A note on the codal gust factor provisions for concrete chimneys

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The gust factor method, as specified in IS:4998 (Part I)-1992[2], for estimating the along-wind response of reinforced concrete chimneys is a special case of the modal gust factor approach. This is based on considering the variation of the peak fluctuating component of the total wind load with the height above ground to be linear. This paper examines the resulting inaccuracies in the response estimates by comparison with the exact stochastic estimates in case of four example chimneys of different heights. An alternative expression for the F_d load, due to the fluctuating component of wind has been suggested for obtaining more accurate moment estimates. It has been shown through a comparative study that the exact response estimates may change significantly for the wind environment considered in IS:875 (Part 3)-1987[1], and thus, it may be necessary to reformulate the expression of the gust factor as in IS:4998 (Part 1)-1992[2] for an appropriate design wind environment.

With the growing consciousness towards air pollution and the standards on smoke dispersal via chimneys becoming more stringent, chimneys of greater heights are being constructed. Tall chimneys are particularly susceptible to wind pressures, and therefore, for reasons of both safety and economy, it is important to correctly estimate the design wind loads expected to act on them during their lifetimes. Recognising this, the Bureau of Indian Standards revised the code of practice for the design of reinforced concrete chimneys in 1992. The revised code, IS:4998 (Part 1)-1992[2] (hereafter referred to as the 'chimney code'), has adopted the gust factor approach for the calculation of along-wind loads. In this, the design wind load, F_t is obtained by adding the mean wind load, F_m, with the wind load F_d, due to the (randomly) fluctuating component of wind. The suggested expression of F_d is based on the modal gust factor approach. In this approach, the largest peak value of the fluctuating load is obtained from that part of the mean wind load which is in the fundamental mode of the chimney. The suggested expression has been obtained by neglecting the variation of chimney mass with height and by assuming that the fundamental mode shape of the chimney is linear. As a result, the peak dynamic component of the wind load has been considered to be linearly varying with height, in preference to the conventional approach of assuming the same distribution as the mean (static) component. This approach is therefore referred to as the triangular gust factor approach. Even though this approach offers an improvement over the conventional approach as shown by Vickery[3], the modal gust factor approach is an approximate approach. Further, since the assumptions made for the codal expression for F_d are violated for the usually provided chimneys, there is a need to examine the accuracy of this expression in greater detail.

The calculation of the gust factor, G in the chimney code is based on the wind characteristics adopted by Vickery[3] for open terrains. Except for the mean wind velocity profile, these characteristics are substantially different from those adopted in formulating the gust factor provisions for buildings as in IS:875 (Part 3)-1987[1] (hereafter referred to as the 'wind code'). This implies that a concrete chimney at a site may be designed for a different wind environment than what a nearby building would be designed for. Hence, it is desirable to examine

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whether these two different wind environments may lead to substantially different chimney designs and whether there is a need to revise the gust factor expressions in the chimney code for compatibility with the wind code.

In this paper, exact profiles for peak fluctuating moment have been obtained for four example chimneys of different configurations and compared with the approximate profiles obtained according to the triangular gust factor approach. Exact moment profiles have also been obtained for the wind environment considered by the wind code to study the impact of a different environment on the response estimates. For completeness in the paper, a review of the exact stochastic formulation, the gust factor approach, and of the wind environments used in the chimney and wind codes, is also included.

Review of formulation

It is convenient to formulate the along-wind response of cylindrical chimneys by using random vibration principles. These structures have line-like configurations and thus, their aerodynamic properties are well defined by the local cross-sections. However, the stochastic formulation may not be simple enough for practical design calculations, and therefore, this formulation has been further simplified to the gust factor approach (Davenport[4], Vellozzi and Cohen[5], Vickery[6]). In the following sub-sections, the exact stochastic formulation and its simplification to the gust factor approach are reviewed in detail.

Exact stochastic formulation

Let us model a chimney as fixed-base Euler-Bernoulli beam of circular cross section which is subjected to a drag wind force, $F(z,t)$ per unit height. On ignoring the contributions of the air mass acceleration, this may be written as

$$F(z,t)=\frac{1}{2}\rho C_D(z)D(z)\left\{U(z,t)-\dot{X}(z,t)\right\}^2 \qquad (1)$$

where $X(z,t)$ is the along-wind displacement response of chimney at height, z, and time, t, $C_D(z)$ and $D(z)$ are the local drag coefficient and diameter respectively at height, z, and ρ is the mass density of air. Taking $X(z,t) = X(z) + x(z,t)$, $U(z,t) = U(z) + u(z,t)$, and on ignoring the second order terms, equation (1) becomes

$$F(z,t)=\frac{1}{2}\rho C_D(z)D(z) \times \{U^2(z)+2U(z)u(z,t)-2U(z)\dot{x}(z,t)\} \qquad (2)$$

Thus, the mean component, $F(z)$, of $F(z,t)$ becomes

$$F(z)=\frac{1}{2}\rho C_D(z)D(z)\,U^2(z) \qquad (3)$$

and, the fluctuating component, $\hat{f}(z,t)$, becomes

$$\hat{f}(z,t)=f(z,t)-\rho C_D(z)D(z)U(z)\dot{x}(z,t) \qquad (4)$$

with

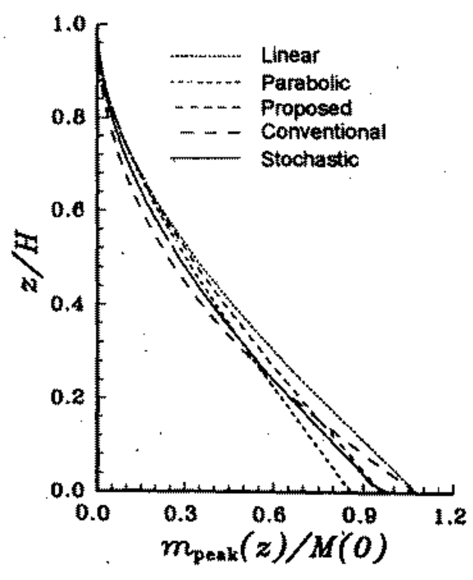

Fig 1 Comparison of the profiles for normalised peak fluctuating moment, Chimney no 1

$$f(z,t)=\rho C_D(z)D(z)U(z)u(z,t) \qquad (5)$$

The equation of motion for the chimney can be written as

$$\rho_m A(z)\ddot{X}(z,t)+c(z)\dot{X}(z,t) + \frac{\partial^2}{\partial z^2}\left[EI(z)X''(z,t)\right]=F(z,t) \qquad (6)$$

where $\rho_m A(z)$ and $c(z)$ respectively represent the mass and damping per unit height, and $EI(z)$ represents the flexural rigidity at height, z. On decomposing $X(z,t)$ and $F(z,t)$ into static (mean) and dynamic (fluctuating) parts, equation (6) leads to the following equations

$$\rho_m A(z)\ddot{x}(z,t)+c(z)\dot{x}(z,t) + \frac{\partial^2}{\partial z^2}\left[EI(z)x''(z,t)\right]=\hat{f}(z,t) \qquad (7)$$

$$\text{and}\ \frac{d^2}{dz^2}\left[EI(z)X''(z)\right]=F(z) \qquad (8)$$

Equation (8) gives the mean response, $X(z)$, from a static analysis of the chimney. Further, on using equation (4), equation (7) may be alternatively written as

$$\rho_m A(z)\ddot{x}(z,t)+\left\{c(z)+\rho C_D(z)D(z)U(z)\right\}\dot{x}(z,t) + \frac{\partial^2}{\partial z^2}\left[EI(z)x''(z,t)\right]=f(z,t) \qquad (9)$$

On expanding $x(z,t)$ in terms of mode shapes, $\phi_1(z)$, $\phi_2(z)$,....., and normal coordinates, $q_1(t)$, $q_2(t)$,...., that is

$$x(z,t)=\sum_i \phi_i(z)q_i(t) \tag{10}$$

in equation (9), multiplying both sides by $\phi_j(z)$, by taking the integral of both sides for the entire height, H, of the chimney, and on using modal orthogonality relationships, we get the decoupled equation of motion in the ith mode as

$$M_i\ddot{q}_i+C_i\dot{q}_i+K_iq_i=F_i; \qquad i=1,2,3,..... \tag{11}$$

Here,

$$M_i=\int_0^H \rho_m A(z)\phi_i^2(z)dz \tag{12}$$

$$C_i=\int_0^H \{c(z)+\rho C_D(z)D(z)U(z)\}\phi_i^2(z)dz=4\pi\zeta_i M_i n_i \tag{13}$$

$$K_i=\int_0^H \frac{d^2}{dz^2}\left[EI(z)\phi_i''(z)\right]\phi_i(z)dz=4\pi^2 n_i^2 M_i \tag{14}$$

$$F_i=\int_0^H f(z,t)\phi_i(z)dz$$

$$=\rho\int_0^H C_D(z)D(z)U(z)u(z,t)\phi_i(z)dz \tag{15}$$

are respectively the modal mass, damping, stiffness and force in the ith mode. Further, ζ_i and n_i respectively represent the damping ratio and natural frequency in this mode. In writing equation (11), coupling between different modal equations due to damping has been assumed to be weak and has thus been ignored. It may be noted that the modal damping ratio ζ_i, is equal to the summation of the damping inherent in the structure and the aerodynamic damping.

Equations (10) and (11) describe the fluctuating response, $x(z,t)$ of the chimney for a given time-history of the wind velocity fluctuations, $u(z,t)$. However, since $u(z,t)$ constitutes a random process, it is useful to characterise the response process, $x(z,t)$, by the largest peak amplitude for a given probability of exceedance. Let both these processes be assumed to be zero-mean Gaussian processes. Following equation (10), the spectral density function, $S_x(z,n)$, for the response process, $x(z,t)$, may be written as

$$S_x(z,n)=\sum_i\sum_j \phi_i(z)\phi_j(z)S_{q_iq_j}(n) \tag{16}$$

where, $S_{q_iq_j}(n)=H_i(n)H_j^*(n)S_{F_iF_j}(n)$ (17)

is the cross-spectral density function of the modal coordinate processes, $q_i(t)$ and $q_j(t)^T$. In equation (17),

$$H_i(n)=\frac{1}{4\pi^2 n_i^2 M_i\left[\left(1-\frac{n^2}{n_i^2}\right)+i2\zeta_i\frac{n}{n_i}\right]} \tag{18}$$

is the transfer function relating the relative displacement of the equivalent (single-degree-of-freedom) oscillator in the ith mode to the ith modal force. Further,

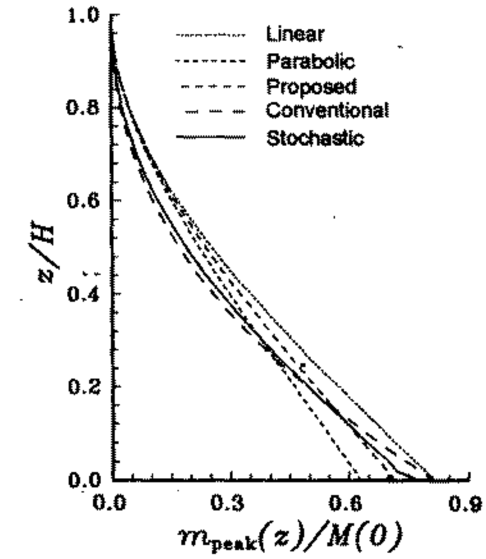

Fig 2 Comparison of the profiles for normalised peak fluctuating moment, Chimney no 2

$$S_{F_iF_j}(n)=\rho^2\int_0^H\int_0^H C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)U(z_2)$$
$$\times\ S_u(z_1,z_2,n)\phi_i(z_1)\phi_j(z_2)dz_1dz_2 \tag{19}$$

is the cross-spectral density function of the modal force processes, $F_i(t)$ and $F_j(t)$, where $S_u(z_1,z_2,n)$ is the cross-spectral density of the velocity fluctuation processes at heights, z_1 and z_2. The cross-spectrum, $S_u(z_1,z_2,n)$, can be written as

$$S_u(z_1,z_2,n)=\gamma(z_1,z_2,n)\sqrt{S_u(z_1,n)S_u(z_2,n)} \tag{20}$$

where $S_u(z,n)$ is the velocity spectrum at height, z and $\gamma(z_1,z_2,n)$ is the normalised cross-spectrum or the coherence function. On substituting equations (17), (19) and (20) in equation (16), we can express the mean-square spectral density of the fluctuating displacement response process at height, z as

$$S_x(z,n)=\rho^2\sum_i\sum_j\phi_i(z)\phi_j(z)H_i(n)H_j^*(n)$$

$$\int_0^H\int_0^H C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)U(z_2)\gamma(z_1,z_2,n)$$

$$\sqrt{S_u(z_1,n)S_u(z_2,n)}\phi_i(z_1)\phi_j(z_2)dz_1dz_2 \tag{21}$$

On neglecting modal cross-correlation, equation (21) is simplified to

$$S_x(z,n)=\rho^2\sum_i\phi_i^2(z)|H_i(n)|^2$$

$$\int_0^H \int_0^H C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)U(z_2)\gamma(z_1,z_2,n)$$

$$\sqrt{S_u(z_1,n)S_u(z_2,n)}\phi_i(z_1)\phi_i(z_2)dz_1dz_2 \qquad (22)$$

Similarly, the spectral density function, $S_m(z,n)$, for the fluctuating bending moment response, $m(z,t)$, can be written as

$$S_m(z,n) = \rho^2 \sum_i \{EI(z)\phi_i''(z)\}^2 |H_i(n)|^2$$

$$\int_0^H \int_0^H C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)U(z_2)\gamma(z_1,z_2,n)$$

$$\sqrt{S_u(z_1,n)S_u(z_2,n)}\ \phi_i(z_1)\phi_i(z_2)dz_1dz_2 \qquad (23)$$

The root-mean-square (r.m.s.) value, σ of a zero-mean process is obtained from the spectral density, $S(n)$ of the process as

$$\sigma(z) = \sqrt{\int_0^\infty S(z,n)dn} \qquad (24)$$

and the largest peak amplitude is obtained by multiplying the r.m.s value with a peak factor. For the non-zero mean processes, $X(z,t)$ and $M(z,t)$, the largest peak amplitudes may be thus expressed as

$$X_{peak}(z) = X(z) + g_{fx}(z)\sigma_x(z) \qquad (25)$$

$$M_{peak}(z) = M(z) + g_{fm}(z)\sigma_m(z) \qquad (26)$$

where, $\sigma_x(z)$ and $\sigma_m(z)$ are the r.m.s. values of the processes, $x(z,t)$ and $m(z,t)$ respectively, as obtained by using equation (24). Further, $g_{fx}(z)$, and $g_{fm}(z)$ denote the peak factors obtained from the zeroth, second and fourth moments of the spectral densities, $S_x(z,n)$ and $S_m(z,n)$ respectively, for a given level of confidence[8,9,10,11]. For a large number of peaks in a single realization of a process with the spectral density, $S(z,n)$, the peak factor, g_f for the expected largest peak in the process may be approximated as[12]

$$g_f = \sqrt{2 ln \nu T} + \frac{0.5772}{\sqrt{2 ln \nu T}} \qquad (27)$$

where

$$\nu = \sqrt{\frac{\int_0^\infty n^2 S(z,n)dn}{\int_0^\infty S(z,n)dn}} \qquad (28)$$

is the expected rate of positive crossings in the process and T is the duration of the process. Since it is a common practice to adopt the hourly mean wind velocity as the mean wind velocity, $U(z)$ at height, z, T is taken to be 1 hour. A similar procedure involving equations (24)-(28) can be followed for finding the largest peak in any other response process, once the spectral density function is determined as in equations (22) and (23) for the displacement and moment responses.

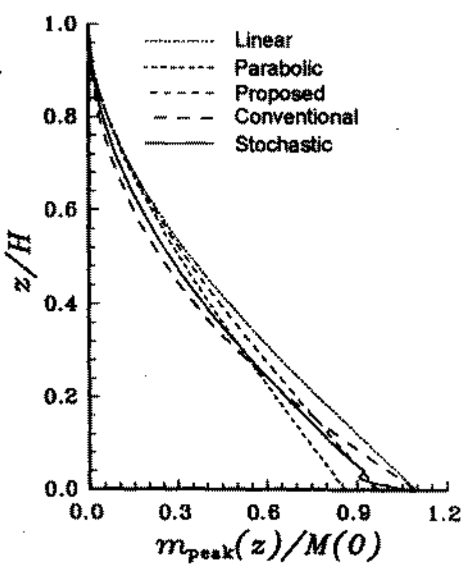

Fig 3 Comparison of the profiles for normalised peak fluctuating moment, Chimney no3

Gust factor approach

The gust factor approach seeks to estimate the expected largest peak response from the mean response, say $X_{peak}(z)$ from $X(z)$, by simply multiplying it with a height-independent factor called as the gust factor. As shown in the *Appendix*, this factor may be expressed as

$$G_x = 1 + g_{fx} r \left[B + \frac{SE}{\zeta_1} \right]^{\frac{1}{2}} \qquad (29)$$

where, g_{fx} represents the (height-independent) peak factor; r represents a measure of ground roughness vis-a-vis the height of the stack; B represents the background excitation factor; S represents the size reduction factor; and E is a measure of the available energy in the wind flow at the natural frequency of the structure. The expressions of these parameters are given in the *Appendix*. The expression of G_x as in equation (29) may be multiplied with the mean displacement response to correctly estimate the largest peak displacement response within the accuracy of the fundamental mode dominance in the chimney response and the accuracy of the considered wind characteristics model. Even though equation (29) gives the gust factor for the displacement response, this will also be applicable for any other response process, say for the moment process, $M(z,t)$, without any further modifications. Therefore, we may alternatively write equation (29) as

$$G = 1 + g_f r \left[B + \frac{SE}{\zeta_1} \right]^{\frac{1}{2}} \qquad (30)$$

to represent the gust factor to be used with all the mean responses where, the various parameters on the right hand side are obtained for the displacement response as explained above. Conventionally, various world codes have been based on the calculation of the design wind load by multiplication of the mean wind load, $F(z)$ with the gust factor, G so that a static analysis of the structure may be carried out for this design load. The accuracy of this approach depends on whether the fundamental mode dominates the profile of a given response function. Since the stress response function have greater domination of the higher modes, this approach gives 'not so accurate' response estimates in such cases as it does in the case of the displacement response. It can be shown for a chimney with uniform mass distribution that the mean displacement response is assured to be in the fundamental mode if the mean wind load distribution is similar to the fundamental mode shape. Hence, it has been considered appropriate to obtain the largest peak value of the fluctuating load from that part of the mean load which is in the fundamental mode, rather than from the total mean wind load itself[3]. On expanding the total mean load $F(z)$ as

$$F(z)=\alpha_1\phi_1(z)+\alpha_2\phi_2(z)+..., \quad (31)$$

multiplying both sides by $\rho_m A(z)\phi_1(z)$, integrating from 0 to H, and on using the orthogonality relationships, the total mean wind load as in the fundamental mode becomes

$$F_1(z)=\alpha_1\phi_1(z)=\frac{\phi_1(z)\int_0^H \rho_m A(z)\phi_1(z)F(z)dz}{M_1} \quad (32)$$

Multiplying this with $(G-1)$ gives the proportionate value of the peak fluctuating load which can now be added to $F(z)$ to obtain the largest peak in the total wind load as $F(z) + (G\text{-}1)F_1(z)$. This approach is called the modal gust factor approach. $F_1(z)$ can be further simplified on assuming uniform mass properties through the height, that is, $\rho_m A(z) = \rho_m A$, and linear mode shape, $\phi_1(z) = z/H$, to

$$F_1(z)=\frac{3z}{H^3}\int_0^H F(z)zdz \quad (33)$$

In this form, the gust factor approach is called as the triangular gust factor approach[3].

Description of wind characteristics

The application of the gust factor approach through codal provisions requires a proper description of the wind characteristics in the form of velocity fluctuation spectral density function, mean wind velocity profile, and coherence function. For a set of assumed characteristics, the parameters, B, S, E and r, as in equation (29) are expressed graphically or through semi-empirical expressions. For example, the wind code gives graphs to determine these parameters in case of buildings, and the chimney code gives expressions for these in case of chimneys. These two codes however assume different wind characteristics, except for the mean wind velocity profile, as is explained in the following sub-sections.

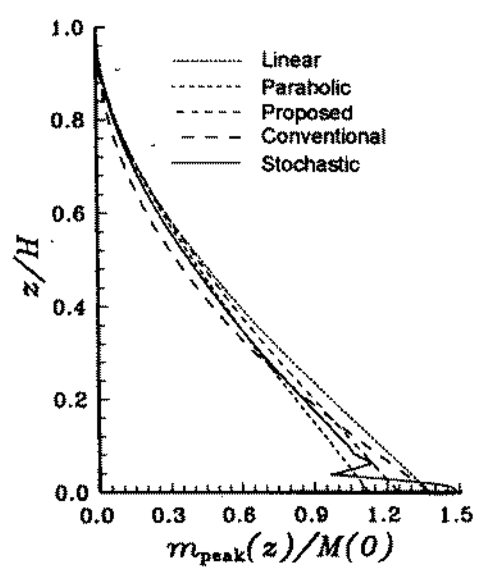

Fig 4 Comparison of the profiles for normalised peak fluctuating moment, Chimney no4

Wind characteristics in IS:875 (Part 3)-1987

In the wind code, IS:875 (Part 3)-1987, the velocity spectrum is assumed to be independent of height, that is, $Su(z,n) \equiv Su(n)$. The following expression for $S_u(n)$, as in Vickery[6], has been used.

$$\frac{nS_u(n)}{\sigma_u^2}=\frac{2}{3}\frac{\chi^2}{(1+\chi^2)^{4/3}} \quad (34)$$

where, σ_u is the r.m.s. value of the (height-independent) velocity fluctuations, and

$$\chi=\frac{nL(H)}{U(H)} \quad (35)$$

In equation (35), $L(H)$ is the integral length scale of turbulence at the top of the structure. The expression in equation (34) was empirically suggested by Davenport[13] on the basis of averaging several wind measurements over terrains of different roughness and at different heights, and by considering

$$L(H)=1200\frac{U(H)}{U(10)} \quad (36)$$

The wind code specifies values of $L(H)$ in graphical form depending on the terrain type and building height, H. The value of σ_u denoting the intensity of velocity fluctuations is considered to be related to the roughness factor, r as (see reference 6)

$$r=\frac{1+\beta+2\alpha}{1+\beta+\alpha}\frac{2\sigma_u}{U(H)} \quad (37)$$

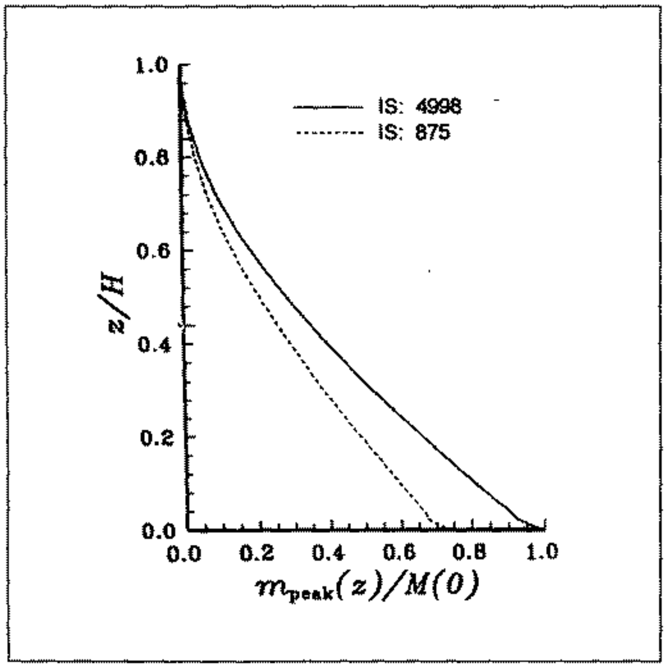

Fig 5 Comparison of the exact moment profiles for different wind environments, Chimney no. 1

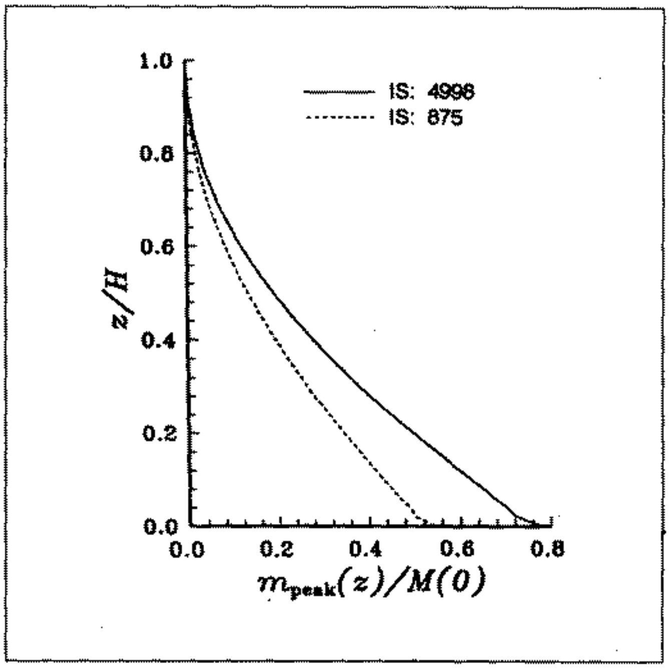

Fig 6 Comparison of the exact moment profiles for different wind environments, Chimney no. 2

where, α and β are the constants used to describe the mean wind velocity profile and fundamental mode shape respectively as in

$$U(z) = U(H)\left(\frac{z}{H}\right)^{\alpha} \tag{38}$$

and

$$\phi_1(z) = \left(\frac{z}{H}\right)^{\beta} \tag{39}$$

The value of α depends on the terrain category, that is, α may be taken as 0.14 for Category 2[6]. The value of β is close to unity for stocky buildings and greater than unity for tall chimneys. The values of the roughness factor, r, have been specified by the wind code in form of the graph for $g_f r$ for different building heights and terrain categories. Finally, the coherence function in the wind code has been considered to be[6]

$$\gamma(M_1, M_2, n) = \exp\left\{\frac{-2n\sqrt{C_y^2(y_1-y_2)^2 + C_z^2(z_1-z_2)^2}}{U(z_1)+U(z_2)}\right\} \tag{40}$$

for the two points, $M_1\,(y_1, z_1)$ and $M_2(y_2, z_2)$, with $C_y = 10$ and $C_z = 12$.

It may be mentioned that though the gust factor calculations are based on the power law variation of the mean wind velocity (see equation (38)), logarithmic law variation has been used for the computation of the mean wind load, F_z. Due to this, the gust factor provisions in the wind code are inconsistent with the specifications on the mean wind load.

Wind characteristics in IS:4998 (Part 1)-1992

The wind characteristics used in the chimney code for the gust factor calculations are the same as considered by Vickery[3] for Category 2. Here, in contrast with the wind code, the velocity spectrum is considered to be the height-dependent von Karman spectrum, that is,

$$\frac{nS_u(z,n)}{\sigma_u^2(z)} = 4\frac{\chi(z)}{[1+70.8\chi^2(z)]^{5/6}} \tag{41}$$

with

$$\chi(z) = \frac{nL(z)}{U(z)} \tag{42}$$

In equation (42), $L(z)$ is the integral length scale of turbulence taken as

$$L(z) = 30z^{0.35} \tag{43}$$

Further, the r.m.s. value of wind velocity fluctuations at height, z is taken as

$$\sigma_u(z) = (0.311 - 0.089 \log z)U(z) \tag{44}$$

Finally, the expression used for the coherence function is

$$\gamma(z_1, z_2, n) = \varepsilon\xi\pi\left[-\frac{10n|z_1 - z_2|}{U\left(\frac{z_1+z_2}{2}\right)}\right] \tag{45}$$

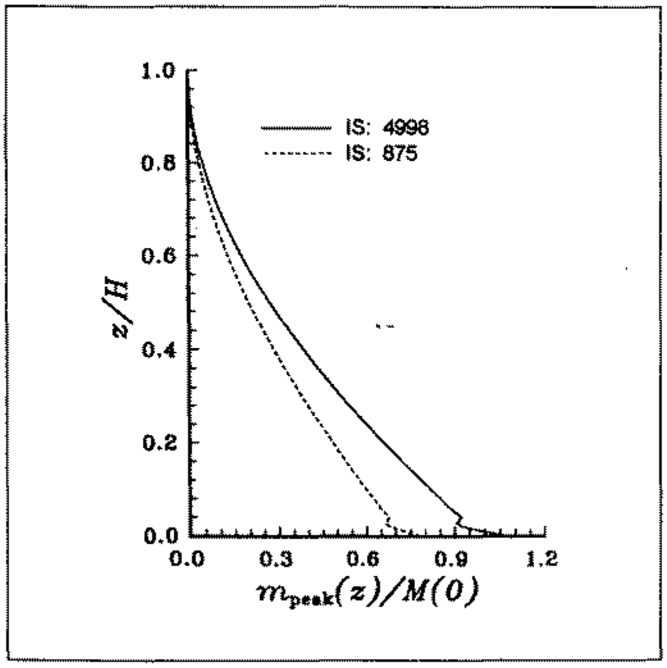

Fig 7 Comparison of the exact moment profiles for different wind environments, Chimney no. 3

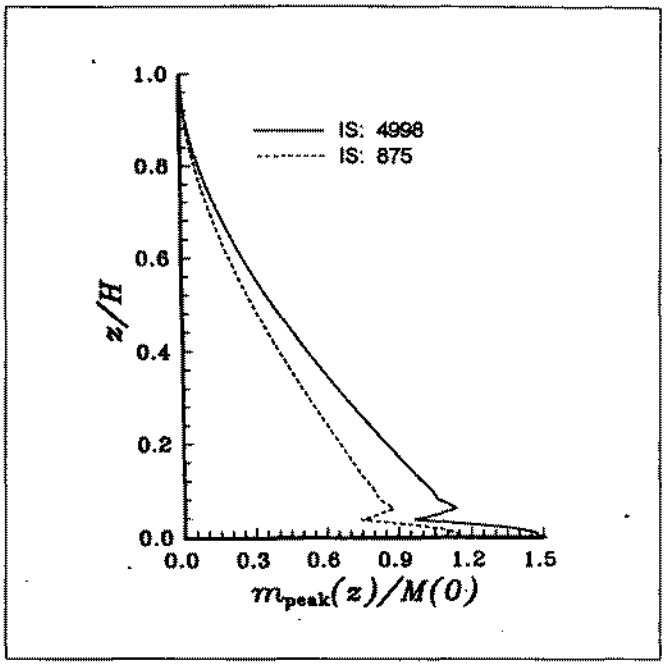

Fig 8 Comparison of the exact moment profiles for different wind environments, Chimney no. 4

It may be noted that in contrast with the wind characteristics model given in the wind code, this model does not have any flexibility regarding the choice of meteorological parameters such as the length scale, and r.m.s. value of fluctuations with respect to the terrain category. Nevertheless, this model uses a more realistic height-dependent spectrum for the velocity fluctuations. The chimney code accounts for different terrain roughness (other than the Category 2) in an adhoc manner through an appropriate change in U(10). However, this is inadequate to describe the required changes in the remaining wind characteristics model and may thus lead to unrealistic gust factors. Moreover, as in the wind code, the gust factor calculations here are inconsistent with the mean wind load calculations due to considering different mean wind velocity variations.

Numerical study

Modal gust factor approach

The simplified form of $F_1(z)$, as in equation (43), is based on the assumption that the chimney has the same cross-section at all heights and that its fundamental mode shape is linear. Since the designs of chimneys usually involve the gradual tapering of the diameter and thickness from base to the top, $\rho_m A(z)$ actually decreases with increasing z. Also, the linear mode shape is not a realistic mode shape for a Euler-Bernoulli beam type of structure. In view of these approximations and because the modal gust factor approach is itself an approximation, it becomes necessary to check the accuracy of the triangular gust factor approach through a comparison of the (approximate) response estimates from this approach with those from an exact stochastic analysis.

For the numerical comparison of the approximate and exact response estimates, four reinforced concrete chimneys of widely different configurations have been considered. Two of these example chimneys, that is, Chimney no. 1 and 2, are the hypothetical chimneys as also considered by Vickery[3]. The particulars of these two chimneys are given in *Table 1*. The other two example chimneys are the existing chimneys. Chimney no. 3 is the 180 m high chimney at Bokaro and Chimney no. 4 is the 120 m high chimney at Obra. The particulars of these two chimneys as considered by Arya and Paul[14] are given in *Table 2*. The modulus of elasticity, *E*, has been taken to be 33,500 N/mm^2 as recommended by the chimney code for M30 concrete. The density of concrete, ρ_m, is taken to be 25 kN/m^3, and the damping ratio is uniformly assumed to be 0.016 in all the modes of vibration.

The wind characteristics considered are the same as those described earlier for the gust factor provisions of the chimney code. Since these characteristics were proposed by Vickery[3] for Category 2 terrain (open terrain), the mean wind velocity profile for Category 2 terrain as proposed by the wind code has been considered here. It may be emphasised again that this profile is different from that considered by Vickery (and thus, by the chimney code) in obtaining the expressions for *B*, *S*, *E* and *r*. The basic wind speed is taken to be 47 m/s, and the mass density of air, ρ, is taken to be 1.2 kg/m^3.

The exact stochastic analysis has been carried out by considering the first six modes in case of each example chimney. In each case, the mode shapes and natural frequencies have been determined by discretizing the chimney into fifty Euler-Bernoulli elements and by carrying out a finite element analysis. The fundamental natural frequencies of all the example chimneys are given in *Tables* 1 and 2. In each case, the (largest) peak fluctuating moment, $m_{peak}(z)(=g_{fm}(z)\sigma_m(z))$ has been calculated at different values of z by the exact analysis as well as by using the triangular gust factor approach. In the latter, the gust factor, G, has been calculated by direct calculation of the ratio, $\sigma_x(z)/X(z)$ as in equation (52) of the Appendix, instead of using the expressions of B, S, E, r and g_f as specified by the chimney code. The various estimates of the peak fluctuating moment in each case have been normalised with respect to the respective mean base moment, $M(0)$. The profiles of the normalised peak moment, $m_{peak}(z)/M(0)$, as obtained for the stochastic and approximate analyses have been compared in *Figs* 1 to 4 respectively for the Chimneys no. 1–4. The results of the approximate analysis are denoted as 'linear' in these figures. Also compared are the estimates from the conventional gust factor approach as obtained by multiplying the mean moment values by $(G-1)$. It may be observed that the conventional approach becomes unconservative for top 70 percent to 80 percent height of the chimneys, while the triangular gust factor approach gives 'too conservative' estimates for almost the entire height of a chimney with as much as 30 percent higher estimates in the top half of the chimney. In order to arrive at an expression of $F_1(z)$ as an alternative to the triangular gust factor approach, we consider the parabolic fundamental mode shape, that is, $\phi_1(z) = z^2/H^2$ in equation (32) and obtain

$$F_1(z)=\frac{5z^2}{H^5}\int_0^H F(z)z^2dz \tag{46}$$

The estimates of $m_{peak}(z)/M(0)$ based on this expression of $F_1(z)$ are shown as 'parabolic' estimates in the figures. It may be seen that for about 30% height towards the base, this expression leads to unconservative estimates. In fact, at the base, the 'parabolic' estimates may be quite unconservative. Considering the trends of the 'linear' and 'parabolic' curves, it appears that a curve in between these two curves may have a reasonable matching with the curve for the stochastic estimates. In view of this, the fundamental mode shape is alternatively considered as $\phi_1(z) = (z/H)^{1.5}$ instead of the linear mode shape, and thus the expression of $F_1(z)$ is taken as

$$F_1(z)=\frac{4z^{1.5}}{H^4}\int_0^H F(z)z^{1.5}dz \tag{47}$$

in place of equation (43). The revised estimates of $m_{peak}(z)/M(0)$ for this expression of $F_1(z)$ as shown in *Figs* 1 – 4 appear to be good approximations to the stochastic estimates. In fact, those are conservative for most of the chimney heights in each case without being 'too conservative'. Thus, it may be appropriate to replace the expression of $F_1(z)$ as in equation (43) by that in equation (47), and thus to consider the following alternate expression of F_{zf} in the gust factor approach adopted by IS: 4998 (Part 1)-1992,

$$F_{zf}=4\frac{(G-1)}{H^{2.5}}\left(\frac{z}{H}\right)^{1.5}\int_0^H F_{zm}z^{1.5}dz \tag{48}$$

This expression may be deemed applicable to the other terrains (than the Category 2 terrain) also since the adhoc U(10)-based variations in the wind model for these terrains may not cause significant qualitative changes in the response

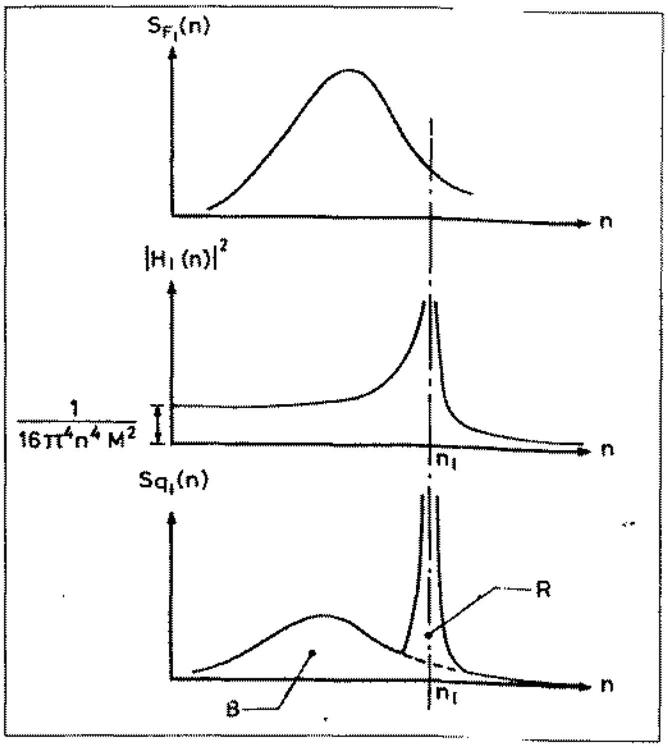

Fig 9 Background and resonant components of the response spectral density function

estimates.

Wind characteristics

As described earlier, the wind characteristics adopted by the wind and chimney codes for the gust factor approaches in case of buildings and chimneys respectively are based on independent sources. There is a fundamental difference between these two sets of characteristics as the wind code assumes a height-independent velocity spectrum and the chimney code provisions are based on a more realistic height-dependent spectrum. It has been shown by Chandra Sekher and Gupta[15] in case of buildings that a height-dependent spectrum may give substantially different gust factors as compared to a height-independent spectrum. However, the wind characteristics model adopted by the wind code is much more flexible as it properly considers the variations due to different types of terrain. The provisions in the chimney code are strictly applicable to the open terrain conditions only. A numerical study has been carried out in this sub-section to appreciate how different these two models are as regards the stochastic response estimated in case of chimneys no. 1–4 situated in an open terrain. For this purpose, the stochastic estimates of $m_{peak}(z)/M(0)$ have been recomputed for the wind characteristics adopted by the wind code, and compared with those shown in *Figs* 1-4. *Figs* 5–8 show the comparison of these two sets of profiles for Chimneys no. 1–4 respectively. It may be noted that the value of α has been taken as 0.14[6], and β has been taken as 2.0 for estimating the value of σ_u from r as in equation (37). Further, the values of r have been estimated from the graphs of $g_f r$ as given in the wind code by estimating g_f which is actually independent of σ_u (see equation (27)).

It may be seen from *Figs* 5 – 8 that the peak fluctuating moment responses for the two wind models are significantly different from each other at all heights of the example chimneys. The estimates for the chimney code are consistently higher than those for the wind code by about 30–40 percent. This implies that the design wind environments as considered by the codal provisions on gust factors for buildings and chimneys are inconsistent. This inconsistency becomes obvious from the fact that one set of provisions is largely based on the work of Vickery in 1970 while the other set is directly taken from the work of the same author in 1985. It is also not obvious that which of these two models appropriately characterises the wind environment in India. Hence, a more comprehensive approach to characterising the wind model and correspondingly formulating the gust factor provisions for different types of structures becomes desirable. It may be mentioned here that in contrast with the code provisions of concrete chimneys, the code provisions for estimating dynamic wind loads on steel chimneys as in IS: 6533 (Part 2)-1989[16] are not even based on the gust factor approach.

Conclusions

The codal specifications on the design fluctuating load as in the gust factor method for the design of reinforced concrete chimneys have been critically looked at. It has been shown by finding the peak fluctuating moments in case of four example chimneys of different heights that the existing codal expression for the load, F_{zf}, leads to consistently conservative estimates. These estimates may be too conservative for the top half of a chimney. A modified expression for F_{zf} has been proposed and it has been shown that this may lead to more reasonable estimates of the bending moments at various levels of a chimney.

The wind environments considered for the gust factor provisions in IS: 875 (Part 3)-1987[1] and IS: 4998 (Part 1)-1992[2] have been compared by comparing the exact peak fluctuating moment profiles due to both environments in case of the example chimneys. It is seen that the two wind models are inconsistent and thus lead to significantly different results despite the use of same mean wind velocity profile. This inconsistency is due to the fact that the two models have been adopted from different sources. Since neither of these models necessarily characterises the wind environment present in India, it is suggested that the expressions for the gust factor as in IS: 4998 (Part 1)-1992[2] be revised by considering a wind model which is known to be representative of Indian wind characteristics. This revision should be based on using the same mean wind velocity profile as considered for calculating the mean wind load, and different expressions should be obtained for different terrain categories.

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Appendix

Derivation of gust factor, G_x

The gust factor approach is based on the assumption that the response of the structure is in its fundamental mode of vibration. Hence, the standard deviation of $x(z,t)$ may be simplified to

$$\sigma_x(z) = \phi_1(z)\left[\frac{\rho}{4\pi^2 n_1^2 M_1}\left\{\int_0^H \int_0^H C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1) U(z_2)R_1(z_1,z_2)\phi_1(z_1)\phi_1(z_2)dz_1dz_2\right\}^{1/2}\right] \quad (49)$$

$$\text{with } R_1(z_1,z_2) = \int_0^\infty \left|\overline{H}_1(n)\right|^2 \gamma(z_1,z_2,n)\sqrt{S_u(z_1,n)S_u(z_2,n)}\,dn \quad (50)$$

In equation (50), $|\overline{H}_1(n)|$ represents the non-dimensional form of $|H_1(n)|$ obtained by multiplying this with $4\pi^2 n_1^2 M_1$. Similarly, the mean response, $X(z)$, as given by equation (8) may be expressed in terms of the first mode as

$$X(z) = \frac{\phi_1(z)}{4\pi^2 n_1^2 M_1}\int_0^H F(z)\phi_1(z)dz$$

$$= \phi_1(z)\left[\frac{\rho}{8\pi^2 n_1^2 M_1}\int_0^H C_D(z)D(z)U^2(z)\phi_1(z)dz\right] \quad (51)$$

It may be observed that in both equations (49) and (51), the multipliers to the mode shape, $\phi_1(z)$ are independent of height, z. Hence, on dividing the r.m.s. response, $\sigma_x(z)$ (as in equation (49)) by the mean response, $X(z)$ (as in equation (51)), we obtain a height-independent ratio as

$$\frac{\sigma_x(z)}{X(z)} = \frac{2\sqrt{\int_0^H\int_0^H\left[\begin{matrix}\langle C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)\rangle\times\\ \langle U(z_2)R_1(z_1,z_2)\phi_1(z_1)\phi_1(z_2)dz_1dz_2\rangle\end{matrix}\right]}}{\int_0^H C_D(z)D(z)U^2(z)\phi_1(z)dz} \quad (52)$$

It may be noted that due to the assumed vibration of the structure in a single mode, the peak factor, $g_p(z)$, for the displacement response process will also be independent of height, z. Thus on rewriting equation 25 as

$$X_{peak}(z) = X(z)\left[1 + g_p(z)\frac{\sigma_x(z)}{X(z)}\right] \quad (53)$$

we obtain the gust factor,

$$G_x = 1 + g_p(z)\frac{\sigma_x(z)}{X(z)} \quad (54)$$

as a height-independent factor which should be multiplied with the mean response, X(z), to obtain the largest peak in the X(z,t) process.

It is possible to further simplify the calculation of gust factor, G_x, by decomposing the numerator on the right hand side of equation (52) into the background and resonant parts. To understand the rationale behind this approximation, let us rewrite equation (49) as

$$\sigma_x(z) = \phi_1(z)\left[\int_0^\infty S_{q1}(n)dn\right]^{1/2} \quad (55)$$

where

$$S_{q1}(n) = |H_1(n)|^2 S_{F1}(n) \quad (56)$$

is the spectral density of the displacement response of the equivalent oscillator in the fundamental mode. In equation 56

$$S_{F1}(n) = \rho^2\int_0^H\int_0^H C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)U(z_2)\gamma(z_1,z_2,n) \times \sqrt{S_u(z_1,n)S_u(z_2,n)}\phi_1(z_1)\phi_1(z_2)dz_1dz_2 \quad (57)$$

is the spectral density of the modal force process in the first mode. Fig 9 shows the spectrum, $S_{F1}(n)$, the mechanical admittance function, $|H_1(n)|^2$ and their product $(= S_{q1}(n))$. It may be seen that the admittance function, $|H_1(n)|^2$ is either equal to a constant value of $1/16\pi^4 n_1^4 M_1^2$ or to zero, except for the frequencies around the natural frequency, n_1. At these frequencies, for small damping, this function attains very high values in form of a sharp spike. Due to these, the spectrum, $S_{q1}(n)$ is characterised by

(i) the same shape as that of $S_{F1}(n)$, at the frequencies much lower than n_1,

(ii) sharp spike at the frequencies around n_1, and

(iii) little energy at the frequencies much higher than n_1.

This is however true only if the peak in the admittance function is not close to the peak in the spectrum of modal force. In such a case, the area under $S_{q1}(n)$ can be thought of comprising of

(i) the area under the spectrum for frequencies much smaller than n_1 as shown by B, and

(ii) the area for frequencies close to n_1 as shown by R in Fig 9.

The first contribution is called as the broad-band, non-resonant or background response and the second contribution is termed as the narrow-band or resonant response. With this understanding, the area under $S_{q1}(n)$ may be approximately written as

$$\int_0^\infty S_{q1}(n)dn \approx \frac{1}{16\pi^4 n_1^4 M_1^2} \times \int_0^\infty S_{F1}(n)dn + S_{F1}(n_1)\int_0^\infty |H_1(n)|^2 dn \quad (58)$$

and since the total area under $|H_1(n)|^2$ is equal to $1/64\pi^3\zeta_1 n_1^3 M_1^2$, we have

$$\int_0^\infty S_x(n)dn \cong \frac{1}{16\pi^4 n_1^4 M_1^2}\int_0^\infty S_{F1}(n)dn + \frac{S_{F1}(n_1)}{64\pi^3\zeta_1 n_1^3 M_1^2} \quad (59)$$

In view of these simplifications, equation (52) becomes

$$\frac{\sigma_x(z)}{X(z)} \sim r\left[B + \frac{SE}{\zeta_1}\right]^{1/2} \quad (60)$$

where

$$B = \frac{\int_0^H\int_0^H\left[\begin{matrix}\left(\bar\gamma(z_1,z_2)C_D(z_1)C_D(z_2)D(z_1)D(z_2)\right)\times\\ \left(U(z_1)U(z_2)\phi_1(z_1)\phi_1(z_2)dz_1dz_2\right)\end{matrix}\right]}{\left[\int_0^H C_D(z)D(z)U^2(z)\phi_1(z)dz\right]^2 / U^2(H)} \quad (61)$$

is called the background excitation factor. It is a measure of slowly varying component of the fluctuating wind load. In equation (61),

$$\bar\gamma(z_1,z_2) = \frac{1}{\sigma_u^2(H)}\int_0^\infty S_u(z_1,z_2,n)dn = \frac{\int_0^\infty S_u(z_1,z_2,n)dn}{\int_0^\infty S_u(H,H,n)dn} \quad (62)$$

is the frequency-independent and normalised measure of correlation between the velocity processes at z_1 and z_2 heights, with $\sigma_u(H)$ denoting the r.m.s. value of the wind velocity fluctuations at $z = H$. Further, in equation (60),

$$S = \frac{\int_0^H\int_0^H\left[\begin{matrix}\left(\frac{S_u(z_1,z_2,n_1)}{S_u(H,n_1)}C_D(z_1)C_D(z_2)D(z_1)\right)\times\\ \left(D(z_2)U(z_1)U(z_2)\phi_1(z_1)\phi_1(z_2)dz_1dz_2\right)\end{matrix}\right]}{\left[\int_0^H C_D(z)D(z)U^2(z)\phi_1(z)dz\right]^2 / U^2(H)} \quad (63)$$

is called the size reduction factor, and

$$E = \frac{\pi n_1}{4} \frac{S_u(H,n_1)}{\sigma_u^2(H)} \qquad (64)$$

is called the gust energy factor. E is a measure of the available energy in the wind flow at the natural frequency of the structure. Further,

$$r = \frac{2\sigma_u(H)}{U(H)} \qquad (65)$$

is a measure of ground roughness vis-a-vis the height of the stack. It may be seen that B and SE/ζ_1 are, respectively, the measures of the non-resonant and resonant parts of the total response. Depending upon the model used for wind characteristics, different expressions or graphs are used to describe these parameters.

On using the approximation as in equation (60), the expression for the gust factor, G_x, as in equation (34) may be written as

$$G_x = 1 + g_{fx} r \left[B + \frac{SE}{\zeta_1} \right]^{1/2} \qquad (66)$$

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