ChE381 Process Dynamics & Control **Jan-Apr 2013**

End-semester exam **3 hours; 100 points**

Write answers to each question on a new page. Write your solutions in a clear and legible handwriting.

Problem 1 Qualitatively sketch the Nyquist plots for the following systems. In all your plots, you must draw a unit circle (with dotted lines) centered at the origin as a reference, and must clearly mark the points $(1,0)$ and $(-1,0)$ on the real axis. Also show the direction of increase in frequency with an arrow mark in your plot. Comment on the stability of each of the systems using the Nyquist criterion. [15 points]

a. A first order system with the transfer function $g(s) = 1/(1+s)$.

b. A pure time-delay system with $g(s) = \exp[-s]$.

c. A pure integrator with transfer function $g(s) = 1/s$.

d. For a proportional controller with unity gain.

e. For a PI controller with unity gain and unity integral time constant.

f. For a PD controller with unity gain and unity derivative time constant.

Problem 2 Consider the process described by the following transfer function [15 points]

$$g(s) = \frac{1}{s^3 + 4s^2 + s - 6}$$

a. Use the Routh method to find the range of PI controller parameters (K_c, τ_I) for which the *closed-loop* system is stable.

b. For the same process but now employing only a proportional controller, use the direct substitution method to find the ultimate gain and frequency.

Problem 3 Consider a counter-current heat exchanger in which the hot water inlet flow rate F_h is manipulated to control the outlet cold water temperature T_c. For this process: [20 points]

a. Draw a schematic diagram for this control system showing the temperature transducer and temperature controllers.

b. Using the above schematic, construct a block diagram showing relevant transfer functions and signals.

c. Assume $g_m = K_m = 1$ and the following open-loop transfer function that relates cold water temperature to the flow rate of hot water:

$$\frac{T_c(s)}{F_h(s)} = g(s) = g_p g_v g_m = 4\frac{3s-1}{(s+1)(3s+1)}$$

Here, the subscripts p refers to the process, v to the final valve, and m to the measurement sensor. Show that the above $g(s)$ yields the ultimate gain $K_{cu} = -1/3$ and the ultimate frequency $\omega_u = \sqrt{7}/3$.

d. Calculate the closed-loop poles with $K_c = -1/3$. Determine the response of the closed-loop system (with $K_c = -1/3$) to a step change in the set point.

Problem 4 Consider the following model describing a two-input two-output system. The input (manipulated) variables are V and Q, and the output (i.e. controlled) variables are x_L and T. (This is actually a simplified and linearized unsteady model for a flash distillation unit.) [25 points]

$$\begin{aligned} \frac{\mathrm{d}x_L}{\mathrm{d}t} &= -5x_L + 0.3V \\ \frac{\mathrm{d}T}{\mathrm{d}t} &= -2T + 60V + Q \end{aligned} \tag{1}$$

a. Write the above set of equations in the Laplace domain, and find the transfer function matrix.

b. Using the relative-gain array method, determine the best possible input-output pairings for the above process.

c. Assume the variables V is paired with x_L. Use the method of direct synthesis to design a controller described by transfer function $g_c(s)$ for this pair. Assume that the closed-loop response that is desired is $1/(0.1s+1)$. What is the format of the controller thus obtained ? Find the constants associated with the controller transfer function $g_c(s)$.

d. Assume now Q is paired with T. Determine the range of controller gains K_c for which the closed-loop system is stable with this pair. Consider a PI controller with $K_c = -1$, $\tau_I = 1/4$. Will the PI controller yield a stable closed-loop response for this pair ?

Problem 5 Consider the block diagram (shown in figure 1) that shows the combination of a cascade control with a feed forward controller designed to counteract the disturbance d_1. For this block diagram: [25 points].

a. Derive the closed-loop relation between $y_2(s)$ and y_{2sp} and d_2. Call the transfer functions in this relation as g_1 and g_2, i.e. $y_2(s) = g_1 y_{2sp} + g_2 d_2$. Similarly derive the closed-loop relation between $y_1(s)$ and y_{1sp} and d_1.

b. Consider $g_{p2} = 1/(1+s)$ and $g_{c2} = K_2 = 1$. Then derive an expression for y_2/y_{2sp}.

c. Consider further (with g_{p2} given in part b above) $g_{p1} = 10/(10s+1)$. Design the controller g_{c1} so that $y_1/y_{1sp} = 1/(5s+1)$. Write the controller transfer function g_{c1} in the standard PID format, and determine the constants K_c, τ_I and τ_D.

d. Consider $g_{d1} = 5/(5s+1)$. Design the controller g_f so that there is complete rejection of the disturbance d_1. What type of controller is g_f ?

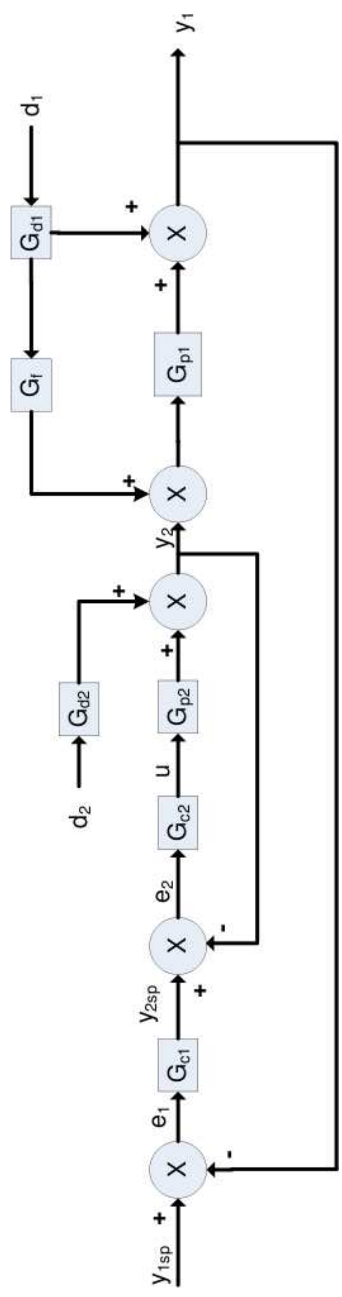

Figure 1: **Problem 5**