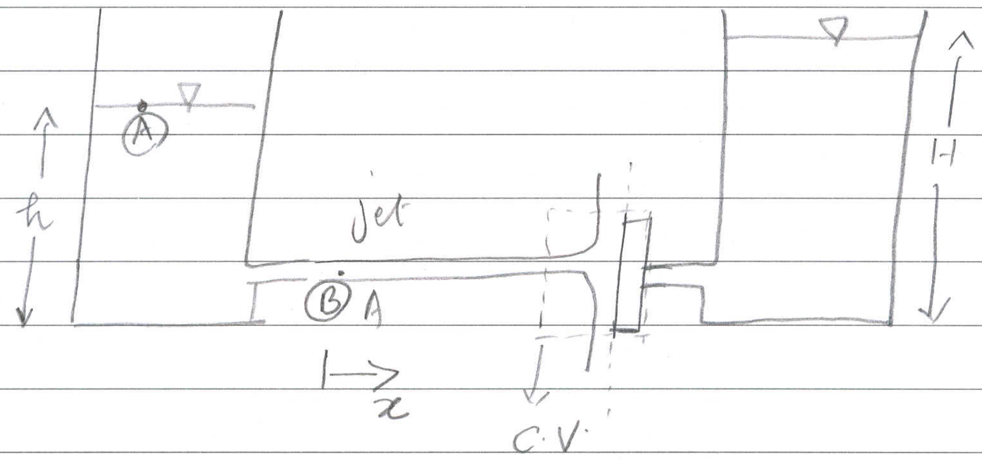
ESO 212 End-Sem Exam Solution Key

(1)



(Steady) Integral momentum balance :

$$\underline{F} = \int_{C.S.} \rho \underline{v} \; \underline{v}.\underline{n} \, dA$$ ——— $\boxed{2 \text{ points}}$

x-component : (uniform flow)

$$F_x = \int_{C.S.} \rho v_x (-v_x) \, dA$$

$$F_x = -\rho v_x^2 A$$

$v_x = V_{jet}$

$F_x = -\rho g H$

$\boxed{2 \text{ points}}$ $-\rho g H A = -\rho V_{jet}^2 A$ ——— (1)

By applying Bernoulli eqn between (A) and (B)

$$\Rightarrow \frac{P_A}{\rho} + \frac{V_A^2}{2} + g z_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + g z_B$$

$P_A = P_B = P_{atm}$; $V_A \approx 0$ $\quad z_A - z_B = h$, (1)

$V_B = V_{jet}$

$\Rightarrow$ Bernoulli eqn gives

$$V_{jet}^2 = 2gh \qquad \boxed{2 \text{ points}}$$

Use this in Eq. ① of prev page:

$$gH = V_{jet}^2$$
$$gH = 2gh \quad \Rightarrow \quad \boxed{h = \frac{H}{2}}$$

$\boxed{2 \text{ points}}$

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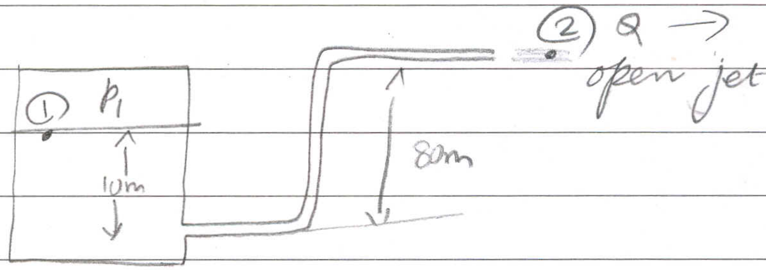
(2)



apply energy balance between ① and ②:

$$\frac{p_1}{\rho} + \frac{\alpha_1 V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + g z_2 + f \frac{L}{D} \frac{V_2^2}{2g} + \sum_i K_i \frac{V_2^2}{2g}$$

$\boxed{2 \text{ points}}$

$p_1 - p_2 = p_{1g}$; $V_1 \approx 0$

$V_2 = V_{pipe}$ (by mass balance)

$V_1 \approx 0$

$z_1 = 10\ m$

$z_2 = 80m$

$$\Rightarrow \frac{p_{1g}}{\rho} = \frac{V^2}{2}\left[\alpha_2 + f\frac{L}{D} + K_{ent} + 2\,K_{90^\circ}\right] + g \cdot 70$$ 2 points

$$Q = 60\,\frac{m^3}{hr} = \frac{60}{3600}\,\frac{m^3}{s} = \frac{1}{60}\,\frac{m^3}{s}$$ 2 points

$$V = \frac{Q}{A} = \frac{1}{60} \cdot \frac{4}{\pi\, 0.05^2} = \boxed{8.49\, m/s}$$

$$\Rightarrow Re \text{ in the pipe} = \frac{\rho V D}{\mu} = \frac{10^3\ 8.49\ 0.05}{10^{-3}} = 4.245 \times 10^5$$

$\Rightarrow$ flow is turbulent $\Rightarrow$ from f-Re chart,

$\boxed{f \approx 0.0136}$ 2 points

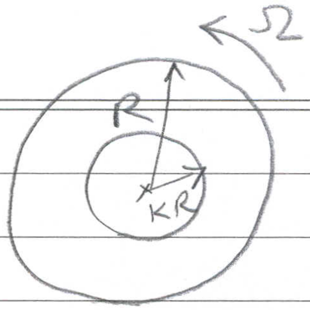
So

$$p_{1g} = \frac{\rho V^2}{2}\left[1 + \frac{0.0136\ 170}{0.05} + 0.5 + 2 \times 0.95\right] + \rho g 70$$

$$= 2.475 \times 10^6\ Pa$$

$\boxed{p_{1g} = 2.475\ MPa}$ 2 points

(3)



Only non-zero velocity component $v_\theta(r)$.

Steady, axisymmetric flow in θ-direction.

$v_\theta,\ p \Rightarrow$ indep of θ

$$\mu \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right] = 0 \qquad \boxed{2\ \text{points}}$$

$$\Rightarrow \quad \frac{\partial}{\partial r}(r v_\theta) = C_1 r$$

$$r v_\theta = C_1 \frac{r^2}{2} + C_2$$

$$\text{or} \quad \boxed{v_\theta = C_1 \frac{r}{2} + \frac{C_2}{r}} \qquad \boxed{2\ \text{points}}$$

B.C $\quad v_\theta(r = kR) = 0$

$\quad v_\theta(r = R) = \Omega R$

$$v_\theta = \frac{\Omega R\left[\frac{r}{R} - \frac{k^2 R}{r}\right]}{(1-k^2)} \qquad \boxed{2\ \text{points}}$$

$$\tau_{r\theta} = \mu r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)$$

$$\tau_{r\theta} = \frac{\mu \Omega R}{\left(\frac{1}{K} - K\right)} \frac{2KR}{r^2}$$

Stress on the inner cylinder at $r = KR$

$$\tau_{r\theta}\Big|_{r=KR} = \frac{2\mu\Omega}{(1-K^2)}$$ $\boxed{\text{2 points}}$

Torque at the cylinder $(r = KR)$

$$= \tau_{r\theta}\Big|_{r=KR} \times 2\pi KRL \times KR$$

$$\boxed{\text{Torque on the inner cylinder} = \frac{4\pi\mu\Omega K^2R^2L}{(1-K^2)}}$$ $\boxed{\text{2 points}}$

2

(4) (a) Potential flow:

$$\phi = Ar^2\cos 2\theta$$

$$v_r = \frac{\partial\phi}{\partial r} = 2Ar\cos 2\theta$$ — $\boxed{\text{1 point}}$

$$v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -2Ar\sin 2\theta$$ — $\boxed{\text{1 point}}$

$$v_\theta = -\frac{\partial\psi}{\partial r} \; ; \qquad v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

$$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = 2Ar\cos 2\theta$$

$$\Rightarrow \psi = Ar^2 \sin 2\theta + C_1(r) \quad - \boxed{\text{2 point}}$$

$$\cancel{-}\frac{\partial \psi}{\partial r} = \cancel{-}\,2Ar\sin 2\theta$$

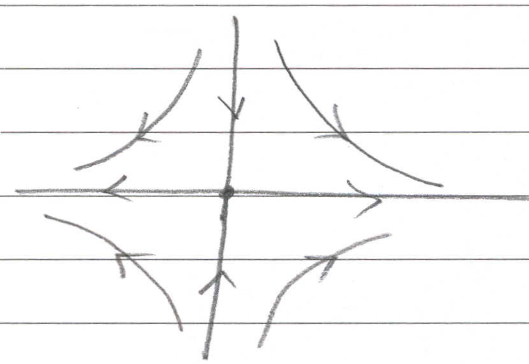
$$\Rightarrow \psi = Ar^2 \sin 2\theta + C_2(\theta) \quad - \boxed{\text{2 point}}$$

$$\Rightarrow \psi = Ar^2 \sin 2\theta$$

Consider the stream line $\psi = 0$.

$\Rightarrow \sin 2\theta = 0 \qquad \Rightarrow 2\theta = n\pi$

$\theta = \frac{n\pi}{2}, \qquad n = 0, 1, 2 \ldots$



$\boxed{\text{4 points}}$

Flow near a stagnation point

4(b) $$C_f = \frac{0.664}{Re_x^{1/2}}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\left(\frac{\rho U}{\mu}\right)^{1/2} x^{1/2}}$$

$$\frac{\tau_w(x)}{\frac{1}{2}\rho U^2} = \left(\frac{\mu}{\rho U}\right)^{1/2} \frac{0.664}{x^{1/2}} \quad \text{— } \boxed{2 \text{ points}}$$

Force on a single plate $= a\int_0^L \tau_w(x)\,dx$

$$= \frac{1}{2}\rho U^2 a \left(\frac{\mu}{\rho U}\right)^{1/2} 0.664 \int_0^L \frac{dx}{x^{1/2}}$$

$$= 0.664\, a \left(\frac{\mu}{a U}\right)^{1/2} \sqrt{L}\; \rho U^2$$

$$= 0.664\, a\, \mu^{1/2} \rho^{1/2} U^{3/2} L^{1/2}$$

F on one plate $= 0.664\,(\rho\mu L)^{1/2} a\, U^{3/2}$ — $\boxed{2 \text{ points}}$

F on four plates $= 4\; 0.664\,(\rho\mu L)^{1/2} a\, U^{3/2}$.

By an integral momentum balance:

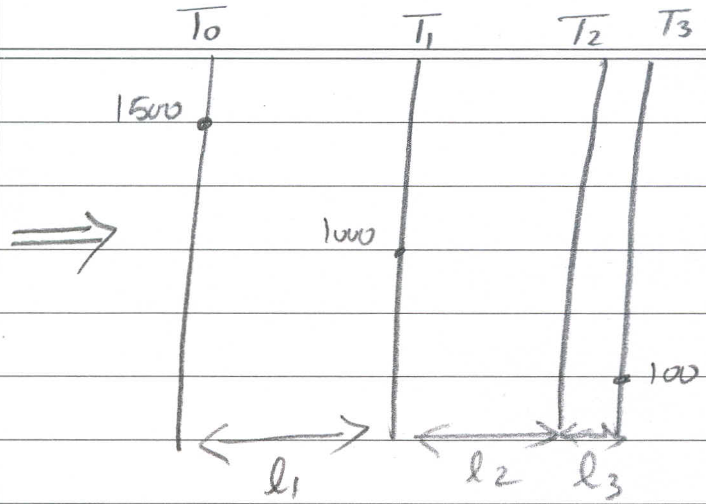
$$\Delta p = \frac{F}{a^2} = \frac{4 \times 0.664\,(\rho\mu L)^{1/2} a\, U^{3/2}}{a^2}$$

$\boxed{\Delta p = 6.17 \text{ Pa}}$ — $\boxed{4 \text{ points}}$

5(a)

q

$2000 \frac{W}{m^2}$



$l_3 = 0.01$

$$q = k_1 \frac{\Delta T}{l_1} \quad \text{— } \boxed{2 \text{ points}}$$

$$\Rightarrow l_1 = \frac{k_1 (T_0 - T_1)}{q}$$

$$l_1 = \frac{4 \times 500}{2000} \Rightarrow \boxed{l_1 = 1\,m} \quad \text{— } \boxed{2 \text{ points}}$$

$$q = \frac{T_1 - T_3}{\left[\frac{l_2}{k_2} + \frac{l_3}{k_3}\right]}$$

$$2000 = \frac{900}{\left[\frac{l_2}{2} + \frac{0.01}{50}\right]}$$

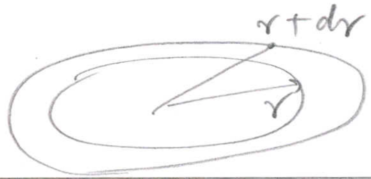
$$\Rightarrow \frac{l_2}{2} + \frac{0.01}{50} = \frac{900}{2000}$$

$$\Rightarrow \boxed{l_2 = 0.8996\,m} \quad \text{— } \boxed{2 \text{ points}}$$

$$2000 = \frac{T_2 - \overset{100°C}{T_3}}{\frac{0.01}{50}}$$

$$\Rightarrow \boxed{T_2 = 100.4°C}$$

$\boxed{2 \text{ points}}$



5(b) heat in at $r \Rightarrow q_r (2\pi r)(2B)\Big|_r$ $\boxed{2 \text{ points}}$

heat out at $r+dr \Rightarrow q_r (2\pi r)(2B)\Big|_{r+dr}$ $\boxed{2 \text{ points}}$

heat lost by convection $\Rightarrow 2 \times (2\pi r)dr\, h(T-T_a)$

at steady-state: $\left[q_r (2\pi r)(2B)\right]_r - \left[q_r (2\pi r)(2B)\right]_{r+dr}$

$$- 2 \times 2\pi r\, dr\, h(T-T_a) = 0.$$

$\div$ by $dr \Rightarrow$ $\boxed{2 \text{ points}}$

$$-\frac{d}{dr}(r q_r) - \frac{r h (T-T_a)}{B} = 0$$

$q_r = -k\frac{dT}{dr} \Rightarrow k\frac{d}{dr}\left(r\frac{dT}{dr}\right) - \frac{r h (T-T_a)}{B} = 0$

$$\Rightarrow \boxed{\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) - \frac{h}{kB}(T-T_a) = 0}$$

$\boxed{2 \text{ points}}$

B.C. at $r = R_0$, $T = T_w$
at $r = R_0 + R_1$, $\frac{dT}{dr} = 0$ $\Big\}$ $\boxed{2 \text{ points}}$

6 (a) Transient Conduction:

$$\Theta = \frac{T - T_0}{T_1 - T_0} \quad ; \quad \tilde{t} = \frac{t\alpha}{R^2}$$

$$\Rightarrow \frac{\partial \Theta}{\partial t} = \frac{1}{\tilde{r}^2} \frac{\partial}{\partial \tilde{r}} \left(\tilde{r}^2 \frac{\partial \Theta}{\partial \tilde{r}} \right)$$

IC: $\Theta = 0$ at $\tilde{t} = 0$

BC: $\Theta = 1$ at $\tilde{r} = 1$.

let $\Theta(\tilde{r} = 0, \tilde{t} = \tilde{\tau}) = 0.99$ $\boxed{\text{2 points}}$

$\boxed{\text{2 points}}$ $\tilde{\tau}$ will be the same for different spheres.

$\boxed{\text{2 points}}$ $$\frac{\tau_1 \alpha}{R_1^2} = \frac{\tau_2 \alpha}{R_2^2}$$

$m = \frac{4}{3}\pi R^3 \rho$

$R \propto m^{1/3}$

$$\frac{\tau_2}{\tau_1} = \frac{R_2^2}{R_1^2} = \left(\frac{m_2}{m_1}\right)^{2/3}$$

if $m_2 = 2m_1$ $\boxed{\tau_2 = \tau_1 \, 2^{2/3}}$ $\boxed{\text{2 points}}$

(or)

$\boxed{\tau_2 = \tau_1 \, 4^{1/3}}$

6(b)

$$D \frac{d^2 C_A}{dz^2} - k C_A = 0$$

$\boxed{\text{2 points}}$

BC $z = 0, \quad C_A = C_{A0}$

$z = L, \quad \left.\frac{dC_A}{dz}\right|_{z=L} = 0$

$\boxed{\text{2 points}}$

$$\frac{d^2 C_A}{dz^2} - \frac{k}{D} C_A = 0$$

let $\Gamma = \frac{C_A}{C_{A0}}$; $\tilde{z} = \frac{z}{L}$.

$$\Rightarrow \frac{d^2 \Gamma}{d\tilde{z}^2} - \phi^2 \Gamma = 0 \qquad \phi^2 = \frac{L^2 k}{D}$$

$$\Gamma = C_1 e^{\phi z} + C_2 e^{-\phi z}.$$

$\boxed{\text{2 points}}$

B.C $\Gamma(\tilde{z} = 0) = 1$

$\frac{d\Gamma}{d\tilde{z}}(\tilde{z} = 1) = 0$

$$\boxed{\Gamma(\tilde{z}) = \frac{e^{-\phi}}{e^{\phi} + e^{-\phi}} \left[e^{\phi \tilde{z}} - e^{-\phi \tilde{z}} \right] + e^{-\phi \tilde{z}}}$$

$\boxed{\text{2 points}}$