ESO 212 Quiz - 1. Paper A. Solution Key.

1.

P. A stream line is perpendicular to the local velocity vector in the fluid $\Rightarrow$ False.

Q. Path lines and streak lines are the same in an unsteady flow $\Rightarrow$ False.

R. Streak lines are produced by continuously injecting dye at a point, and observing its consequent motion $\Rightarrow$ True.

S. Stream lines and streak lines are the same in a steady flow $\Rightarrow$ True.

Only R & S are true $\Rightarrow$ Correct Ans: (B)

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2. $\underline{v} = 5t\,\underline{i} + 2xz\,\underline{j} + ty^2\,\underline{k}$.

$$\underline{a} = \frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + v_x \frac{\partial \underline{v}}{\partial x} + v_y \frac{\partial \underline{v}}{\partial y} + v_z \frac{\partial \underline{v}}{\partial z}.$$

$$\frac{\partial \underline{v}}{\partial t} = 5\,\underline{i} + y^2\,\underline{k}$$

$$\frac{\partial \underline{v}}{\partial x} = 2z\,\underline{j}\;; \quad v_x \frac{\partial \underline{v}}{\partial x} = 10tz\,\underline{j}.$$

$$\frac{\partial \underline{v}}{\partial y} = 2ty\,\underline{k}\;; \quad v_y \frac{\partial \underline{v}}{\partial y} = 4xyzt\,\underline{k}.$$

$$\frac{\partial \underline{v}}{\partial z} = 2x\,\underline{j}\;; \quad v_z \frac{\partial \underline{v}}{\partial z} = 2xy^2t\,\underline{j}.$$

PCS

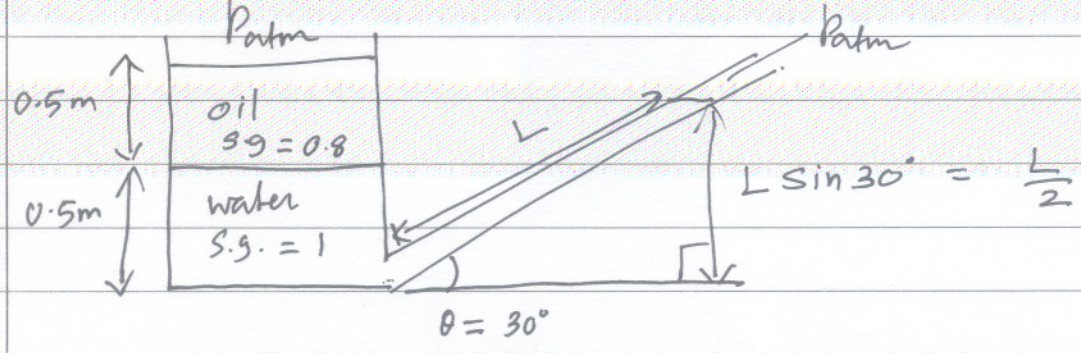
$$\underline{a} = 5\,\underline{i} + (10tz + 2xy^2t)\,\underline{j} + (y^2 + 4xyzt)\,\underline{k}$$

$$\underline{a}\Big|_{\substack{x=1\\y=1\\z=-1\\t=2}} = 5\,\underline{i} + (-20 + 4)\,\underline{j} + (1 - 8)\,\underline{k}$$

$$\underline{a}\Big|_{\substack{x=1\\y=1\\z=-1\\t=2}} = 5\,\underline{i} - 16\,\underline{j} - 7\,\underline{k}$$

$\Rightarrow$ Correct Ans: (A)

3.



For this static arrangement:

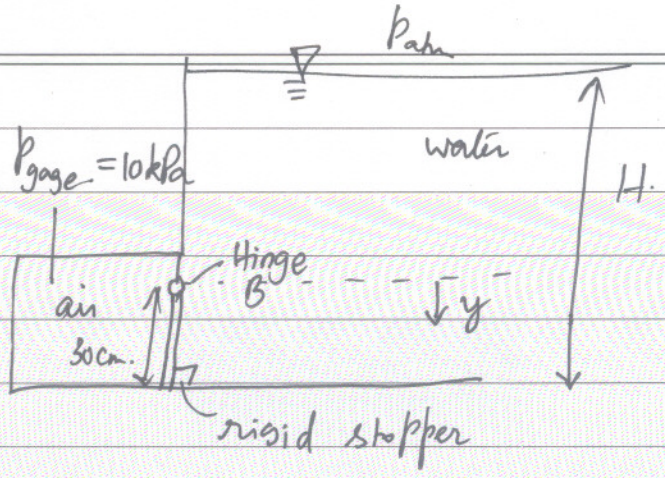
$$P_{atm} + \rho_{oil}\, g\, 0.5(m) + \rho_{water}\, g\, 0.5(m) = P_{atm} + \rho_{water}\, g\, \frac{L}{2}$$

$$\Rightarrow 800 \times 9.8 \times 0.5 + 1000 \times 9.8 \times 0.5 = 1000 \times 9.8 \times \frac{L}{2}$$

$$\Rightarrow 0.8 \times 0.5 + 0.5 = \frac{L}{2} \Rightarrow L = 1.8\,m$$

$\Rightarrow$ Correct Ans. (B)

(4)



Force due to air pressure = $10^4 \times 0.3 \times 0.6 = 1800N$

Moment of this force about B = $1800 \times 0.15 = 270\ N\text{-}m.$

In the fluid (water) : $P_{gage} = S_w g[(H-0.3)+y]$.

Moment due to the distributed force on the fluid side :

$$0.6 \int_{y=0}^{y=0.3} S_w g\,[(H-0.3)+y]\, y\, dy$$

$$= 0.6\ 10^3 \times 9.8 \left[(H-0.3)\frac{y^2}{2} + \frac{y^3}{3} \right]_0^{0.3}$$

Equating the moments

$$\Rightarrow \quad 270 = 0.6 \times 10^3 \times 9.8 \left[(H-0.3)\frac{(0.3)^2}{2} + \frac{(0.3)^3}{3} \right]$$

$$0.0459 = \left[(H-0.3)\,0.045 + 0.009 \right]$$

$$\Rightarrow \quad H = 1.12\ m.$$

$\Rightarrow$ Correct Ans : (C)