assumed to be described by height-dependent wind velocity spectra[10]. A logarithmic law profile as given in Indian Standard Code[11], with 2.0 m roughness length, 10 m zero plane displacement and 2.26m/sec friction velocity, u_*, has been used for calculating the mean wind velocity. The alongwind correlation factor, $N(n)$ has been assumed to be same as given by Vellozzi and Cohen[3]. The mean pressure coefficients, c_w and c_l, on the windward and leeward sides have been respectively assumed as 0.8 and 0.5. Further, $R(x_1, x_2, z_1, z_2; n)$ has been taken to be the same as proposed by Davenport[12], with the exponential decay coefficients taken as 16 and 10 respectively in the horizontal and vertical directions[4]. It may be noted that the qualitative conclusions of this study will remain unchanged irrespective of the assumed data.

For a similar building as above, Simiu and Scanlan[8] have reported just 0.1% contribution of the second and third modes to the root-mean-square value, $\sigma_y(z)$ of the deflection response, when the natural frequencies, n_1, n_2 and n_3 are taken equal to 0.1, 0.25 and 0.5 Hz respectively. This contribution becomes 7% only, if n_2 and n_3 are respectively taken as 0.12 and 0.15 causing the first three modes to be very close. This relatively insignificant rise in the contribution of higher modes to the total response implies that there is little energy in the wind excitation for the second and third modes even when the transmission of the excitation energy in these modes is comparable to that in the first mode. To understand the reasons, let us closely examine the term, $\sigma_y(z)$ (see Eq. (5)). In general, the value of $|H_r(n)|^2$ decreases rapidly for increasing natural frequencies, thus causing lower transmission of the excitation energy to the structural response in higher modes. More significantly, however, the modal force density, $S_{F_r}(n)$ at any frequency is substantially greater for the fundamental mode shape as compared to that for higher mode shapes. This is due to the close resemblance between the first mode shape and the distribution of mean wind pressures acting along the height which does not lead to any mutual cancellation of the wind pressure contributions at different heights to the total energy in the mode (notice the multiplication of terms $\mu_r(.)$ and $\bar{u}(.)$ in Eq. (7)). In higher mode shapes, however, such cancellation causes modal energy to be reduced to a smaller value. These observations are parallel to those of Jennings and Bielak[13], and Gupta and Trifunac[14] on the domination of the foundation rocking by the first fixed-base mode in their study of the soil-structure interaction during the seismic response of multistoried buildings. To illustrate further, the modal force spectra for the first three modes have been calculated in case of the example building for the following two cases (Figs.2 and 3), (i) the mean wind velocity profile being described by the logarithmic law, and (ii) the mean wind velocity being hypothetically assumed as constant with height. The latter case corresponds to the case of uniform mean wind load along the height of the building, and hence is analogous to that of earthquake loading in which the building is subjected effectively to a uniform forcing function with height during the base acceleration. The figures clearly show that with the other data remaining same, the second case is going to be associated with relatively much greater contributions from the higher modes to the overall building response. This is also consistent with the present approach on the increased contribution of higher modes to the seismic response of buildings. It is thus implied that the higher modes may play much less important role in the along-wind response of tall buildings even when those are close to the fundamental mode.

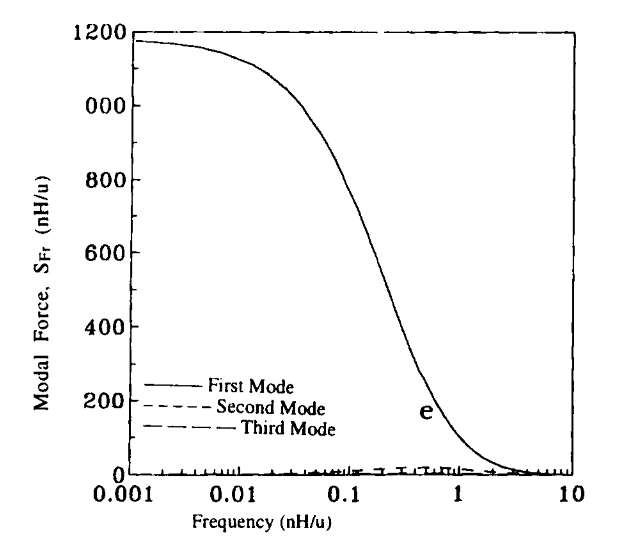

FIG.2 MODAL FORCE SPECTRA FOR VARIOUS MODES (LOGARITHMIC PROFILE)

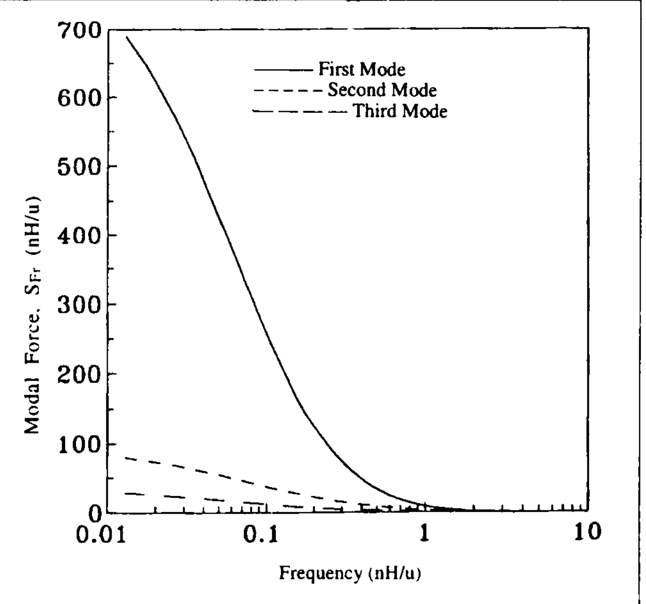

FIG.3 MODAL FORCE SPECTRA FOR VARIOUS MODES (HYPOTHETICAL PROFILE)

Though the above conclusions have been based on a specific study only, they do not lack generality in view of the underlying reasons. Further, these conclusions are not applicable to those structures whose lateral dimension in the x-direction varies with height. In such cases, for example those of stacks and cooling towers, the higher modes may actually play much more significant role, particularly in evaluating the stress responses.

CONCLUSIONS

This study has shown that due to i) the mean wind velocity profile being very similar to the first mode shape of the tall buildings, and ii) the lateral dimension of the building on the windward side being largely invariant with height, the along-wind response of such structures will almost be in the fundamental mode. It is thus quite appropriate to neglect the contributions of the higher modes in calculating the gust factors for the along-wind response of regular tall buildings.

REFERENCES

1. Davenport, A.G., ''The Application of Statistical Concepts to the Wind Loading of Structures'', *Proc. Inst. Civil Engrs.* London, V.19, 1961, pp. 446-472.
2. Davenport, A.G., ''Gust Loading Factors'', *J. Struct. Div. Proc. ASCE,* V. 93, No. ST3, 1967, pp.11-33.
3. Vellozzi, J. and Cohen, E., '' Gust Response Factors'', *J. Struct.Div. Proc. ASCE,* V.94, No. ST6, 1968,pp.1295-1313.
4. Vickery, B.J., ''On the Reliability of Gust Loading Factors'', *Proc. Symp. Wind Loads on Buildings and Struct.,* Washington, D.C., 1970, pp.93-104.
5. Simiu, E., ''Wind Spectra and Dynamic Alongwind Response'', *J. Struct. Div., Proc. ASCE,* V. 100, No.ST9, 1974, pp. 1879-1910.
6. Simiu, E.,''Revised Procedure for Estimating Along-Wind Response'', *J. Struct. Div., Proc. ASCE,* V.106, No. ST1,1980, pp.1-10.

7. Solar, G., ''Alongwind Response Estimation: Closed Form Solution'', *J. Struct. Div. Proc. ASCE*, V. 108, No.ST1, 1982, pp.225-244.

8. Simiu, E. and Scanlan, R.H., ''*Wind Effects on Structures*'', John Wiley & Sons, New York, 1986.

9. Davenport, A.G., ''The Spectrum of Horizontal Gustiness Near the Groundin High Winds'', *Quart. J. Roy. Met. Soc.*, V. 87, 1961, pp. 194-211.

10. Simiu, E., ''Equivalent Static Wind Loads to Tall Building Design'', *J. Struct. Div. Proc. ASCE*, V. 102, No. ST4, 1976, pp. 719.-736.

11. *IS:875 (Part 3), ''Code of Pratice for Design Loads (Other than Earthquake) for Buildings and Structures, Part 3, Wind Loads*'', Bureau of Indian Standard, New Delhi, 1987.

12. Davenport, A.G., ''The Dependence of Wind Load upon Meteorological Parameters'', *Proc. Int. Res. Seminar on Wind Effects on Buildings and Struct.*, University of Toronto Press, Tornoto, Canada, 1968, pp. 19-82.

13. Jennings, P.C. and Bielak, J., ''Dynamics of Buildings-Soil Interaction'', *Bull, Seism. Soc. Amer.* V. 63, 1973, pp.9-48.

14. Gupta, V.K. and Trifunac, M.D., ''Investigation of Building Response to Translational and Rotational Earthquake Excitations'', *Report CE 89-02*, University of Southern California, Los Angeles, U.S.A., 1989.