ESO 212 I Midsem Soln. key. (2011).

1 (a). Apply energy eqn. between ① and ②.

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_\ell$$

$p_1 = p_2 \approx P_{atm}$; $V_1 = V_2 \approx 0$.

$$\Rightarrow (z_1 - z_2) = h_\ell$$
$$= 5.4 \frac{V_{tube}^2}{2g}$$

$$15m = \frac{5.4 \times V_t^2}{2 \times 9.8}$$

Ans part (a). $\Rightarrow \boxed{V_t = 7.38 \ m/s.}$ [2 points].

(b) $$\frac{\cancel{p_1}}{\cancel{\rho g}} + \frac{\cancel{V_1^2}}{\cancel{2g}} + z_1 = \frac{\cancel{p_2}}{\cancel{\rho g}} + \frac{\cancel{V_2^2}}{\cancel{2g}} + z_2 - h_p + h_\ell$$

pump. head

$$\underbrace{(z_1 - z_2)}_{15m} = \frac{5.4 V_t^2}{2 \times 9.8} - h_p$$

$\frac{w_p}{g}$

Db $V_t = 10 \ m/s \Rightarrow h_p = 12.55m.$

w_s = rate of which pump does work on the fluid / mass flow rate

Power reqmt. of pump = Rate at which pump does work on the fluid

$$= \dot{m} \times w_p$$

$$= \dot{m} \times h_p \times g$$

$$= \rho \dot{Q} \, h_p \, g$$

$$= 10^3 \times \frac{\pi}{4} (0.05)^2 \times 10 \times 12.55 \times 9.8$$

Rate at which pump does work on the fluid: $= \boxed{2415 \text{ Joules/sec}}$

Ans Part B. [2 points]

(c). Flow is now reversed:

$$z_2 = z_1 - h_p + h_e$$

$$h_p = h_e + (z_1 - z_2)$$

$$= \frac{5.4 \times 10^2}{2 \times 9.8} + 15 = 42.55 \text{ m.}$$

$w_p = h_p g$; $\dot{W}_p = \dot{m} w_p = \dot{m} h_p g$ [2 points].

$$= \rho \dot{Q} h_p g$$

Ans Part C. 2 $= \boxed{8187 \text{ J/s}}$

PCS

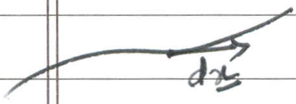
(d). Bernoulli between ① & ② results in

$$g z_1 = g z_2 \Rightarrow \text{a contradiction}$$

since $z_1 \neq z_2$.

The reason for this contradiction is that Bernoulli eqn. neglects all losses; which is not true here. The gravitational potential energy is converted to internal energy by viscous losses h_e, which is set to zero ~~here~~ in Bernoulli eqn. [2 points].

② (a) $\underline{v} \times d\underline{x} = 0$ along a streamline.



$$\Rightarrow \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0.$$

$$\Rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

[2 points].

(b) 2-D flow: $u = \frac{\partial \psi}{\partial y}$; $v = -\frac{\partial \psi}{\partial x}$.

$\psi = \psi(x, y)$

$$\Rightarrow d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy.$$

$\Rightarrow d\psi = -v\,dx + u\,dy$

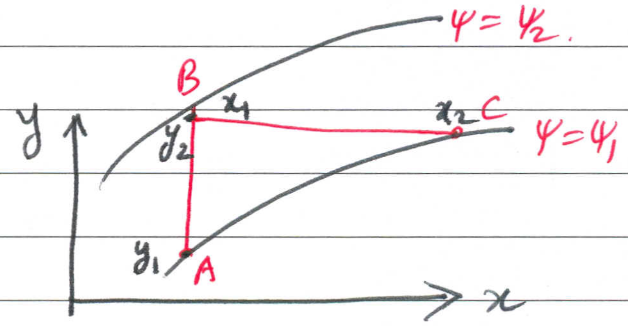
But $\dfrac{dx}{u} = \dfrac{dy}{v} \Rightarrow u\,dy - v\,dx = 0$

So $d\psi = 0$ along a streamline

$\Rightarrow \psi$ is a constant along a streamline.

[3 points]

(C).



along AB $\quad Q$ (per width) $= \displaystyle\int_{y_1}^{y_2} u\,dy$

$$= \int_{y_1}^{y_2} \frac{\partial \psi}{\partial y}\,dy$$

Since $x = \text{const}$ at AB

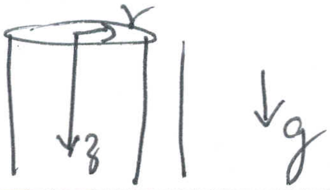
$$Q = \int_{y_1}^{y_2} d\psi$$

$$= \psi(y_2) - \psi(y_1) = \psi_2 - \psi_1.$$

along BC $\quad Q = \displaystyle\int_{x_1}^{x_2} v\,dx = -\int_{x_1}^{x_2} \frac{\partial \psi}{\partial x}\,dx = -\int_{x_1}^{x_2} d\psi$

$$= \psi(x_1) - \psi(x_2)$$

(4). $= \psi_2 - \psi_1$. [2 points].



z.

Simplifications of Continuity eqn. $\Longrightarrow$ [1 point]

~~1 point~~

(3)

(a) $v_z = u_z(r)$

BC
$p = p_{atm}$ at $r = aR$
$\tau_{rz} = \mu \dfrac{dv_z}{dr} = 0$ at $r = aR$
$v_z(r) = 0$ at $r = R$

[1 point] $\Longleftarrow$

r-mom: $\dfrac{\partial p}{\partial r} = 0 \Rightarrow p = p(z)$ only

But $p = p_{atm}$ @ $r = aR \Rightarrow p = p_{atm}$ everywhere inside the liquid.

[1 point]

$\Rightarrow \dfrac{\partial p}{\partial z} = 0.$

z-mom: $\mu \dfrac{1}{r}\dfrac{\partial}{\partial r}\left[r \dfrac{\partial v_z}{\partial r}\right] + \rho g = 0$ — [1 points]

$$\Rightarrow \frac{\partial}{\partial r}\left\{r \frac{\partial v_z}{\partial r}\right\} = -\frac{\rho g}{\mu} r$$

$$\Rightarrow v_z = -\frac{\rho g r^2}{4\mu} + c_1 \ln r + c_2.$$ — ~~1 point~~

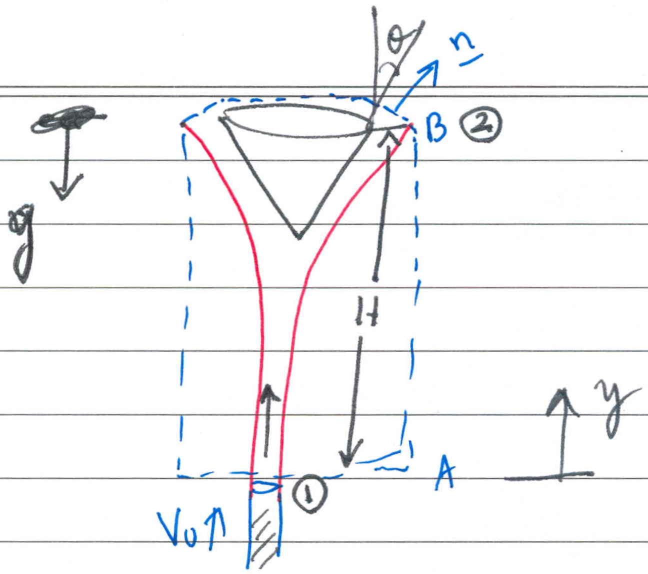
after using B.C's.

$$v_z = \frac{\rho g R^2}{4\mu}\left[1 - \frac{r^2}{R^2} + 2a^2 \ln \frac{r}{R}\right]$$ — [1 point]

(b) $\tau_{rz}\Big|_{r=R} = \mu \dfrac{dv_z}{dr}\Big|_{r=R} = \dfrac{\rho g R}{2}(a^2 - 1)$ — [2 points]

(5)

4



Mass : $V_1 A_1 = V_2 A_2$. — $\boxed{\text{1 point}}$

y-comp. Momentum : $F_y = \int_{c.s.} v_y \, \rho \underline{v} \cdot d\underline{A}$

$\boxed{\text{2 points}}$ — $Mg = \left[V_2 \cos\theta \, \rho V_2 A_2 \right] - \left[V_1 \rho V_1 A_1 \right]$

Bernoulli : $V_2^2 = V_1^2 - 2gH$. — $\boxed{\text{2 points}}$

$V_1 = V_0$;

$\Rightarrow \quad M = \dfrac{(V_0 - V_2 \cos\theta) \, \rho V_0 A_1}{g}$

$\boxed{\text{2 points}}$

$M = \left[\dfrac{V_0 - \sqrt{V_0^2 - 2gH} \, \cos\theta}{g} \right] \rho V_0 A_1$

$H = 1 m$

$V_0 = 10 m/s$

$D = 100 mm = 0.1 m$

$\theta = 30^0$

$\Rightarrow \quad \boxed{M = 17.9 \text{ kg.}}$ — $\boxed{\text{1 point}}$