ChE381 Process Dynamics & Control **Jan-Apr 2014**

End-semester exam **3 hours; 100 points**

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1. A second-order system is given by the transfer function

$$G(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

(a) Determine the response of this system to a unit step input for $\zeta = 1$. [7 points]

(b) Determine the response of this system to a unit step input for the case $0 < \zeta < 1$. [7 points]

(c) Derive an expression for the overshoot for this process when $\zeta < 1$. *Hint:* Overshoot is defined as the ratio of the maximum amount by which the transient response exceeds the ultimate value to the ultimate value. [6 points]

2. A first order process with $G_p(s) = 1/(s+3)$ and $G_d(s) = 1/(s+3)$ is controlled with a PI controller:

(a) Find the values of controller gain K_c and integral time τ_I such that the closed-loop gain to a unit step change in disturbance is 2. [5 points]

(b) Find the values of controller gain K_c such that the decay ratio of the closed-loop response is 1/4, when $\tau_I = 0.01$. [5 points]

Hint: Use the result you derived for question 1(c), and the relation decay ratio = (overshoot)^2.

3 (a). Is the direct substitution method for finding the limits of stability equivalent to the Bode stability criterion ? Justify your result by a short mathematical proof. [4 points]
3 (b). Derive expressions for frequency response, i.e. amplitude ratio and phase angle, for a pure capacitive process. [4 points]

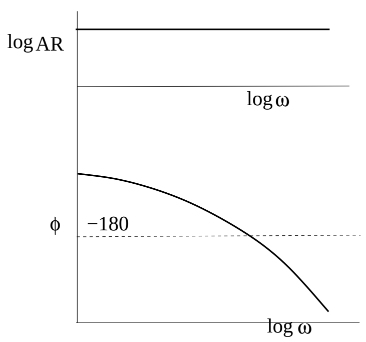

Figure 1: **Problem 3c**

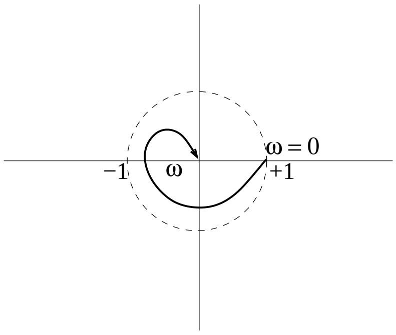

Figure 2: **Problem 3d**

3 (c). Determine the process that corresponds to the Bode plot sketched in figure 1. Give reasons for your answer. [4 points]

3 (d). Determine the process that corresponds to the Nyquist plot sketched in figure 2. Give reasons for your answer. [4 points]

3 (e). A process described by the transfer function $2/(s-4)$ is to be controlled by a proportional controller with gain K_c. Assuming $G_v = G_m = 1$, sketch the root locus of the closed-loop system. Find the range of K_c for which the closed-loop response is stable. For K_c such that the closed-loop pole is zero, determine the response of the closed-loop process to a unit step change. Is the response stable or unstable ? [9 points]

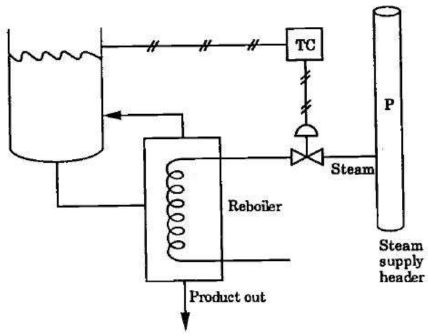

Figure 3: **Problem 4**

4. The bottoms temperature of a distillation column (y in deviation variables) is controlled by manipulating the steam flow rate to the reboiler (u_1 in deviation variables). This purely feedback control strategy is shown in figure 3. An approximate transfer function model for this process is given as:

$$y(s) = \frac{0.25s}{10s+1} u_1(s)$$

However, the steam flowrate itself depends on the percent valve opening (u_2 in deviation variables), and the steam supply pressure (d_1 in deviation variables) which is known to fluctuate in an unpredictable (but measurable) fashion. These process variables are related according to the approximate model:

$$u_1(s) = \frac{2.2}{2s+1}u_2(s) + \frac{1.5}{0.5s+1}d_1(s)$$

(a) Implement a cascade control strategy to control problems created by steam pressure fluctuations. Draw a block diagram for this process under the cascade control strategy. Include all the given transfer functions (and controllers), and label all the signals in the block diagram. [3 points]

(b) Calculate the ultimate (i.e. $t \to \infty$) closed-loop response to a unit step change in the disturbance variable d_1 with and without the cascade control assuming proportional controllers for the master and slave loops. [7 points]

(c) Determine the ratio of the ultimate steady state response with and without the cascade control. [2 points]

(d) If the steam supply pressure is measured, propose a design for feedback-feedforward control *instead* of the cascade control. Draw a block diagram for this new configuration, and obtain an expression for the feedforward controller to be implemented. [8 points]

5. Consider a two-input, two-output process with manipulated variables m_i and controlled variables y_i ($i = 1, 2$). The open-loop dynamics is described by the relations:

$$\begin{aligned} y_1(s) &= G_{11}(s)m_1(s) + G_{12}(s)m_2(s) \\ y_2(s) &= G_{21}(s)m_1(s) + G_{22}(s)m_2(s) \end{aligned}$$

(a) If loop 1 pairs y_1 with m_1 (with controller G_{c1}) and loop 2 pairs y_2 with m_2 (with controller G_{c2}) under feedback control, draw the block diagram for the feedback control of this process. Assume the set points to be y_{1SP} and y_{2SP} respectively for y_1 and y_2. Assume no disturbance variables, and the transfer functions of measuring devices and valves to be unity. [3 points]

(b) Derive the relation between $y_1(s)$ and $y_{1SP}(s)$ if loop 1 is closed, but loop 2 is kept open. What is the characteristic equation of this closed loop process ? [4 points]

(c) If both loops (1 and 2) are closed, derive the following relation under closed-loop, and thereby explicitly derive the expressions for $P_{ij}(s)$ ($i, j = 1, 2$): [5 points]

$$\begin{aligned} y_1(s) &= P_{11}(s)m_1(s) + P_{12}(s)m_2(s) \\ y_2(s) &= P_{21}(s)m_1(s) + P_{22}(s)m_2(s) \end{aligned}$$

(d) What is the characteristic equation of the closed-loop transfer functions ? If we tune G_{c1} by keeping loop 1 closed and loop 2 open, and G_{c2} by keeping loop 2 closed and loop 1 open, would it guarantee the stability of the process with both loops closed ? Explain why with the help of your derivations. [5 points]

(e) If $G_{11} = 1/(0.1s + 1)$, $G_{12} = 5/(s + 1)$, $G_{21} = 1/(0.5s + 1)$, and $G_{22} = 2/(0.4s + 1)$, and assuming simple proportional controllers for G_{c1} and G_{c2}, find the ultimate values of the controller gains if each loop is tuned independently (i.e. loop 1 is closed and loop 2 is open for tuning G_{c1}, and the reverse for G_{c2}). [3 points]

(f) For the transfer functions given in the previous part, we would like to tune the controllers with both loops closed. If we are given $K_{c2} = 4$, is there any restriction on the values of K_{c1} that you would choose ? *Hint: Use Routh array test.* [5 points]
