ChE381 Process Dynamics & Control **Jan-Apr 2013**

Mid-semester exam **2 hours; 70 points**

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1. Consider the *parallel* arrangement of two first-order systems, each described by their individual transfer functions $g_1(s) = K_1/(\tau_1 s + 1)$ and $g_2(s) = K_2/(\tau_2 s + 1)$. [15 points]

- Write down the overall transfer function for this composite system, and derive an expression for its gain.
- How many poles and zeros does this transfer function have, and derive expressions for the same.
- Derive an expression for the response of the composite system to a unit step input.
- If $K_1 = 1$, $K_2 = 2$, and $\tau_1 = 1$ and $\tau_2 = 2$ (in appropriate units), will the response of the composite system to a unit step input exhibit an overshoot ? Justify your answer with a calculation.
- If $K_1 = 2$, $K_2 = -1$ and $\tau_1 = 10$, $\tau_2 = 1$, what type of response would the composite system exhibit ? Justify with algebraic deviation.

2. Consider an isothermal CSTR wherein a first-order reaction $A \to B$ is occurring. The feed stream (containing species A) to this CSTR is transported to the reactor at uniform, constant velocity V by a long cylindrical pipe of length L with uniform cross-section area. No reaction takes place in the pipe, and reaction starts as soon as the feed enters the CSTR. The concentration of A at the inlet of the CSTR is c_1 with flow rate F, while the concentration at the inlet of the pipe is c_0. [15 points]

- Derive the dynamical model for this system, and obtain the transfer function that relates the concentration at the outlet of the CSTR to the concentration at the inlet of the pipe.
- Derive the response of this transfer function to a unit step input.
- Derive the response of this transfer function to an unit impulse change in c_0.

3. Consider a CSTR in which a simple exothermic reaction $A \to B$ is occurring. The reactor is cooled by a coolant that flows through a jacket around the reactor. Assume that the volume of the reacting mixture remains a constant at V. The rate of the reaction is given by $r_A = -k_0 \exp[-E/RT] c_A$. Assume that the heat of reaction $H_A - H_B = -\triangle H_r$, and the overall heat transfer coefficient between the jacket and the CSTR is U. [15 points]

- Write down the mass and energy balance for this system.
- Linearize the mass and energy balance about a given steady-state.
- Use deviation variables to write down the linearized dynamical equations in a compact form.

4. Consider a first order system with a time delay given by the transfer function $g(s) = K\exp[-\alpha s]/(1+\tau s)$. Determine the frequency response of this transfer function (i.e. amplitude ratio AR and phase angle ϕ as a function of ω). Determine the low-frequency and high-frequency asymptotes of both AR and ϕ. Qualitatively sketch the Bode plot for this frequency response. [10 points]

5. Consider a constant volume electric water heater, which takes cold water of temperature T_i at the inlet, and gives out hot water of temperature T at the outlet. In a particular operation, cold water at $30°C$ entered the water heater at a rate of 10 lit/min, and hot water from the tank at $80°C$ was withdrawn at the same flow rate. The volume of the liquid inside the tank is 100 liters. Suddenly the heater broke down and stopped supplying heat. At this time, water was still being withdrawn at the rate of 10 lit/min. Cold water at automatically flowed into the tank at precisely the same rate. This went on for exactly 5 min after the heater broke down, and the water withdrawal was stopped, and the cold water also stopped flowing in. By developing an appropriate mathematical model (assuming well-mixed heating tank), show that it is reasonable to consider this as a first-order system. By solving the resulting differential equation, find the final tank water temperature at the end of this 5-minute period. Clearly state your assumptions. [15 points]