MANUFACTURING PROCESSES: (TA-202)

Mechanics of Cutting

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Geometry of chip Formation:

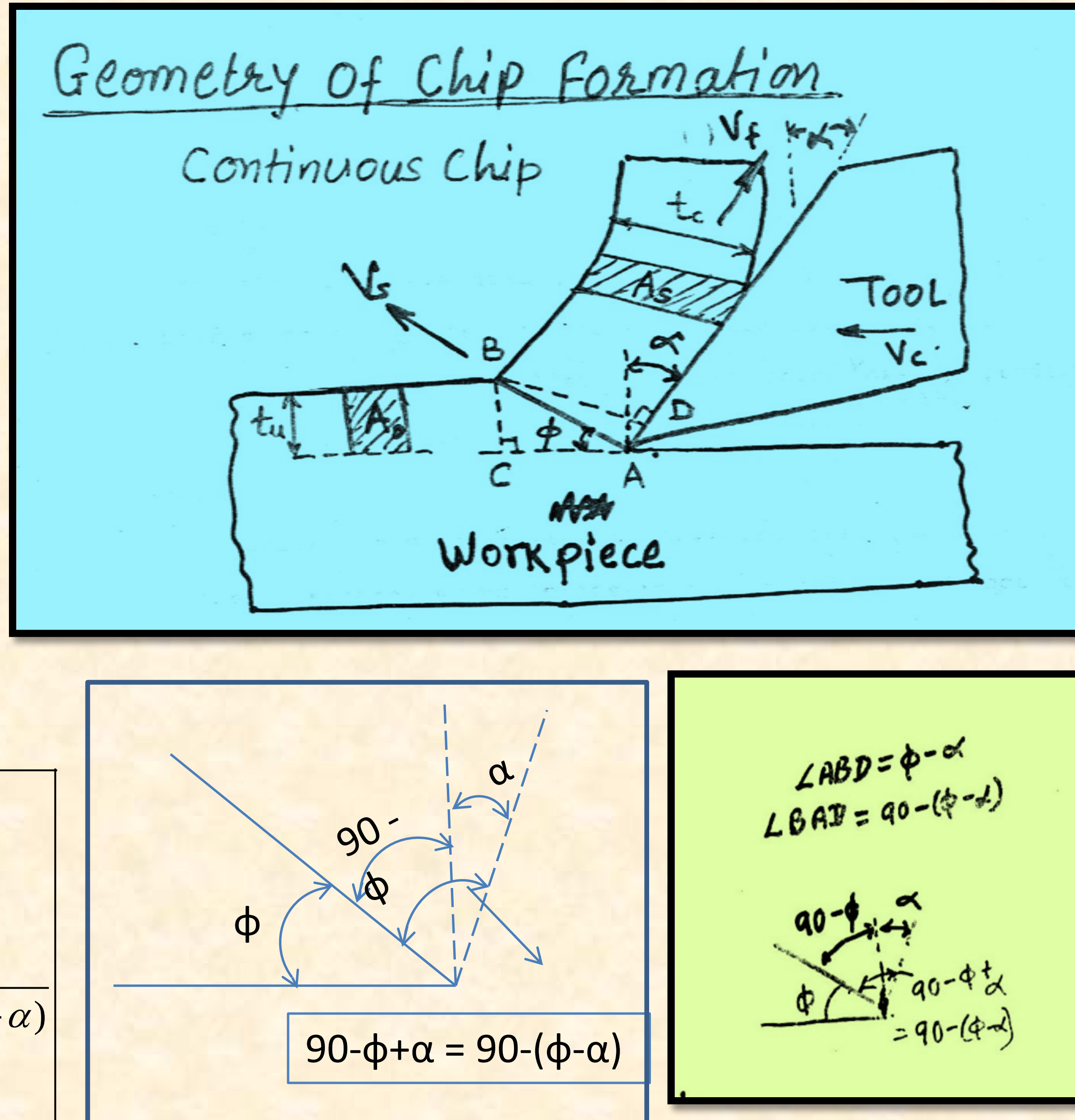


Fig: Schematic of Geometry of chip formation

t_c : *Chip thickness*

t_u : *Uncut chip thickness*

V_f : *Chip Sliding Velocity*

V_s : *Shear Velocity*

V_c : *Cutting Velocity*

ϕ : *Shear Angle*

$\Delta ABC \;\&\; \Delta ABD$

$$AB = \frac{t_u}{\sin\phi}$$

$$also,\ AB = \frac{t_c}{\sin(90-(\phi-\alpha))} = \frac{t_c}{\cos(\phi-\alpha)}$$

$$\frac{t_u}{t_c} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$

SHEAR ANGLE AND CHIP THICKNESS RATIO EVALUATION

$$r_c = \frac{t_u}{t_c} : Chip\ thickness\ Ratio / Coeffinicient$$

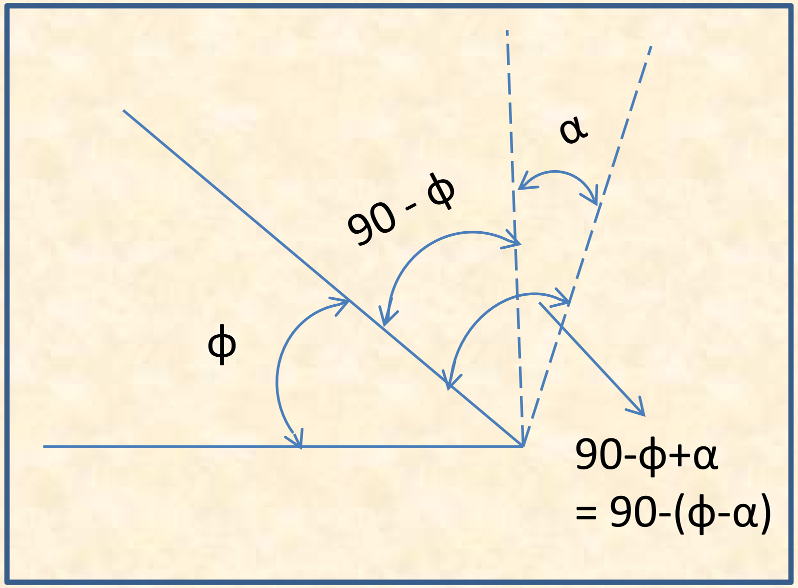
Substitute the value of t_u / t_c from earlier slide and simplify to get:

$$\frac{1}{r_c} = \frac{cos\phi cos\alpha + sin\phi sin\alpha}{sin\phi}$$

$$1 = r_c cot\phi cos\alpha + r_c sin\alpha$$

$$r_c cos\alpha = (1 - r_c sin\alpha)\tan\phi$$

$$\therefore \boxed{\tan\phi = \left(\frac{r_c\ cos\alpha}{1 - r_c sin\alpha}\right)}$$



How to determine ϕ & r_c ?

t_c should be determined from the chip. t_u (= feed) and α are already known.

To determine **t_c** with micrometer, is difficult and not so because of uneven surface. **How?** ***(say, f=0.2 mm/rev. An error of even 0.05 mm will cause an error of 25 % in the measurement of t_c)***

Volume Constancy Condition: Volume of Uncut chip = Volume of cut chip

SHEAR ANGLE AND CHIP THICKNESS RATIO EVALUATION

$$L_u t_u b = L_c t_c b$$

$$\therefore L_c t_c = L_u t_u$$

$$or, r_c = \frac{t_u}{t_c} = \frac{L_c}{L_u}$$

L_c = Chip length

L_u = Uncut chip length

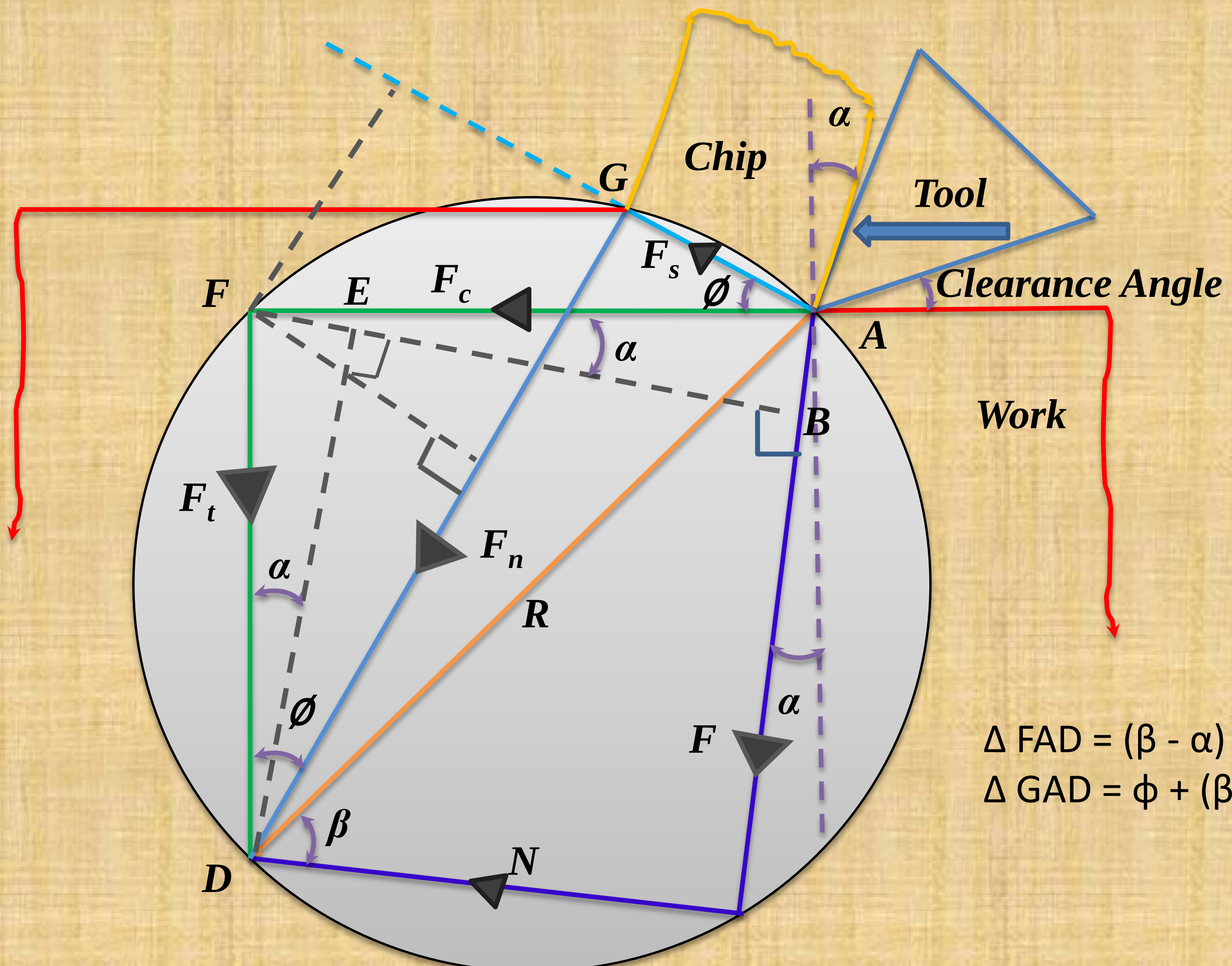
b = Chip width

(2-D Cutting)

LENGTH OF THE CHIP MAY BE MANY CENTIMETERS HENCE THE ERROR IN EVALUTION OF r_c WILL BE COMPARATIVELY MUCH LOWER.

(rc = Lc / Lu)

FORCE CIRCLE DIAGRAM



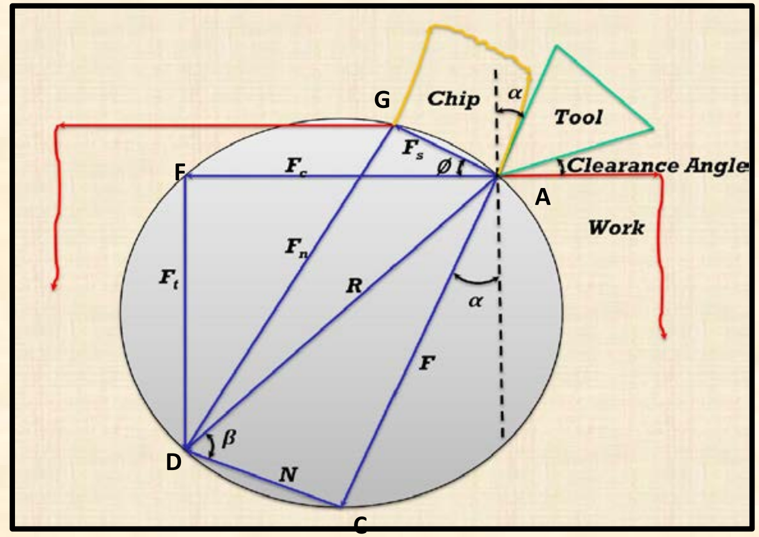
Δ FAD = (β - α)

Δ GAD = ϕ + (β - α)

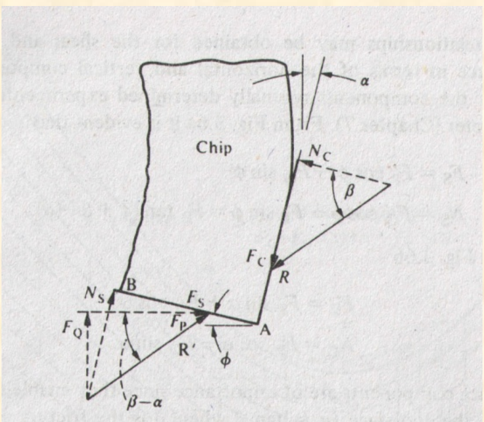
Force Analysis

Forces in Orthogonal Cutting:

- Friction force, F
- Force normal to Friction force, N
- Cutting Force, F_C
- Thrust force, Ft
- Shear Force, F_S
- Force Normal to shear force, F_n
- Resultant force, R



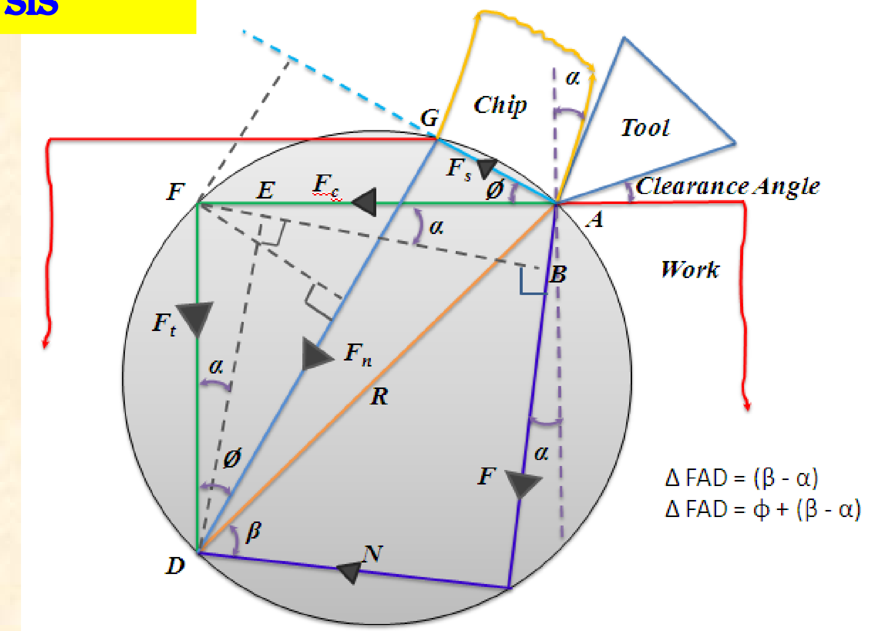
Force Circle Diagram



FREE BODY DIAGRAM

$$R' = \vec{F} + \vec{N}$$

$$R = \vec{F}_s + \vec{F}_N = \vec{F}_c + \vec{F}_t = \vec{R}'$$



$$F = F_t \cos\alpha + F_c \sin\alpha$$

$$N = F_c \cos\alpha - F_t \sin\alpha$$

$$Coefficient\ of\ Friction\ (\mu)$$

$$\mu = \tan\beta = \frac{F}{N} = \frac{F_t \cos\alpha + F_c \sin\alpha}{F_c \cos\alpha - F_t \sin\alpha}$$

$$\beta = Friction\ Angle$$

DIVIDE R.H.S. BY Cos α

$$\mu = \frac{F_t + F_c \tan\alpha}{F_c - F_t \tan\alpha} \quad also,\ \beta = \tan^{-1}(\mu)$$

Foce Analysis

$$F_S = F_c \cos\phi - F_t \sin\phi$$

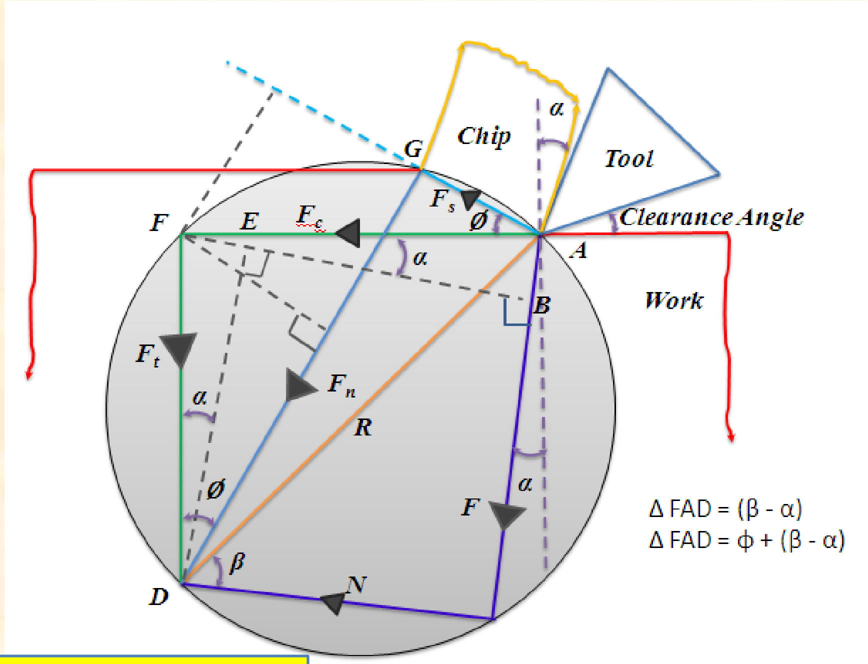
$$F_N = F_t \cos\phi + F_c \sin\phi$$

also,

$$F_C = R \cos(\beta - \alpha)$$

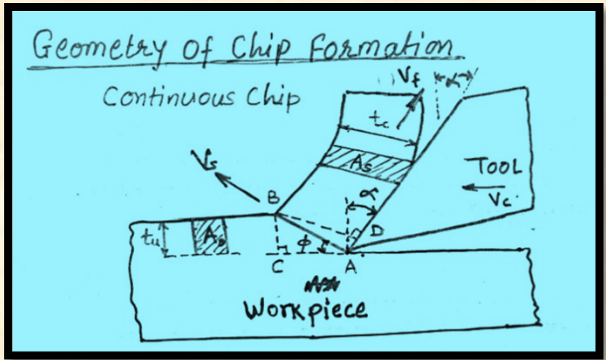
$$F_S = R \cos(\phi + \beta - \alpha)$$

$$\therefore \frac{F_C}{F_S} = \frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$



Δ FAD = (β - α)
Δ GAD = ϕ + (β - α)

$$Shear Plane Area\ (A_S) = \frac{t_u b}{\sin\phi} \left(= \frac{t_u}{\sin\phi} \times b \right)$$



Foce Analysis

Let τ *be the strength of work material*

$$F_S = A_S \tau = \frac{t_u b}{sin\phi}\tau$$

$$F_C = \left(\frac{t_u b\tau}{sin\phi}\right)\left(\frac{cos(\beta-\alpha)}{cos(\phi+\beta-\alpha)}\right) \quad and, \quad R = \left(\frac{t_u b\tau}{sin\phi}\right)\times\left(\frac{1}{cos(\phi+\beta-\alpha)}\right)$$

$$F_t = R\ \sin(\beta-\alpha) = \frac{t_u b\tau}{sin\phi}\times\frac{sin(\beta-\alpha)}{cos(\phi+\beta-\alpha)}$$

$$\boxed{\frac{F_t}{F_c} = \tan(\beta-\alpha)}$$

Foce Analysis

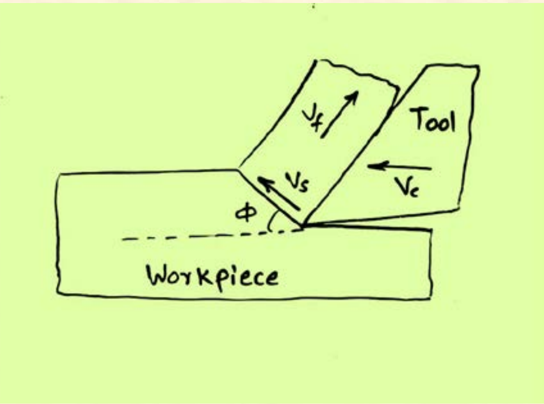
$$Mean\ Shear\ Stress\ (t_{chip}) = \frac{F_S}{A_S}$$
(On Chip)

$$= \frac{(F_c\ cos\phi - F_t\ sin\phi)\sin\phi}{b\ t_u}$$

$$Mean\ Normal\ Stress\ (\sigma_{chip}) = \frac{F_N}{A_S}$$
(On Chip)

$$= \frac{(F_t\ cos\phi + F_c\ sin\phi)\sin\phi}{b\ t_u}$$

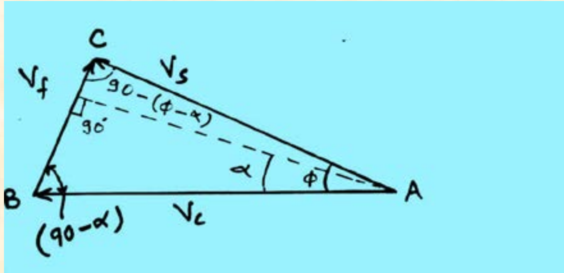
VELOCITY ANALYSIS



V_c : *Cutting velocity of tool relative to workpiece*

V_f : *Chip flow velocity*

V_s : *Shear velocity*



Using sine Rule:

$$\frac{V_c}{\sin(90-(\phi-\alpha))} = \frac{V_f}{\sin\phi} = \frac{V_s}{\sin(90-\alpha)}$$

$$\frac{V_c}{\cos(\phi-\alpha)} = \frac{V_f}{\sin\phi} = \frac{V_s}{\cos\alpha} \qquad and \quad V_f = \frac{V_c \sin\phi}{\cos(\phi-\alpha)} = V_c \cdot r_c$$

$$V_s = \frac{V_c \cos\alpha}{\cos(\phi-\alpha)} \Rightarrow \frac{V_s}{V_c} = \frac{\cos\alpha}{\cos(\phi-\alpha)}$$

Shear Strain & Strain Rate

Two approaches of analysis:

Thin Plane Model:- Merchant, PiisPanen, Kobayashi & Thomson

Thick Deformation Region:- Palmer, (At very low speeds) Oxley, kushina, Hitoni

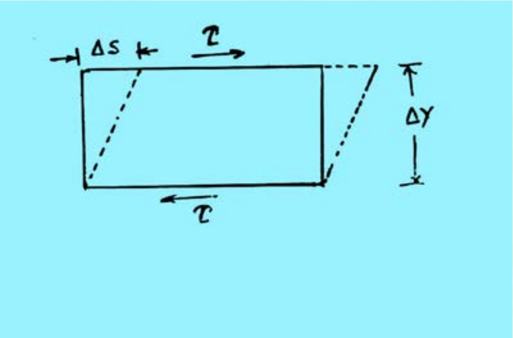
Thin Zone Model: Merchant

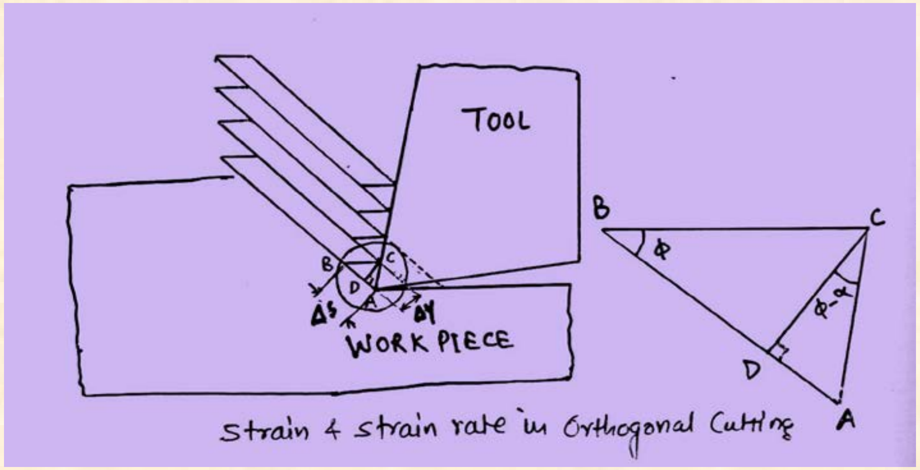
ASSUMPTIONS:-

- Tool tip is sharp, No Rubbing, No Ploughing
- 2-D deformation.
- Stress on shear plane is uniformly distributed.
- Resultant force R on chip applied at shear plane is equal, opposite and collinear to force R' applied to the chip at tool-chip interface.

Expression for Shear Strain

The deformation can be idealized as a process of block slip (or preferred slip planes)





$$ShearStrain(\gamma) = \frac{deformation}{Length}$$

$$\gamma = \frac{\Delta s}{\Delta y} = \frac{AB}{CD} = \frac{AD}{CD} + \frac{DB}{CD} \qquad = \tan(\phi - \alpha) + \cot\phi$$

$$\frac{\sin(\phi-\alpha)\sin\phi + \cos\varphi\cos(\phi-\alpha)}{\sin\phi\cos(\phi-\alpha)},$$

$$\therefore \gamma = \frac{\cos\alpha}{\sin\phi\cos(\phi-\alpha)}$$

Expression for Shear Strain rate

In terms of shear velocity (V_s) and chip velocity (V_f), it can be written as

$$\therefore \quad \gamma = \frac{V_s}{V_c \sin\phi} \qquad \left(\text{since} \frac{V_s}{V_c} = \frac{\cos\alpha}{\cos(\phi-\alpha)} \right)$$

$$Shear\ strain\ rate\ (\dot{\gamma})$$

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{\left(\frac{\Delta s}{\Delta y}\right)}{dt} = \left(\frac{\Delta s}{\Delta y}\right)\frac{1}{\Delta y}$$

$$= \frac{V_s}{\Delta y} = \frac{V_c \cos\alpha}{\cos(\phi-\alpha)\Delta y}$$

$$where,\ \Delta y : Mean\ thickness\ of\ PSDZ$$

Shear angle relationship

- Helpful to predict position of shear plane (angle ϕ)

- **Relationship between-**
  - ✓Shear Plane Angle (ϕ)
  - ✓Rake Angle (α)
  - ✓Friction Angle(β)

Several Theories

Earnst-Merchant(Minimum Energy Criterion):

Shear plane is located where least energy is required for shear.

Assumptions:-

- Orthogonal Cutting.
- Shear strength of Metal along shear plane is not affected by Normal stress.
- Continuous chip without BUE.
- Neglect energy of chip separation.

Shear angle relationship

Assuming No Strain hardening:

Condition for minimum energy, $\frac{dF_c}{d\phi} = 0$

$$\frac{dF_c}{d\phi} = \tau \; t_u \; b \cos(\beta - \alpha)\left[\frac{\cos\phi\cos(\phi+\beta-\alpha) - \sin\phi\sin(\phi+\beta-\alpha)}{\sin^2\phi\cos^2(\phi+\beta-\alpha)}\right] = 0$$

$$\therefore \cos\phi\cos(\phi+\beta-\alpha) - \sin\phi\sin(\phi+\beta-\alpha) = 0$$

$$\cos(2\phi+\beta-\alpha) = 0$$

$$2\phi+\beta-\alpha = \frac{\pi}{2}$$

$$\boxed{\therefore \phi = \frac{\pi}{4} - \frac{1}{2}(\beta-\alpha)}$$

THANK YOU