\( g(s) = \frac{K_p}{s^2 + 2\zeta \omega_n s + \omega_n^2 + 1} \)

a) \( \zeta = 1, \text{ step input} \)

\[ y(s) = y'(s) \frac{1}{s} \]

\( \zeta = 1, \]

\[ s^2 + 2\zeta \omega_n s + \omega_n^2 = (\zeta s + 1)^2 \]

\[ y(s) = \frac{K_p}{(\zeta s + 1)^2} \frac{1}{s} \]

Laplace inversion:

\[ y(t) = K_p \left[ 1 - e^{-t/\tau} - \frac{t}{\tau} e^{-t/\tau} \right] \]
1) \( 0 < \zeta < 1 \)

\[ y(s) = \frac{k_p}{\zeta s^2 + 2\zeta s + 1} \frac{1}{s} \]

\[ \zeta s^2 + 2\zeta_cs + 1 = 0 \]

\[ s = \frac{-\zeta \pm i\beta}{\zeta} \quad ; \quad \beta = \sqrt{1-\zeta^2} \]

Laplace inversion: \( \phi = \tan^{-1}\left(\frac{\beta}{\zeta}\right) \)

\[ y(t) = k_p \left[ 1 - \frac{1}{\beta} e^{-\zeta \frac{t}{\zeta}} \sin \left(\frac{\beta t}{\zeta} + \phi\right) \right] \]
1c) overshoot:

\[ y(t) = k_p \left[ 1 - \frac{1}{\beta} e^{-\frac{t}{\tau}} \sin \left( \beta \frac{t}{\tau} + \phi \right) \right] \]

\[ \phi = \tan^{-1} \left( \frac{\beta}{\xi} \right) \]

\[ \beta = \sqrt{1 - \xi^2} \]

\[ \tan \phi = \frac{\beta}{\xi} = \frac{\sin \phi}{\cos \phi} = \frac{\sqrt{1 - \xi^2}}{\xi} \]

Maximizing:

\[ \frac{dy}{dt} = 0 \]

\[ -\beta \cos \left( \beta \frac{t}{\tau} + \phi \right) + \xi \sin \left( \phi + \beta \frac{t}{\tau} \right) = 0 \]

\[ \frac{\sin \left( \phi + \beta \frac{t}{\tau} \right)}{\cos \left( \phi + \beta \frac{t}{\tau} \right)} = \frac{\beta}{\xi} \]
\[
\tan \left( \frac{\beta t + \phi}{2} \right) = \tan \phi
\]

\[t = \frac{\pi \pm \beta}{\beta}\] for max.

\[
y(t)_{\text{max}} = K_p \left[1 + \frac{1}{\beta} e^{\frac{-T_1}{\beta}} \sin \phi\right]
\]

\[
y(t)_{\text{max}} = K_p \left[1 + e^{\frac{-t}{\beta}}\right]
\]

**overshoot:** \[
\frac{y_{\text{max}} - y(t \to \infty)}{y(t \to \infty)}
\]

\[= e^{\frac{-T_1}{\beta}}\]

\[\beta = \sqrt{1 - \xi^2}\]

**overshoot** = \[\exp \left( \frac{-\xi \pi}{\sqrt{1 - \xi^2}} \right)\]
2a) closed-loop response to unit step change in disturbance

\[ y(s) = \frac{1}{s+3} \frac{1}{1 + \frac{1}{s+3} k_c \left( 1 + \frac{1}{s+3} \right)} \cdot \frac{1}{s} \]

\[ y(t \to \infty) = \lim_{s \to 0} \left[ \frac{1}{s+3} \frac{1}{1 + \frac{1}{s+3} k_c \left( 1 + \frac{1}{s+3} \right)} \right] \]

\[ \to 0 \]

Closed-loop ultimate gain = 0 so there are no values of $K_c$ and $T_z$ for which there is a finite steady gain to $d(s)$. 
2b) charac eqn:

\[ 1 + K_c \left( \frac{\frac{Ts + 1}{Ts}}{Ts + 3} \right) \frac{1}{Ts + 3} = 0 \]

\[ \frac{2}{T} s^2 + \frac{2}{T} (K_c + 3) s + K_c = 0 \]

\[ \frac{2}{T} s^2 + \frac{2}{T} \frac{K_c + 3}{K_c} s + 1 = 0 \]

Decay ratio: \( \exp \left( \frac{\sqrt{\frac{2\pi}{1 - \xi^2}}}{\sqrt{1 - \xi^2}} \right) = \frac{1}{4} \)

\( \xi = 0.215 \)

\[ \xi = \frac{\frac{2}{T} \sqrt{\frac{K_c}{Ts}}}{2} \left[ \frac{\frac{2}{T} (K_c + 3)}{K_c} \right] = 0.215 \]

\[ K_c = 0.7677, 11.722 \]
3a) \[ 1 + G(s) = 0 \]
\[ G(s)(j\omega) = -1 \]
\[ |AR| e^{j\phi} = -1 \]
\[ |AR| = 1 \]
\[ \phi = -\pi \]

Direct Substitution:
put \( s = j\omega \) in
\[ 1 + G(s)(j\omega) = 0 \]

So direct substitution and Bode criterion are equivalent.
3b) \[ G_p(s) = \frac{K}{s} \]

\[ G(j\omega) = \frac{K}{j\omega} = -j\frac{k}{s} \]

\[ \text{LAR} = \frac{K}{\omega} \]

\[ \phi = \tan^{-1}(-\infty) \]

\[ = -\frac{\pi}{2} \]

3c) Bode plot corresponds to a pure lag process with \[ G(j\omega) = e^{-j\omega} \]

\[ \text{LAR} = 1 \]

\[ \phi = -\omega \]
3d) Nyquist plot corresponds to a 3rd order branch.

Since as $\omega \to 0$, $|AR| \to 0$

and $\phi$ is in the range $-\pi < \phi < -\frac{3\pi}{2}$

3e) Closed loop: $$\frac{2Kc}{s-4}$$ \[1 + \frac{2Kc}{s-4}\]

$$= \frac{2Kc}{s+(2Kc-4)}$$

Charac eqn: $(4-2Kc) = 0$
For \( K_c = 2 \), \( b0k = 0 \)

\[
y(s) = \frac{2K_c}{s} \cdot \frac{1}{s}
\]

\[
y(t) = 2K_c t
\]

unbounded response

\( \rightarrow \) unstable
unit step to $d_1$:

$$u_1 = \frac{g_1c_2 \frac{2.2}{2s+1}}{1 + g_1c_2 \frac{2.2}{2s+1}}$$

$m_1 + \frac{1.5}{0.5s+1}

m_1 = 0, \quad d_1 = \frac{1}{5}$

$g_1c_2 = k_c_2$

$g_1c_1 = k_c_1$

$y(s) = \frac{0.25}{10s+1} \frac{q_d^*}{1 + g_1c_1 g_2c_2} \frac{0.25}{10s+1}$

$$\lim_{s \to 0} s y(s) = \frac{0.25 \times 1.5}{1 + k_c_2 \frac{2.2}{2(1 + 0.25 k_c_1)}}$$
Without cascade:

\[ y(s) = \frac{0.25}{10s+1} \times \frac{1.5}{0.5s+1} \]

\[ 1 + Kc_1 \times \frac{2-2}{2s+1} \times \frac{0.25}{10s+1} \]

\[ \lim_{s \to 0} y(s) = \frac{0.25 \times 1.5}{1 + Kc_1 \times 2-2 \times 0.25} \]
response with cascade

without cascade

\[
\frac{1 + K_c 2.2 0.25}{1 + 2.2 K_c (1 + 0.25 - K_{c1})}
\]

If \( K_{c2} \gg 1 \) then the response with cascade reduces the output due to a step change in \( u \) significantly.
\[ \text{gff} = \frac{-1.5}{0.5s + 1} \]

\[ = \frac{-0.6816 \times (2s + 1)}{(0.5s + 1)} \]
\[ y_1(s) = P_{11}(s) \cdot y_{1sp} + P_{12}(s) \cdot y_{2sp} \]

\[ y_2(s) = P_{21}(s) \cdot y_{1sp} + P_{22}(s) \cdot y_{2sp} \]

\[ P_n = H_{11} G_{C1} + G_{C1} G_{C2} (H_{11} H_{22} - H_{12} H_{21}) \]

\[ Q(s) = H_{12} G_{C2} \]

\[ P_{22} = H_{22} G_{C2} + G_{C1} G_{C2} (H_{11} H_{22} - H_{12} H_{21}) \]

\[ Q(s) = (1 + H_{11} G_{C1}) (1 + H_{22} G_{C2}) - H_{21} H_{12} G_{C1} G_{C2} \]
Charac eqn for both loops closed:

\[ Q(s) = (1 + H_{ii} G_{c1})(1 + H_{22} G_{c2}) \]

\[ - H_{12} H_{21} G_{c2} G_{c1} = 0 \]

If we keep loop 1 closed and loop 2 open:

\[ y_1 = \frac{h_{ii} G_{c1}}{1 + h_{ii} G_{c1}} y_{isp} \]

\[ y_2 = \frac{h_{21} G_{c1}}{1 + h_{ii} G_{c1}} y_{isp} \]

\( m_2: \text{const} \)

(assume) \[ \text{Part 6} \]
d) If $K_1$ is tuned keeping loop 2 open, and if $K_2$ is tuned keeping loop 1 open, it will NOT guarantee the stability of the process with both loops closed. The characteristic curves are very different.
5 e) tuning each loop separately:

\[ 1 + H_{11} C_{1} = \frac{1 + K_{C_1}}{0.15 + 1} = 0 \]

characteristic eqn: \((1 + K_{C_1}) + 0.15 = 0\)

pole \(s = -(1 + K_{C_1})\)

\(\) \underline{always stable for \(K_{C_1} > 0\)}

Similarly, if loop 2 is closed with loop 1 open, for any \(K_{C_2} > 0\), stable.
5f) Both loops closed:

\[
\left( 1 + \frac{Kc_1}{0.15+1} \right) \left( 1 + \frac{2Kc_2}{0.45+1} \right) \]

\[
- \frac{5}{s+1} \quad - \frac{1}{0.55+1} \]

\[Kc_1 \quad Kc_2 = 0\]

for instability (Routh test).

\[1 + 2Kc_2 + Kc_1 - 3Kc_1 \quad Kc_2 < 0\]

\[Kc_2 = 4\]

\[9 - 11Kc_1 < 0\]

So \[Kc_1 > \frac{11}{9}\] means instability for \[Kc_2 = 4\].