ChE381 Process Dynamics & Control **Jan-Apr 2014**

Mid-semester exam **2 hours; 100 points**

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1. Solve the following ODE using Laplace transform:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 2$$

with $y(0) = y'(0) = 0$. [10 points]

2. Consider a continuous stirred tank reactor (CSTR) in which a first order reaction is taking place with rate $r_A = -kC_A$ with $k = 0.040$ min^{-1}. The volume V of the reactor is constant, and the reactor is operating under isothermal conditions. The input volumetric flow rate is 0.085m^3/min, and $V = 2.1$m^3. The input concentration of species A $C_{A0} = 0.925$ mol/m^3. [20 points]

a. Write down the mathematical model for the isothermal CSTR.

b. Determine the steady state concentration at the output of the CSTR.

c. At time $t = 10$ min, the inlet concentration C_{A0} undergoes a step increase of magnitude 0.925 mol/m^3. Determine how the concentration of the output of the CSTR changes as a function of time.

d. What is the ultimate steady value of the output concentration after the step jump ? Qualitatively sketch the response of the output concentration as a function of time.

e. What is the time constant of the process ?

3. A storage tank (shown in figure 1) is fed by an input flow rate $F_i(t)$, and a steady rate of liquid (constant density) withdrawal is maintained by the *constant speed* pump at the tank outlet with flow rate F_0. The tank's cross-sectional area is 2.5 m^2. The incoming flow rate fluctuates around its nominal steady-state value in a sinusoidal fashion with a maximum deviation of 10 m^3/hr, and the frequency of the sinusoidal fluctuation is denoted by ω. [25 points]

a. Derive from first principles the mathematical model (i.e. the ODE) for the change in height of liquid level in the tank.

b. If there is a steady state, what is $F_i(t)$?

c. Derive the response of the liquid level in the tank (as a deviation from the steady state) as a function of time and frequency of the input flow rate.

d. If $\omega = 0.2$ radians/hr, what is the maximum and minimum values of the deviation in the liquid level from its nominal operating value ?

e. If the tank is 10m high and that the nominal operating level is $h = 5$ m, what condition must the frequency of the input flow rate satisfy to guarantee that during operation the tank does not overflow ? Can the tank become empty at any frequency ?

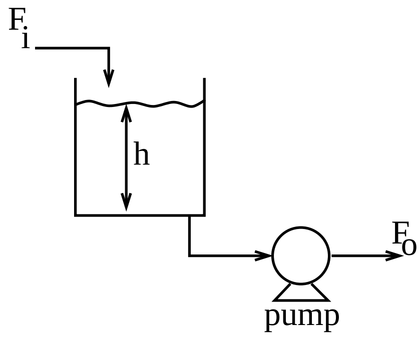

Figure 1: **Problem 3**

4. Consider a second-order system that has a single zero:

$$G(s) = \frac{K(\tau_a s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

If $\tau_1 > \tau_2$, and for a unit step input: [20 points]

a. Determine under what condition(s) will the output $y(t)$ show an extremum (maximum or minimum).

b. Determine when overshoot can occur.

c. Determine when inverse response can occur.

d. If an extremum in $y(t)$ exists, determine the time at which the extremum occurs.

5. In the lectures, it was shown that if there is a general linear relation between output and input variables $y(s) = g(s)u(s)$, and if $u(t) = A\sin(\omega t)$, then the output $y(t) = A_0 \sin(\omega t + \phi)$, where $A_0 = |g(j\omega)|$ and $\phi = \arg(g(j\omega))$. Here $g(s)$ is an arbitrary n^{th} order transfer function. Can a similar relation be derived if $g(s)$ has time delay, i.e. if $g(s) = g_1(s)\exp(-\alpha s)$ where $g_1(s)$ is an n^{th} order transfer function without time delay ? Justify your conclusion by a mathematical proof similar to what was carried out in the lectures. [25 points]