ESO212 Fluid Mechanics & Rate Processes **July-Nov 2011**

Mid-semester Exam **2 hours; 30 points**

Use $g = 9.8$ m/s^2, density of water = 10^3 kg/m^3, viscosity of water = 10^{-3} Pa s.

1. Two large water tanks are connected with a pipe of diameter $D = 5$ cm and length 2 m as shown in figure 1. The kinetic energy correction factor for flow in the pipe $\alpha = 1$.

   (a) If the frictional loss head (h_l, in meters) for the pipe is given by $5.4\frac{V_{tube}^2}{2g}$ (V_{tube} in m/s, and g in m/s^2), calculate the velocity V_{tube} in the pipe. [2 points]

   (b) If a pump is installed in the pipe (with h_l given by the same expression as in part a) to have a velocity $V_{tube} = 10$ m/s, calculate the rate at which the pump does work on the fluid, if the flow is from A to B. [2 points]

   (c) Calculate the rate at which the pump does work on the fluid if the flow is from B to A, for $V_{tube} = 10$ m/s, with h_l given by the same expression as in part a. [2 points]

   (d) What is the inconsistency in using the Bernoulli equation between points 1 and 2 on the free surface of the two tanks ? [2 points]

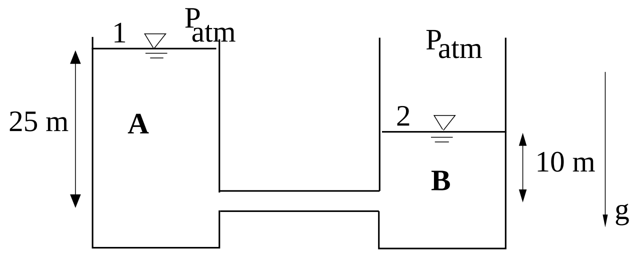


Figure 1: **Problem 1**

2. (a) Derive the equation that governs a streamline using the definition of the streamline. [2 points]

   (b) For a 2-D incompressible flow, show that stream function is a constant along a streamline. [3 points]

   (c) Show that for a 2-D incompressible flow, the volumetric flow rate (per unit width) between two streamlines is given by the difference in the values of their stream function. [2 points]

3. Consider the axi-symmetric, steady, fully-developed, laminar flow of a liquid film (density ρ and viscosity μ) **outside** a circular tube driven by gravity, as shown in figure 2. The outer radius of the tube is R, and the thickness of the annular liquid film is $(a-1)R$. The Navier-Stokes equations in cylindrical coordinates are provided at the end of this question paper.

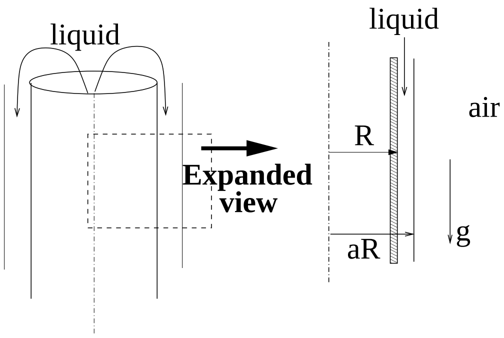


Figure 2: **Problem 3**

(a) Derive the expression for the velocity distribution in the falling liquid film outside the tube. [5 points]

(b) Derive the expression for the shear stress exerted on the outer wall of the tube. [2 points]

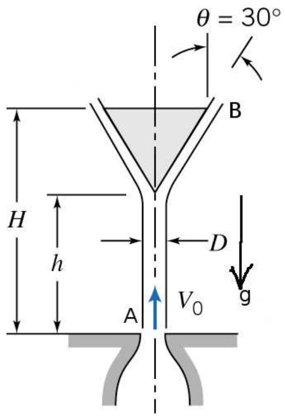
4. A water jet of diameter D flows out of a nozzle at a velocity V_0. This jet is used to support a cone-shaped object as shown in figure 3(a). The jet forms a uniform film surrounding the cone. Assume friction-less, steady, incompressible flow. Choose an appropriate control volume (CV) with control surfaces cutting across locations A and B.

    (a) Simplify the integral mass balance for the CV.

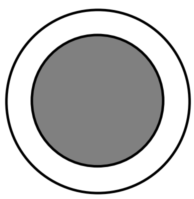
    (b) Simplify the integral momentum balance for the CV.

    (c) Apply Bernoulli equation between points A and B.

    Using the above three equations, derive an expression for the combined mass (M) of the cone and water in the CV that can be supported by the jet. For $V_0 = 10$ m/s, $H = 1$ m, $h = 0.8$ m, $D = 100$ mm, $\theta = 30^\circ$, what is the numerical value of M (in kg) ? [8 points]



(a) Cross-section view



(b) Top view

Figure 3: **Problem 4**

Navier-Stokes equations in cylindrical coordinates:

Continuity:

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$

r-momentum:

$$\rho\left[\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right]=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]+\rho g_r$$

z-momentum:

$$\rho\left[\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right]=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$