1. A second-order system is given by the transfer function

\[ G(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta \tau s + 1} \]

(a) Determine the response of this system to a unit step input for \( \zeta = 1 \). [7 points]

(b) Determine the response of this system to a unit step input for the case \( 0 < \zeta < 1 \). [7 points]

(c) Derive an expression for the overshoot for this process when \( \zeta < 1 \). \textit{Hint:} Overshoot is defined as the ratio of the maximum amount by which the transient response exceeds the ultimate value to the ultimate value. [6 points]

2. A first order process with \( G_p(s) = 1/(s+3) \) and \( G_d(s) = 1/(s+3) \) is controlled with a PI controller:

(a) Find the values of controller gain \( K_c \) and integral time \( \tau_I \) such that the closed-loop gain to a unit step change in disturbance is 2. [5 points]

(b) Find the values of controller gain \( K_c \) such that the decay ratio of the closed-loop response is 1/4, when \( \tau_I = 0.01 \). \textit{Hint:} Use the result you derived for question 1(c), and the relation decay ratio = (overshoot)^2. [5 points]

3 (a). Is the direct substitution method for finding the limits of stability equivalent to the Bode stability criterion? Justify your result by a short mathematical proof. [4 points]

3 (b). Derive expressions for frequency response, i.e. amplitude ratio and phase angle, for a pure capacitive process. [4 points]

Figure 1: Problem 3c
3 (c). Determine the process that corresponds to the Bode plot sketched in figure 1. Give reasons for your answer. [4 points]

3 (d). Determine the process that corresponds to the Nyquist plot sketched in figure 2. Give reasons for your answer. [4 points]

3 (e). A process described by the transfer function \( \frac{2}{s - 4} \) is to be controlled by a proportional controller with gain \( K_c \). Assuming \( G_v = G_m = 1 \), sketch the root locus of the closed-loop system. Find the range of \( K_c \) for which the closed-loop response is stable. For \( K_c \) such that the closed-loop pole is zero, determine the response of the closed-loop process to a unit step change. Is the response stable or unstable? [9 points]

4. The bottoms temperature of a distillation column \((y\) in deviation variables\) is controlled by manipulating the steam flow rate to the reboiler \((u_1\) in deviation variables\). This purely feedback control strategy is shown in figure 3. An approximate transfer function model for this process is given as:

\[
y(s) = \frac{0.25s}{10s + 1}u_1(s)
\]

However, the steam flowrate itself depends on the percent valve opening \((u_2\) in deviation variables\), and the steam supply pressure \((d_1\) in deviation variables\) which is known to fluctuate in an unpredictable (but measurable) fashion. These process variables are related
according to the approximate model:

\[ u_1(s) = \frac{2.2}{2s+1}u_2(s) + \frac{1.5}{0.5s+1}d_1(s) \]

(a) Implement a cascade control strategy to control problems created by steam pressure fluctuations. Draw a block diagram for this process under the cascade control strategy. Include all the given transfer functions (and controllers), and label all the signals in the block diagram. [3 points]

(b) Calculate the ultimate (i.e. \( t \to \infty \)) closed-loop response to a unit step change in the disturbance variable \( d_1 \) with and without the cascade control assuming proportional controllers for the master and slave loops. [7 points]

(c) Determine the ratio of the ultimate steady state response with and without the cascade control. [2 points]

(d) If the steam supply pressure is measured, propose a design for feedback-feedforward control instead of the cascade control. Draw a block diagram for this new configuration, and obtain an expression for the feedforward controller to be implemented. [8 points]

5. Consider a two-input, two-output process with manipulated variables \( m_i \) and controlled variables \( y_i \) (\( i = 1, 2 \)). The open-loop dynamics is described by the relations:

\[
\begin{align*}
y_1(s) &= G_{11}(s)m_1(s) + G_{12}(s)m_2(s) \\
y_2(s) &= G_{21}(s)m_1(s) + G_{22}(s)m_2(s)
\end{align*}
\]

(a) If loop 1 pairs \( y_1 \) with \( m_1 \) (with controller \( G_{c1} \)) and loop 2 pairs \( y_2 \) with \( m_2 \) (with controller \( G_{c2} \)) under feedback control, draw the block diagram for the feedback control of this process. Assume the set points to be \( y_{1SP} \) and \( y_{2SP} \) respectively for \( y_1 \) and \( y_2 \). Assume no disturbance variables, and the transfer functions of measuring devices and valves to be unity. [3 points]

(b) Derive the relation between \( y_1(s) \) and \( y_{1SP}(s) \) if loop 1 is closed, but loop 2 is kept open. What is the characteristic equation of this closed loop process? [4 points]

(c) If both loops (1 and 2) are closed, derive the following relation under closed-loop, and thereby explicitly derive the expressions for \( P_{ij}(s) \) (\( i, j = 1, 2 \)): [5 points]

\[
\begin{align*}
y_1(s) &= P_{11}(s)m_1(s) + P_{12}(s)m_2(s) \\
y_2(s) &= P_{21}(s)m_1(s) + P_{22}(s)m_2(s)
\end{align*}
\]

(d) What is the characteristic equation of the closed-loop transfer functions? If we tune \( G_{c1} \) by keeping loop 1 closed and loop 2 open, and \( G_{c2} \) by keeping loop 2 closed and loop 1 open, would it guarantee the stability of the process with both loops closed? Explain why with the help of your derivations. [5 points]
(e) If $G_{11} = 1/(0.1s + 1)$, $G_{12} = 5/(s + 1)$, $G_{21} = 1/(0.5s + 1)$, and $G_{22} = 2/(0.4s + 1)$, and assuming simple proportional controllers for $G_{c1}$ and $G_{c2}$, find the ultimate values of the controller gains if each loop is tuned independently (i.e. loop 1 is closed and loop 2 is open for tuning $G_{c1}$, and the reverse for $G_{c2}$). [3 points]

(f) For the transfer functions given in the previous part, we would like to tune the controllers with both loops closed. If we are given $K_{c2} = 4$, is there any restriction on the values of $K_{c1}$ that you would choose? *Hint: Use Routh array test.* [5 points]