Subject ChE 381 End-Sem
solution key.

① $G(s) = \dfrac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$

a) $\zeta = 1$, step input

$y(s) = G(s)\,\dfrac{1}{s}$

$\zeta = 1$, $\tau^2 s^2 + 2\tau s + 1 = (\tau s + 1)^2$

$$y(s) = \frac{K_p}{(\tau s+1)^2}\,\frac{1}{s}$$

Laplace inversion:

$$\boxed{y(t) = K_p\left[1 - e^{-t/\tau} - \frac{t}{\tau}e^{-t/\tau}\right]}$$

1b) $0 < \zeta < 1$

$$y(s) = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1} \quad \frac{1}{s}$$

$$\tau^2 s^2 + 2\zeta\tau s + 1 = 0$$

$$s = \frac{-\zeta}{\tau} \pm \frac{i\beta}{\tau} \; ; \quad \beta = \sqrt{1-\zeta^2}$$

Laplace inversion: $\phi = \tan^{-1}\left(\frac{\beta}{\zeta}\right)$

$$\boxed{y(t) = K_p\left[1 - \frac{1}{\beta} e^{-\zeta t/\tau} \sin\left(\frac{\beta t}{\tau} + \phi\right)\right]}$$

1c) overshoot:

$$y(t) = K_p\left[1 - \frac{1}{\beta} e^{-\zeta t/\tau} \sin\left(\beta \frac{t}{\tau} + \phi\right)\right]$$

$$\phi = \tan^{-1}\left(\frac{\beta}{\zeta}\right) \qquad \beta = \sqrt{1-\zeta^2}$$

$$\tan\phi = \frac{\beta}{\zeta} = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

maxm: $\frac{dy}{dt} = 0$

$$-\beta \cos\left[\beta \frac{t}{\tau} + \phi\right] + \zeta \sin\left(\phi + \frac{\beta t}{\tau}\right) = 0$$

$$\frac{\sin\left(\phi + \frac{\beta t}{\tau}\right)}{\cos\left(\phi + \frac{\beta t}{\tau}\right)} = \frac{\beta}{\zeta}$$

$$\tan\left(\frac{\beta t}{\tau} + \phi\right) = \tan\phi$$

$$t = \pi\frac{\tau}{\beta} \quad \text{for maxm.}$$

$$y(t)\Big|_{max} = K_p\left[1 + \cancel{\frac{1}{\beta}}\, e^{-\frac{\xi\pi}{\beta}}\, \cancel{\sin\phi}\right]$$

$$y(t)\Big|_{max} = K_p\left[1 + e^{-\frac{\xi\pi}{\beta}}\right]$$

$$\text{Overshoot:} \quad \frac{y_{max} - y(t\to\infty)}{y(t\to\infty)}$$

$$= e^{-\frac{\xi\pi}{\beta}}$$

$$\beta = \sqrt{1-\xi^2}$$

$$\boxed{\text{Overshoot} = \exp\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]}$$

2a) closed loop response to unit step change in disturbance.

$$y(s) = \frac{\frac{1}{s+3}}{1 + \frac{1}{s+3} k_c \left(1 + \frac{1}{\tau_I s}\right)} \cdot \frac{1}{s}$$

$$y(t \to \infty) = \lim_{s \to 0} \left[\frac{\frac{1}{s+3}}{1 + \frac{1}{s+3} k_c \left(1 + \frac{1}{\tau_I s}\right)} \right]$$

$$\to 0$$

closed-loop ultimate gain $= 0$
so there are no values of k_c and τ_I for which there is a finite steady gain to $d(s)$.

2b) charac eqn:

$$1 + K_c \left(\frac{\tau_I s + 1}{\tau_I s}\right) \frac{1}{s+3} = 0$$

$$\tau_I s^2 + \tau_I (K_c + 3) s + K_c = 0$$

$$\underbrace{\frac{\tau_I}{K_c}}_{\tau^2} s^2 + \underbrace{\tau_I \frac{K_c+3}{K_c}}_{2\zeta\tau} s + 1 = 0$$

decay ratio $= \exp\left[\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right] = \frac{1}{4}$

$$\zeta = 0.215$$

$$\zeta = \frac{\tau_I}{2}\sqrt{\frac{K_c}{\tau_I}} \left[\frac{\tau_I}{K_c}(K_c+3)\right] = 0.215$$

$$\boxed{K_c = 0.7677,\ 11.722}$$

3a) $1 + G_{OL} = 0$

$$G_{OL}(j\omega) = -1$$

Bode

$$|AR|\, e^{j\phi} = -1$$

$$|AR| = 1$$

$$\phi = -\pi$$

Direct substitution:

put $s = j\omega$ in

$$1 + G_{OL}(j\omega) = 0$$

So direct substitution and Bode criterion <u>are equivalent</u>

3b) $$G_p(s) = \frac{K}{s}$$

$$G(j\omega) = \frac{K}{j\omega} = -j\frac{k}{\omega}$$

$$|AR| = \frac{K}{\omega}$$

$$\phi = \tan^{-1}(-\infty)$$

$$= -\frac{\pi}{2}$$

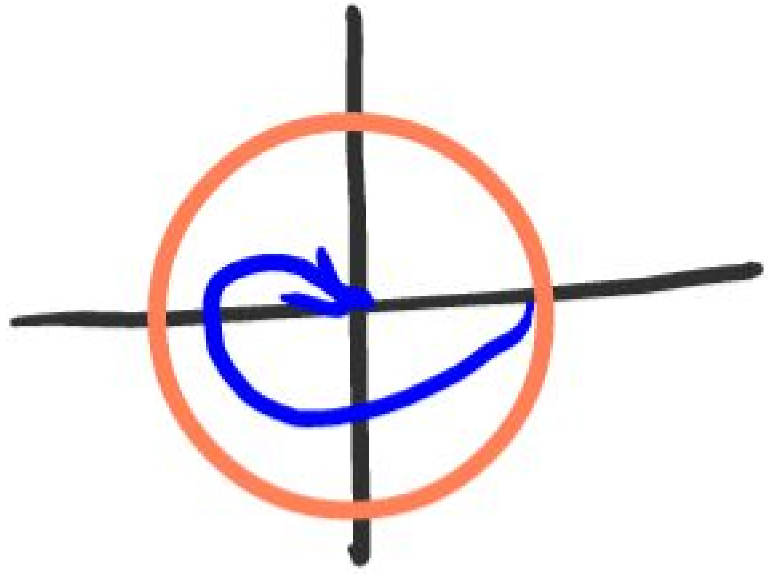
3c) Bode plot corresponds to a pure lag process

with $G(j\omega) = e^{-j\omega}$

$$|AR| = 1$$

$$\phi = -\omega$$

3d)



Nyquist plot corresponds to a <u>3rd order process</u>

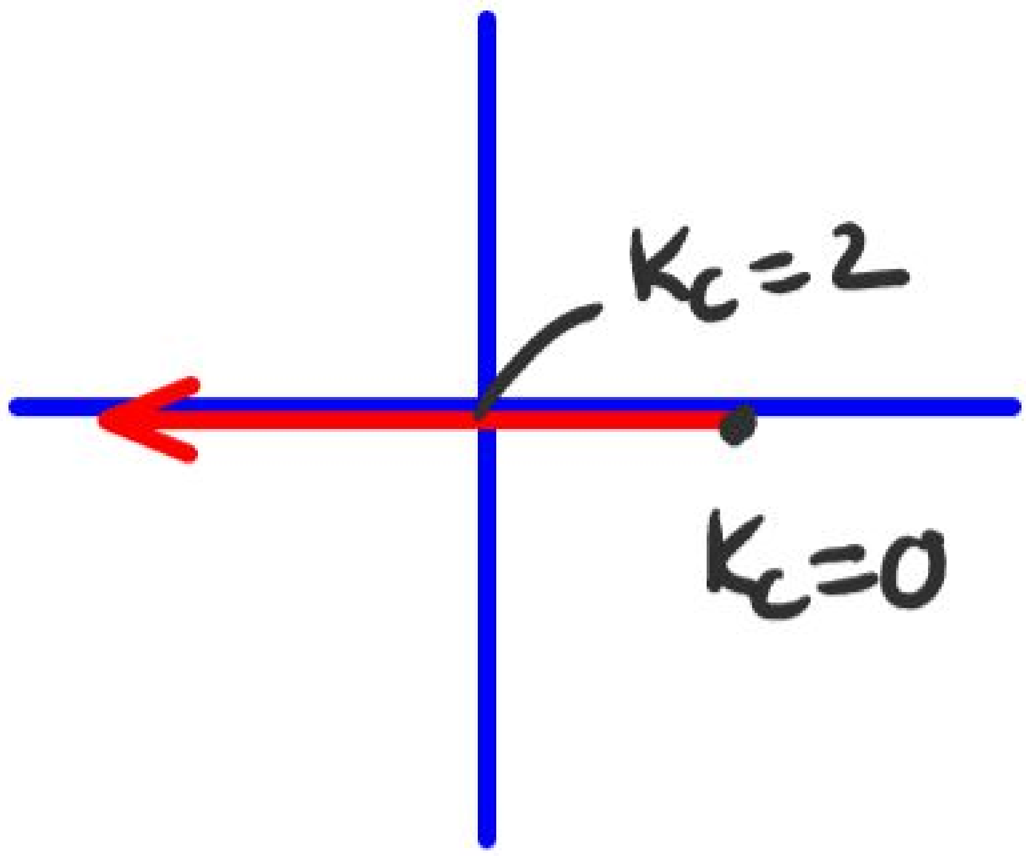
Since as $\omega \to 0$, $|AR| \to 0$

and ϕ is in the range $-\pi < \phi < -\frac{3\pi}{2}$

3e) Closed loop: $\dfrac{\frac{2K_c}{s-4}}{1+\frac{2K_c}{s-4}}$

$$= \frac{2K_c}{s+(2K_c-4)}$$

Charac eqn:
$(4-2K_c)=0$



Root-locus plot.

For $K_c = 2$, pole $= 0$

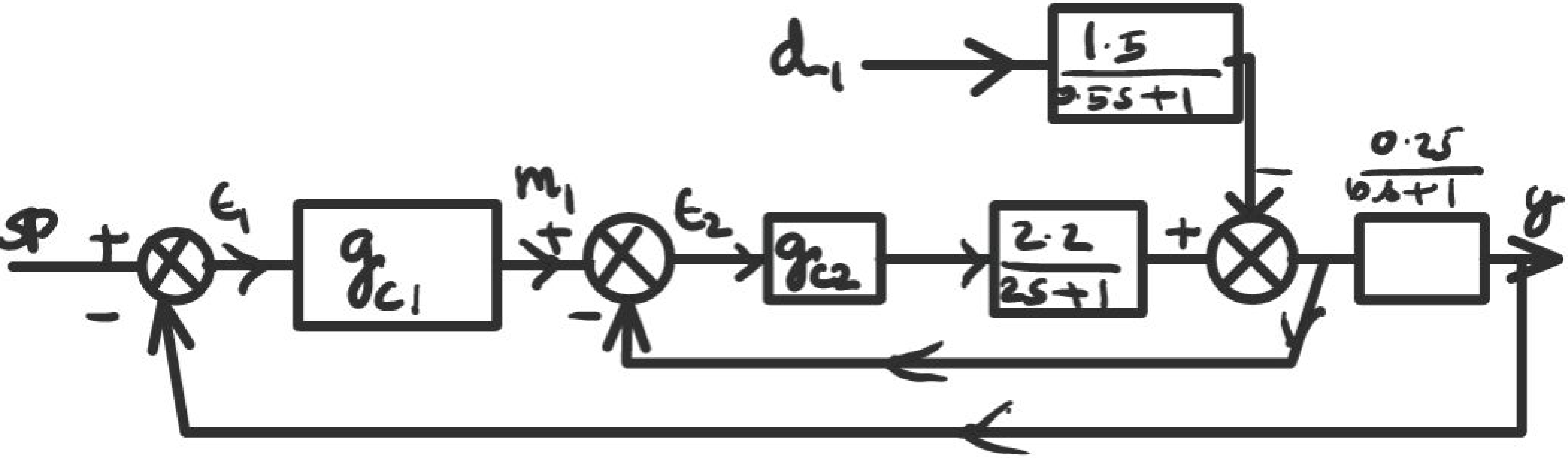
$$y(s) = \frac{2K_c}{s} \cdot \frac{1}{s} \quad \text{unit step}$$

$$\boxed{y(t) = 2K_c t}$$

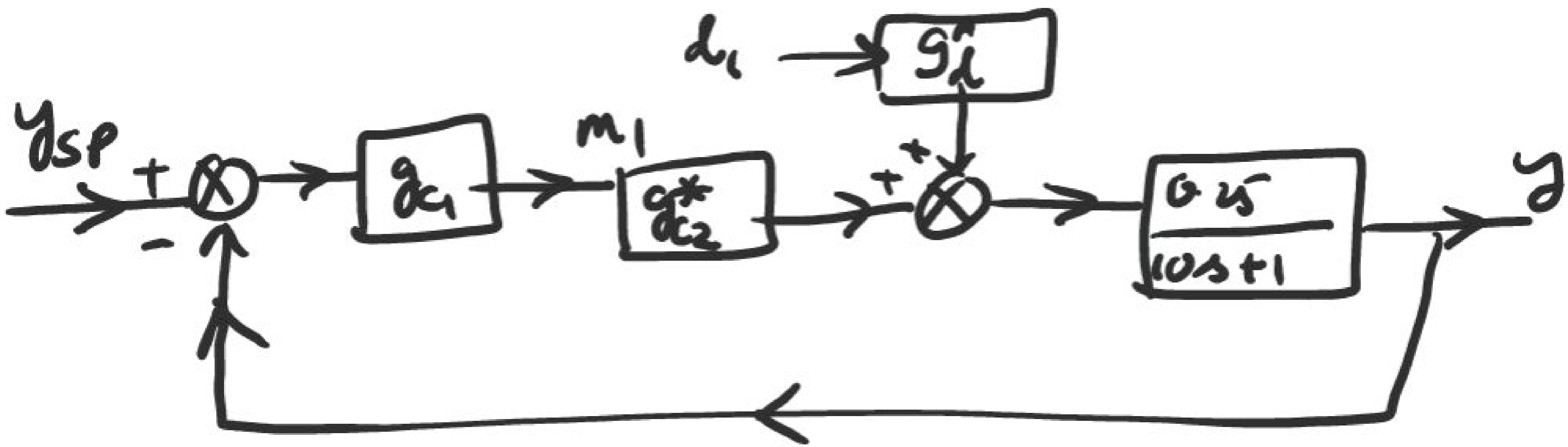
unbounded response

$\rightarrow$ <u>unstable</u>

4)



d_1
$\frac{1.5}{0.5s+1}$
$\frac{0.25}{6s+1}$
SP
+
−
ε_1
g_{c1}
m_1
+
−
ε_2
g_{c2}
$\frac{2.2}{2s+1}$
+
−
y



$$g^*_{c2} = \frac{g_{c2}\dfrac{2.2}{2s+1}}{1 + g_{c2}\dfrac{2.2}{2s+1}} \; ; \quad g^*_d = \frac{\dfrac{1.5}{0.6s+1}}{1 + g_{c2}\dfrac{2.2}{2s+1}}$$

unit step to d_1:

$$u_1 = \frac{g_{c2}\frac{2.2}{2s+1}}{1+g_{c2}\frac{2.2}{2s+1}} m_1 + \frac{\frac{1.5}{0.5s+1}}{1+g_{c2}\frac{2.2}{2s+1}} d_1$$

g_{c2}^* $\quad$ g_d^*

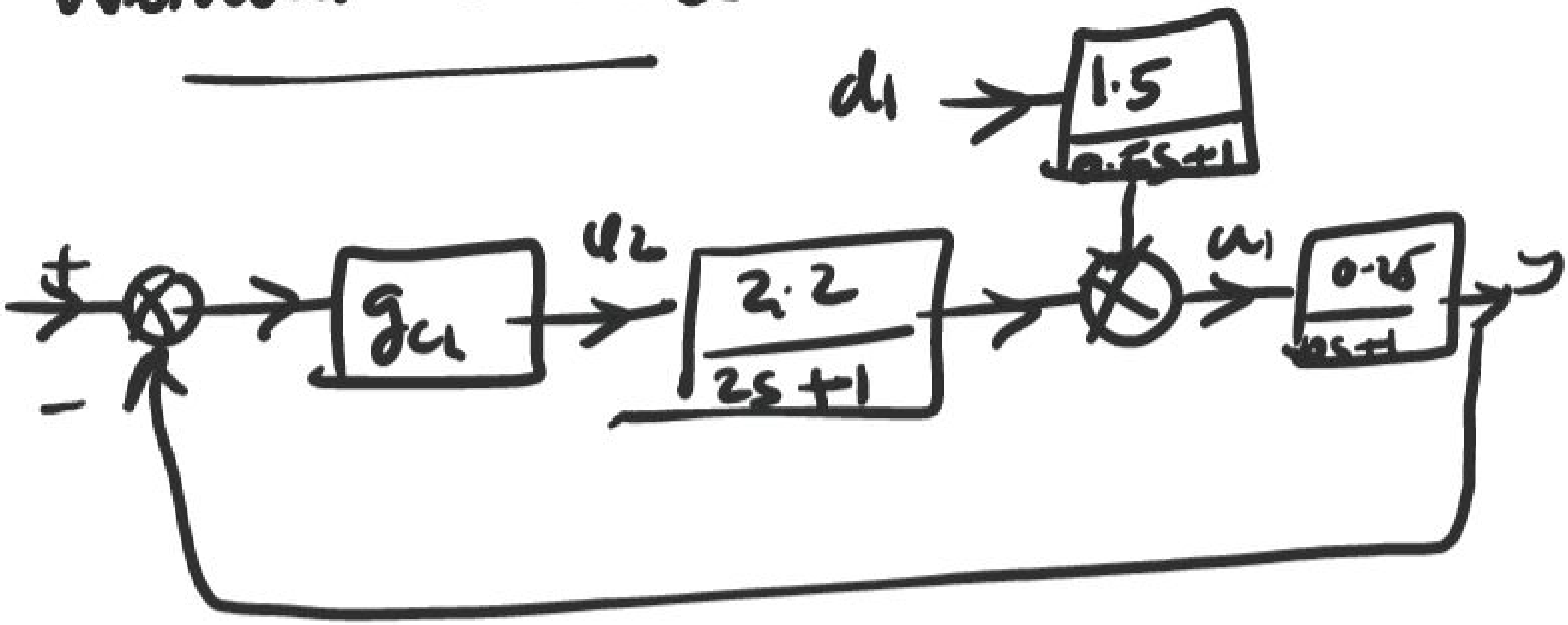
$m_1 = 0, \quad d_1 = \frac{1}{s} \qquad g_{c2} = K_{c2}$

$g_{c1} = K_{c1}$

$$y(s) = \frac{\frac{0.25}{10s+1} g_d^*}{1+g_{c1} g_{c2}^* \frac{0.25}{10s+1}} \cdot \frac{1}{s}$$

$$\lim_{s\to 0} s\,y(s) = \frac{0.25 \times 1.5}{1+K_{c2}\,2.2(1+0.25K_{c1})}$$

Without cascade:



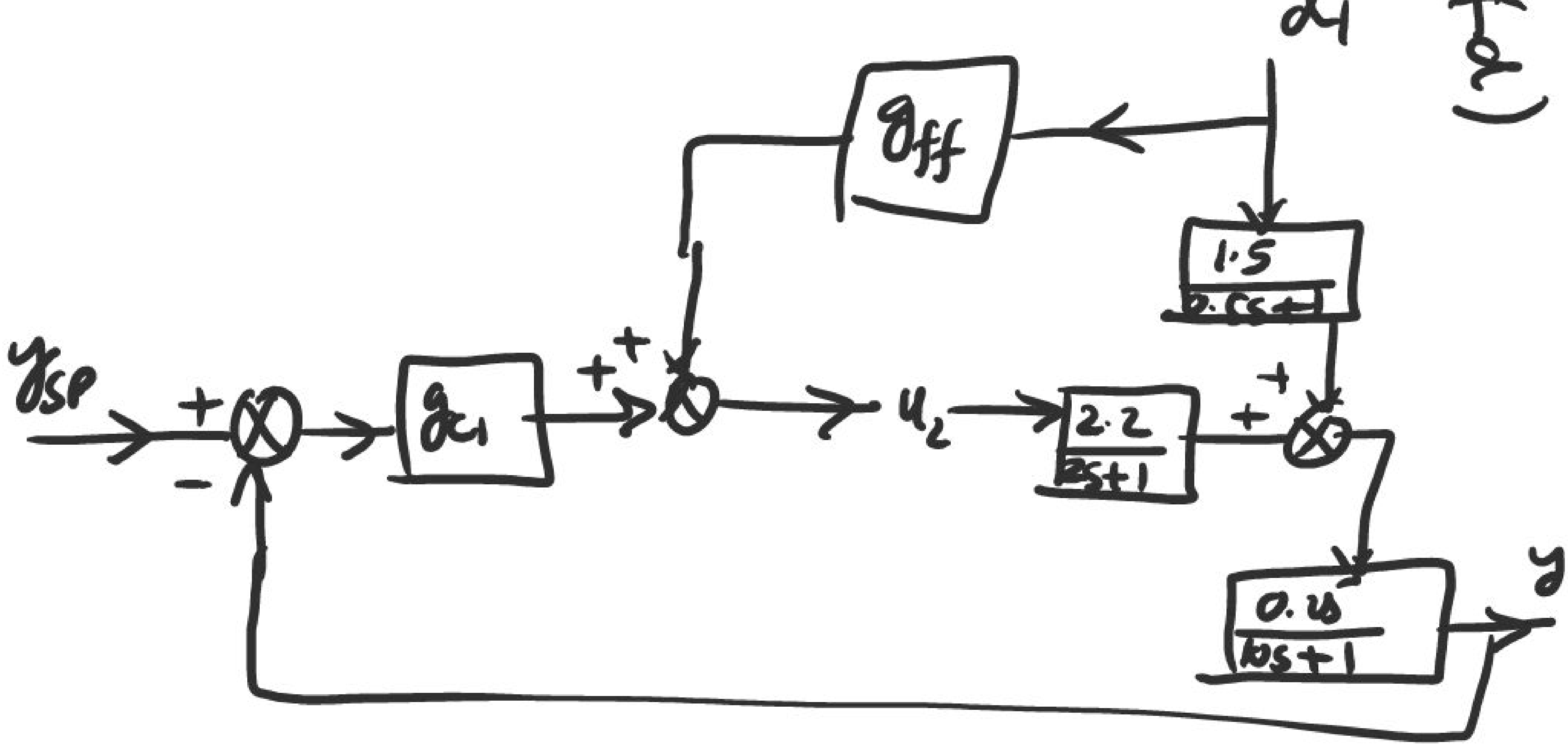
$$y(s) = \frac{\frac{0.25}{10s+1}\,\frac{1.5}{0.5s+1}}{1 + K_{c1}\,\frac{2.2}{2s+1}\,\frac{0.25}{10s+1}} \qquad d_1 \rightarrow \frac{1}{s}$$

$$\lim_{s\to 0} s\,y(s) = \frac{0.25 \times 1.5}{1 + K_{c1}\,2.2\;0.25}$$

$$\frac{\text{response with cascade}}{\text{without cascade}} = \frac{1 + K_{c1}\, 2.2\, 0.25}{1 + 2.2 K_{c2} (1 + 0.25 K_{c1})}$$

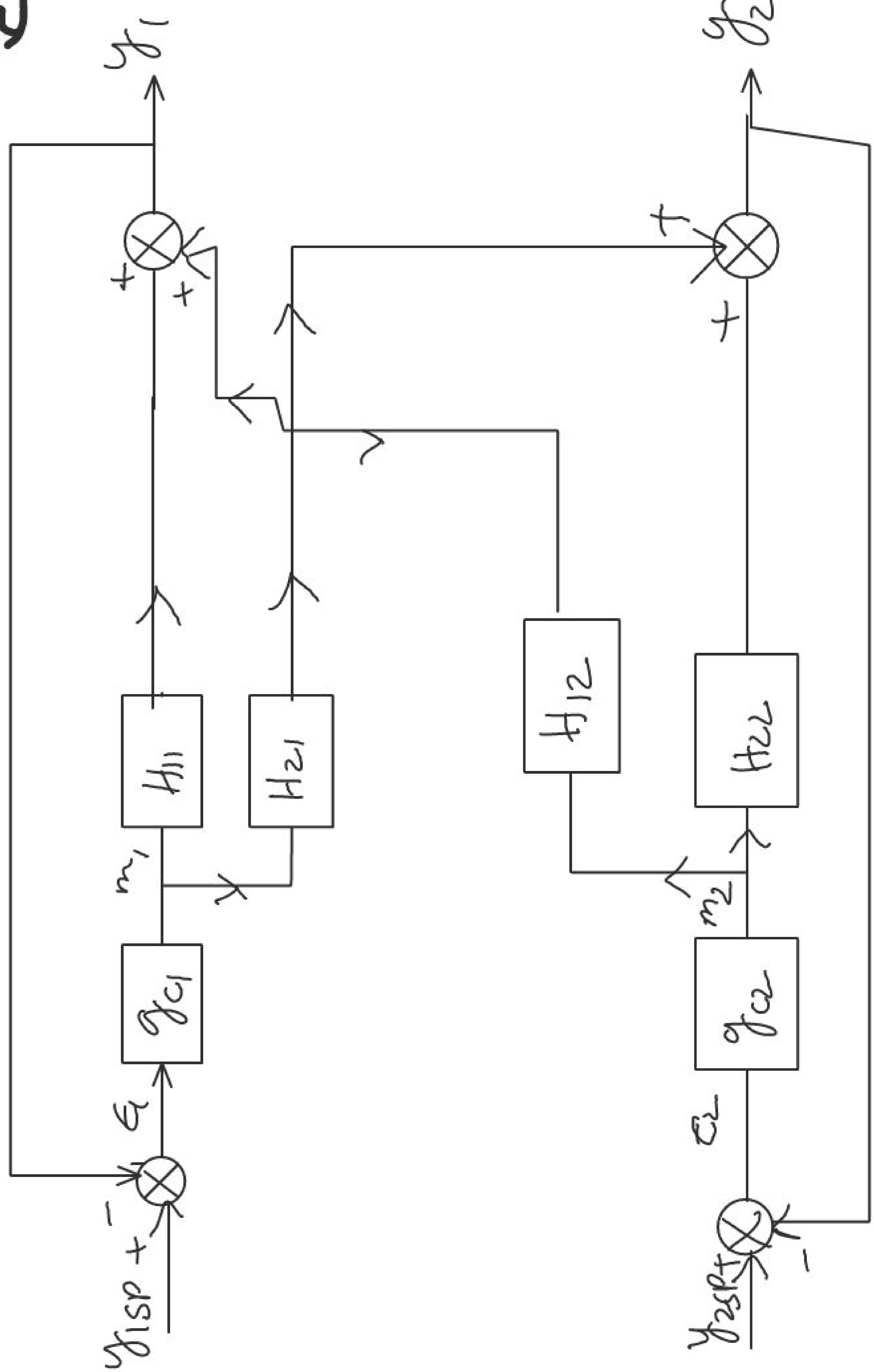
If $K_{c2} \gg 1$ then the response with cascade reduces the output due to a step change in d_1 significantly.

(d)



$$g_{ff} = \frac{\dfrac{-1.5}{0.5s+1}}{\dfrac{2.2}{2s+1}} = \frac{-0.6818 \times (2s+1)}{(0.5s+1)}$$

5a)



y_1
y_2
H_{11}
H_{21}
H_{12}
H_{22}
g_{c1}
g_{c2}
m_1
m_2
ϵ_1
ϵ_2
y_{1SP}
y_{2SP}
+
−

5C

$$y_1(s) = P_{11}(s)\, y_{1sp} + P_{12}(s)\, y_{2sp}$$

$$y_2(s) = P_{21}(s)\, y_{1sp} + P_{22}(s)\, y_{2sp}$$

$$P_{11} = \frac{H_{11} G_{c1} + G_{c1} G_{c2} (H_{11} H_{22} - H_{12} H_{21})}{Q(s)}$$

$$P_{22} = \frac{H_{12} G_{c2}}{Q}; \quad P_{21} = \frac{H_{21} G_{c1}}{Q}$$

$$P_{22} = \frac{H_{22} G_{c2} + G_{c1} G_{c2} (H_{11} H_{22} - H_{12} H_{21})}{Q(s)}$$

$$Q(s) = (1 + H_{11} G_{c1})(1 + H_{22} G_{c2}) - H_{21} H_{12} G_{c1} G_{c2}$$

charac eqn for both loops closed:

$$Q(s) = (1 + H_{11} G_{c1})(1 + H_{22} G_{c2}) - H_{12} H_{21} G_{c2} G_{c1} = 0$$

If we keep loop 1 closed and loop 2 open.

$$y_1 = \frac{h_{11} \, g_{c1}}{1 + h_{11} g_{c1}} \, y_{1sp}$$

$$y_2 = \frac{h_{21} \, g_{c1}}{1 + h_{11} g_{c1}} \, y_{1sp}$$

(m_2 : const)
assume

Part b

d) If G_{c1} is tuned keeping loop 2 open, and if G_{c2} is tuned keeping loop 1 open, it will NOT guarantee the stability of the process with both loops closed. The charac eqns are very different.

5 e) tuning each loop separately:

$$1 + H_{11} G_{c1} = \frac{1 + K_{c1}}{0.1s+1} = 0$$

charac eqn: $(1 + K_{c1}) + 0.1s = 0$

pole $s = -\,\dfrac{(1 + K_{c1})}{0.1}$ for $K_{c1} > 0$

always <u>stable</u>.

Similarly, if loop 2 is closed with loop 1 open,

for any $K_{c2} > 0$, <u>stable</u>.

5f) Both loops closed:

$$\left(1 + \frac{K_{c1}}{0.1s+1}\right)\left(1 + \frac{2K_{c2}}{0.4s+1}\right)$$

$$- \frac{5}{s+1} \quad \frac{1}{0.5s+1} \quad K_{c1} K_{c2} = 0$$

For <u>instability</u> (Routh test).

$$1 + 2K_{c2} + K_{c1} - 3K_{c1} K_{c2} < 0$$

$K_{c2} = 4$

$$9 - 11K_{c1} < 0$$

So $K_{c1} > \frac{11}{9}$ means instability for $K_{c2} = 4$.