ESO212 Fluid Mechanics & Rate Processes **July-Nov 2011**

End-semester Exam **3 hours; 80 points**

• Start each question in a new page of the answer book; Write your answers in legible and clear handwriting.

1. **Integral Momentum Balance:** Two large tanks containing water have smoothly contoured orifices (openings) of equal area (figure 1). A jet of water issues from the orifice of the left tank. Assume that the flow is uniform and **viscous losses are negligible**. The jet impinges on a vertical flat plate covering the opening of the right tank. Choose an appropriate CV to determine the minimum value of the height h required to keep the plate in place over the orifice of the right tank. [8 points]

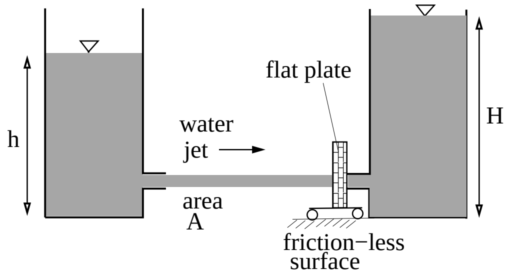

Figure 1: **Problem 1**

2. **Pipe Flows and Losses:** The pipe flow shown in figure 2 is driven by pressurized air (at pressure p_1) in the tank. What gage pressure p_1 is needed to provide water flow rate $Q = 60\, m^3/h$? Assume the entrance from the tank to the pipe be a sharp entrance, and the two bends to be at an angle of 90°. The friction factor chart and the minor loss coefficients are provided in a separate data sheet. [10 points]

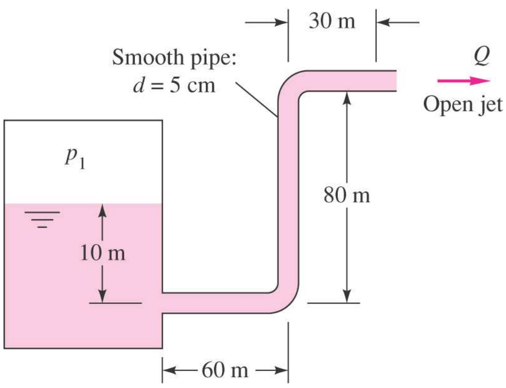

Figure 2: **Problem 2**

3. **Navier-Stokes Equations:** Consider the steady flow of a Newtonian fluid in the annular region between two long cylinders (inner radius κR and outer radius R) as shown in the figure 3. The outer cylinder moves with an angular velocity Ω, while the inner cylinder is stationary.

(a) Simplify the relevant component of the Navier-Stokes momentum equations in cylindrical coordinates. [2 points]

(b) Solve this differential equation, and obtain the velocity profile in the annular gap after utilizing the boundary conditions. [4 points]

(c) Find the torque exerted by the fluid on the inner cylinder. [4 points]

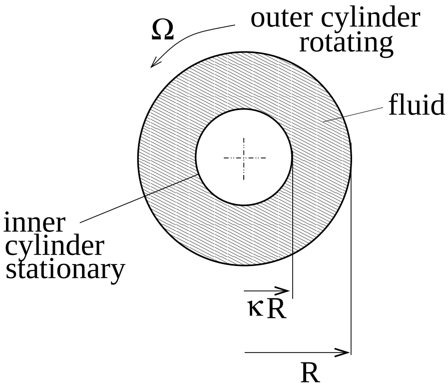

Figure 3: **Problem 3: Top view of the set-up with two concentric cylinders**

4 (a) **Potential Flow:** The velocity potential for a 2-D potential flow is given by $\phi(r,\theta) = Ar^2\cos 2\theta$, where A is a constant with appropriate units, and r and θ are the polar coordinate variables. Determine the expressions for the velocity components and stream function for this flow. By considering the streamline with $\psi = 0$, sketch the geometry of the surface. Based on this, qualitatively plot the streamlines of the flow, and comment on what physical situation the potential flow represents. [10 points]

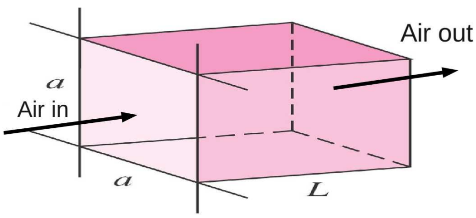

Figure 4: **Problem 4(b): Air flow into a box**

4 (b) **Boundary Layers:** Consider a box made of four plates with length $L = 25$ cm and width $a = 4$ cm as shown in the figure 4. Air flows into this box with a free stream velocity $U_0 = 12\,m/s$.

Assuming steady, laminar flat-plate boundary layer flow over the plates, determine the pressure drop required to maintain the flow in the box. Assume that the interaction between the boundary layers at the corners of the box is negligible. The friction coefficient $C_f = 0.664/Re_x^{1/2}$ for laminar boundary-layer flow over a single flat plate, and Re_x is the Reynolds number based on the distance from the origin of the plate. [8 points]

5 (a) **Steady Conduction:** A furnace wall is to be designed with three layers (figure 5). The first layer is a heat-resistant fire brick (thermal conductivity $k = 4$ W/m-K, thickness l_1), which can withstand a maximum temperature of $1500°C$. The second layer is made of insulating brick (thermal conductivity $k = 2$ W/m-K, thickness l_2) which can withstand a maximum temperature of $1000°C$. The third layer is a steel plate (thermal conductivity $k = 50$ W/m-K, thickness = 10 mm, which is included for mechanical strength. The steel plate is exposed to an atmosphere of temperature 100°C. If the heat loss through the wall is 2000 W/m^2, determine the thickness l_1 and l_2 of the first two layers. Also determine the temperature of the interface between the insulating brick and steel. [8 points]

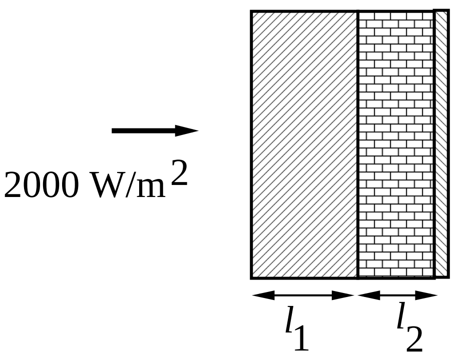

Figure 5: **Problem 5(a): Heat flow into three layers**

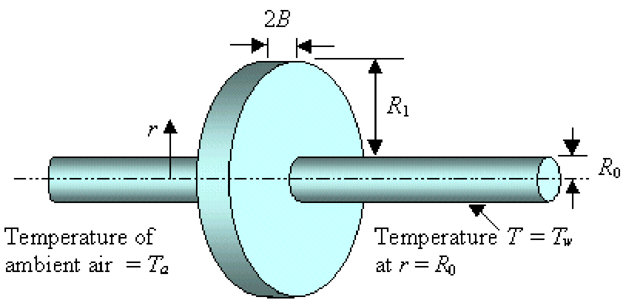

Figure 6: **Problem 5(b): Circular fin fitted to a long cylinder**

5 (b) **Circular fin:** A long cylinder of radius R_0 and temperature T_w is fitted with a circular fin (of radius $(R_0 + R_1)$ and thickness $2B$) as shown in figure 6. The ambient fluid is at temperature T_a, and the heat transfer coefficient between the fin and the fluid is h. Carry out a differential internal energy balance over a thin annular region between r and $r + dr$, and thereby derive a differential

equation for the steady temperature profile $T(r)$ in the fin. Specify the boundary conditions to solve this equation. *Do not solve the equation.* [10 points]

6 (a) **Transient Conduction:** A solid sphere (density ρ, thermal diffusivity α) of mass m_1 and initial temperature T_0 is immersed in a liquid bath which is maintained at a temperature T_1. Let the time taken for the center of the sphere to reach $0.99(T_1 - T_0)$ be τ_1. Another solid sphere (with identical density and thermal diffusivity as the previous sphere) of mass m_2 and initial temperature T_0 is immersed in an identical bath of temperature T_1. Let the time taken for the center of this sphere to reach $0.99(T_1 - T_0)$ be τ_2. If the heat transfer resistance in the liquid side is negligible, and $m_2 = 2m_1$, determine the ratio of the times τ_2/τ_1. [8 points]

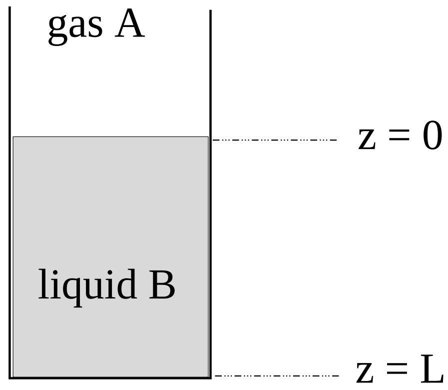

Figure 7: **Problem 6(b): diffusion of gas A in liquid B**

6 (b) **Steady Mass Transfer:** Gas A dissolves in liquid B (present in a beaker, see figure 7) and subsequently undergoes an irreversible reaction wherein the dissolved A reacts with B to give C throughout the liquid in the beaker. The volumetric rate of consumption of A (moles of A consumed per unit volume per unit time) is given by $-kC_A$. The concentration of A at the liquid-gas interface is C_{A0}. Determine the concentration profile $C(z)$ of species A in the liquid. Assume that the concentration of A is in the **dilute regime** [8 points]