2011

ESO 212 QUIZ 3 PAPER A MODEL SOLUTION

(1) line vortex $v_r = 0,\quad v_\theta = \dfrac{K}{\sqrt{(x-x_0)^2 + (y-y_0)^2}}$.

given $x_0 = 2,\ y_0 = 2$.

given $v_\theta = \frac{1}{2}$ at $x = 0$, $y = 0$

$\Rightarrow \dfrac{1}{2} = \dfrac{K}{\sqrt{2^2 + 2^2}} \Rightarrow \dfrac{K}{\cancel{2}\sqrt{2}} = \dfrac{1}{\cancel{2}}$

or, $K = \sqrt{2}$

Then, at $x = 1,\ y = 1$

$$v_\theta = \frac{\sqrt{2}}{\sqrt{(1-2)^2 + (1-2)^2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1.$$

$v_r = 0,\ v_\theta = 1 \Rightarrow$ Correct Ans. A

(2) For potnl flow at A, $v_r = 0,\ v_\theta = 0$

at B, $v_r = 0,\ v_\theta = -2U$.

Correct Ans. D

(3) For potnl flow, $p_A > p_B,\quad p_C > p_D$

Correct Ans. C

(4) FALSE statements on 2-D potential flows:
R & S

$\Rightarrow$ Correct Ans (D)

(5)
$$C_f = \frac{\text{Const}}{\sqrt{Re_x}}$$

$$\tau_w = C_f \, \tfrac{1}{2} \rho U^2$$

$$= (\text{Const}) \, \tfrac{1}{2} \rho U^2 \cdot \frac{1}{\sqrt{\frac{\rho U x}{\mu}}}$$

$$= \underbrace{(\text{Const}) \, \tfrac{1}{2} \rho U^2 \sqrt{\frac{\mu}{\rho U}}}_{\text{all consts.}} \; \frac{1}{x^{1/2}}$$

$$\Rightarrow \tau_w = \frac{K}{x^{1/2}}$$

(A) Force (F_A)
$$= 2 \times \left\{ W \int_0^L \tau_w \, dx \right\} = 2WK \int_0^L \frac{dx}{x^{1/2}}$$
$$= 2WK \, L^{1/2} \times 2$$

(B) Force (F_B)
$$= W \int_0^{2L} \tau_w \, dx = WK \, (2L)^{1/2} \times 2$$
$$= 2^{1/2} \, WK \, L^{1/2} \times 2$$

$$\Rightarrow \frac{F_A}{F_B} = \sqrt{2} \Rightarrow \text{Correct Ans: (A)}$$