$$\frac{\rho g h^2 b}{2} - \left[\frac{\rho g h^2 b}{2} + \frac{\partial}{\partial x}\left(\frac{\rho g h^2 b}{2}\right)\delta x\right] = \frac{\partial}{\partial t}[V\rho h b\,\delta x] - V(\rho h V b)$$

$$+\left[\rho h b V^2 + \frac{\partial}{\partial x}\left(\rho h b V^2\right)\delta x\right] \quad \text{(b)}$$

since there is no horizontal momentum flux associated with the fluid entering the top surface. On simplification Eq. (b) gives

$$\frac{\partial (Vh)}{\partial t} + \frac{\partial\left(hV^2\right)}{\partial x} = -gh\frac{\partial h}{\partial x} \quad \text{(c)}$$

Eqs. (a) and (c) are two equations for the two unknowns, V and h.

When $\overset{\circ}{q}$ is a constant and the flow is steady, Eq. (a) integrates to (see Example 4.7)

$$Vh = \overset{\circ}{q}\, x/b \quad \text{(d)}$$

and Eqs. (c) and (d) give

$$\frac{\partial\left(hV^2\right)}{\partial x} = -g\frac{\partial}{\partial x}\left(h^2/2\right)$$

or

$$hV^2 = -\frac{gh^2}{2} + C_1 \quad \text{(e)}$$

At $x = 0$, $V = 0$ and $h = h_0$ and, therefore, C_1 is $gh_0^2/2$. Eqs. (d) and (e) lead to

$$h^3 - h_0^2\ h + \left(\frac{2\,\overset{\circ}{q}{}^2}{gb^2}\right)x^2 = 0$$

which gives the free surface profile. It can be confirmed that initially (till $h = h_0/\sqrt{3}$) the slope of the free surface is negative, i.e., the level decreases with x!

5.3 MOMENTUM CORRECTION FACTOR

The calculation of the momentum flux across a tube section in Example 5.2 was simplified by assuming that the velocity is constant across the section, i.e., the flow is one-dimensional. The value of the velocity used in the calculations is the average velocity at the given section. The correct momentum flux is *larger* than the value so obtained. When more accurate results are required a correction for this difference is usually done through the use of a *momentum correction factor*. Obtained below is an expression for the momentum correction factor for incompressible flows where the velocity is normal everywhere to the cross-sectional area. This condition holds when the flow is uni-directional.

The actual momentum flux across a cross-section in such a flow is given by $\iint_A V(\rho V\, dA)$, while the one based on the average velocity is $\rho V_{av}^2 A$, where $V_{av} = \frac{1}{A}\iint_A V\, dA$. The momentum correction factor β is defined as the ratio of the actual momentum flux to the one obtained with the 1-D approximation. Thus,

$$\beta = \frac{\iint_A \rho V^2\, dA}{\rho V_{av}^2 A} = \frac{1}{A}\iint_A \left(\frac{V}{V_{av}}\right)^2 dA \qquad \text{...(5.4)}$$

since ρ is constant.

The use of this expression to compute β for fully-developed flow through a circular pipe of radius R is illustrated below. The velocity profile in this case is (see Fig. 1.28)

$$V = V_m\left(1 - \frac{r^2}{R^2}\right), \text{ for laminar flows}$$

and

$$V = V_m\left(1 - \frac{r}{R}\right)^{1/7} \quad \text{for turbulent flows}$$

where V_m is the maximum velocity (at the centre line).

For laminar flows

$$V_{av} = \frac{1}{A}\iint_A V\, dA = \frac{1}{\pi R^2}\int_0^R V_m\left(1 - \frac{r^2}{R^2}\right) 2\pi r\, dr = V_m/2$$

and

$$\beta = \frac{1}{A}\iint_A \left(\frac{V}{V_{av}}\right)^2 dA = \frac{1}{\pi R^2}\int_0^R 4\left(1 - \frac{r^2}{R^2}\right)^2 2\pi r\, dr = 1.33 \qquad \text{...(5.5)}$$

For turbulent flows

$$V_{av} = \frac{1}{A}\iint_A V\, dA = \frac{1}{\pi R^2}\int_0^R V_m\left(1 - \frac{r}{R}\right)^{1/7} 2\pi r\, dr = 49\, V_m/60$$

and

$$\beta = \frac{1}{A}\iint_A \left(\frac{V}{V_m}\right)^2 dA = \frac{1}{\pi R^2}\int_0^R \left(\frac{60}{49}\right)^2 \left(1 - \frac{r}{R}\right)^{2/7} 2\pi r\, dr = 1.020 \qquad \text{...(5.6)}$$

Thus, one-dimensionality assumption is far more acceptable when the flow through a circular tube is turbulent than when it is laminar. This is because the velocity profiles in turbulent flows are much flatter (Fig. 1.28) and, thus, are closer to the 1-D approximation.

Example 5.6. In Example 5.2 the pressure drop across a sudden expansion assuming 1-D flow was calculated. If the flow in the narrower pipe is assumed to be turbulent and fully developed and that in the wider pipe, laminar and fully developed, obtain the correct expression.

Application of momentum equation to the *CV* in Fig. 5.8 (b) gives

$$p_1 A_2 - p_2 A_2 = -\beta_1 \left(\overset{\circ}{m}\ V_{1,\,\text{av}}\right) + \beta_2 \left(\overset{\circ}{m}\ V_{2,\,\text{av}}\right)$$

where $V_{1,\text{av}}$, and $V_{2,\text{av}}$ are the average velocities at sections 1 and 2 respectively, and β_1 and β_2 are the corresponding momentum correction factors. Here $\beta_1 = 1.33$ and $\beta_2 = 1.02$ by Eqs. (5.5) and (5.6). Thus,

$$p_1 - p_2 = \frac{1.33\, \overset{\circ}{m}\, V_{1,\text{av}}}{A_2} \left[\frac{0.77 A_1}{A_2} - 1\right]$$

This is larger in magnitude than that obtained earlier.

5.4 MOMENT-OF-MOMENTUM EQUATION

In many applications such as rotary pumps and turbines, we are interested in torques rather than forces. In these cases it is convenient, at times to use the concept of moment of momentum (or angular momentum). One starts with the application of Newton's law to a single particle of fluid in a flow field to write

$$\delta \mathbf{F} = \frac{D}{Dt}(\delta m\ \mathbf{V})$$

where $\delta\mathbf{F}$ is the external force acting on it, $\mathbf{V}$ is the velocity in an inertial frame of reference and D/Dt is the material derivative. The torque about a fixed point O (Fig. 5.13) is, then,

$$\delta \mathbf{T} = \mathbf{r} \times \delta \mathbf{F} = \mathbf{r} \times \frac{D}{Dt}(\delta m\ \mathbf{V}) \qquad ...(5.7)$$

where $\mathbf{r}$ is the position vector of the particle with origin at O. Application of the chain rule* gives

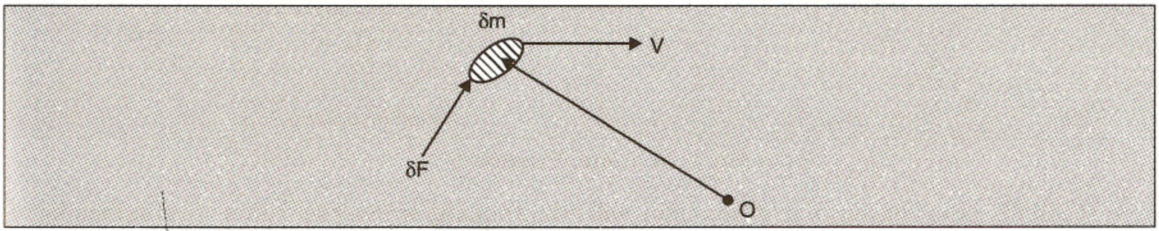


Fig. 5.13. Force δ**F** acting on a fluid particle.

$$\frac{D}{Dt}(\mathbf{r} \times \delta m\ \mathbf{V}) = \mathbf{r} \times \frac{D}{Dt}(\delta m\ \mathbf{V}) + \frac{D\mathbf{r}}{Dt} \times (\delta m\ \mathbf{V})$$

Since $D\mathbf{r}/Dt$ is the velocity $\mathbf{V}$, the second term on the right hand side is zero. Therefore, Eq. (5.7) becomes

$$\delta \mathbf{T} = \frac{D}{Dt}(\mathbf{r} \times \delta m\ \mathbf{V}) \qquad ...(5.8)$$

* It can be shown easily that the chain rule of differentiation is applicable to material derivatives as well.

CHAPTER 5

where $\mathbf{r} \times \mathbf{V}\, \delta m$ is the *moment of momentum* $\delta\mathbf{M}$ of the fluid particle.

On integrating Eq. (5.8) over the fluid body one obtains

$$\mathbf{T} = \frac{D\mathbf{M}}{Dt} \qquad \text{...(5.9)}$$

where $\mathbf{M}$ is the moment of momentum of the whole body. The specific value η of the moment of momentum is $\mathbf{r} \times \mathbf{V}$.

For the control-volume formulation, Reynolds transport theorem (with $\eta = \mathbf{r} \times \mathbf{V}$) is used to obtain $D\mathbf{M}/Dt$ in terms of the rate of accumulation and the net efflux. This gives

$$\mathbf{T} = \frac{\partial}{\partial t}\iiint_{CV}(\mathbf{r}\times\mathbf{V})\rho\, d\forall + \oint\!\!\oint_{CS}(\mathbf{r}\times\mathbf{V})(\rho\mathbf{V}\cdot d\mathbf{A}) \qquad \text{...(5.10)}$$

Thus, the net external torque acting on a *CV* is equal to the rate of change of the moment of momentum contained within it (i.e., the rate of accumulation) *plus* the net efflux of angular momentum across the *CS*.

Given below is an application of this equation. Another application appears in Sec. 8.3.

Example 5.7. The lawn spinkler (Fig. 5.14) has two jets of water (diameter 5 mm) issuing at 3 m/s at 60° to the tangent. The arms of the sprinkler rotate because of the jet reaction. Find the *steady state* angular velocity ω of rotation if the pivot is assumed frictionless. Assume water enters the spinkler axially through a central pipe.

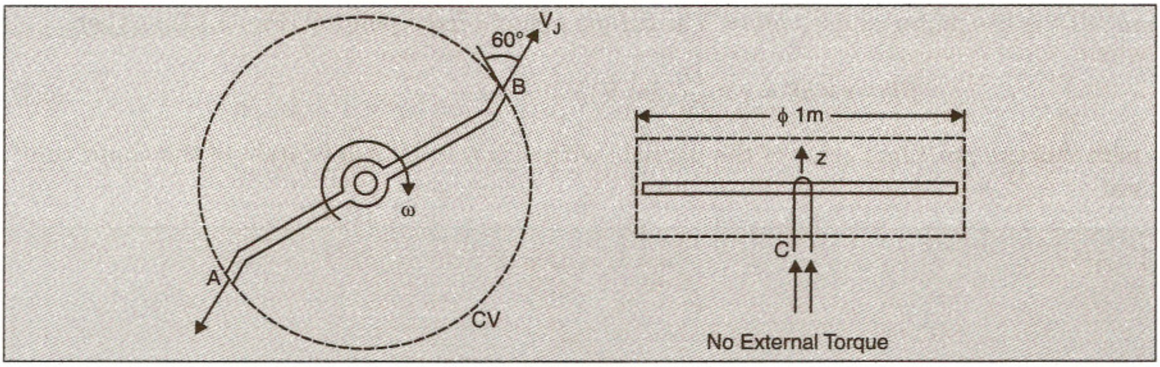


Fig. 5.14. *CV* for Example 5.7.

If the *CV* is so chosen that it coincides with the rotating arms, the frame of reference fixed with the *CV* will be non-inertial and we will not be able to apply the momentum Eq. (5.2) to obtain the forces. The problem is considerably simplified, however, if we take a stationary *CV* enclosing the entire region swept by the arms (Fig. 5.14). As the arms rotate, the water issues in different directions at different times and so the *(linear)* momentum flux changes with time. But the flux of the moment-of-momentum has a constant direction *(along the axis of rotation)* and magnitude. Thus, it is more convenient to work with the *z*-component of the moment-of-momentum Eq. (5.10), rather than with the momentum Eq. (5.2).

Although the flow within the *CV* is unsteady, the total moment of momentum within the *CV* is constant with time. This is because $\mathbf{r} \times \mathbf{V}$ for any segment of the sprinkler arms is independent of the angular position of the arms. Thus, the rate of accumulation of the moment of momentum is zero and, since the external torque in the z-direction is also zero (due to the pivot being frictionless), we have

$$T_z = 0 = \oiint_{CS} (\mathbf{r} \times \mathbf{V})_z (\rho \mathbf{V} \cdot d\mathbf{A})$$

Note that the velocity here is measured in the frame of reference fixed with the stationary *CV*.

Water crosses the *CS* at the two jet orifices *A* and *B* and at the centre *C*. At *C*, there is no contribution to the moment of momentum flux since **V** is normal to **r**. Thus, the efflux of the moment of momentum at the two jet orifices is zero.

The jet velocity of 3 m/s is with respect to the jet orifice and, since the jet orifice itself is moving, the two velocities should be added vectorially to obtain the velocity with respect to the stationary *CS*. This is done using the velocity triangle (Fig. 5.15). The tangential component $V_{\tan}$ of the absolute velocity, $\mathbf{V}_{abs}$ is

$$\begin{aligned} V_{\tan} &= V_j \cos 60° - \omega R \\ &= (3 \cos 60° - 0.5\ \omega) \text{ m/s} \end{aligned}$$

and the radial component V_r is

$$V_r = V_j \sin 60° = 3 \sin 60° \text{ m/s}$$

Only the tangential component of $\mathbf{V}_{abs}$ contributes to the moment of momentum in the z-direction. Thus

$$2 \times \left[\frac{1}{2}(\text{m}) \times (3 \cos 60° - 0.5\ \omega)(\text{m/s}) \times \rho V_j A_j \right] = 0$$

where A_j is the area of the jet. (The reader can verify that $\mathbf{V}_{abs} \cdot \mathbf{A}$ is indeed equal to $V_j A_j$). This gives the steady state angular velocity ω as 3 rad/s.

The torque T_z on the sprinkler arm is zero because the arm has acquired a rotational velocity such that the water coming out of the jets does not have any tangential velocity as seen by a stationary observer. Such an observer, therefore, sees water issuing radially from the sprinkler.

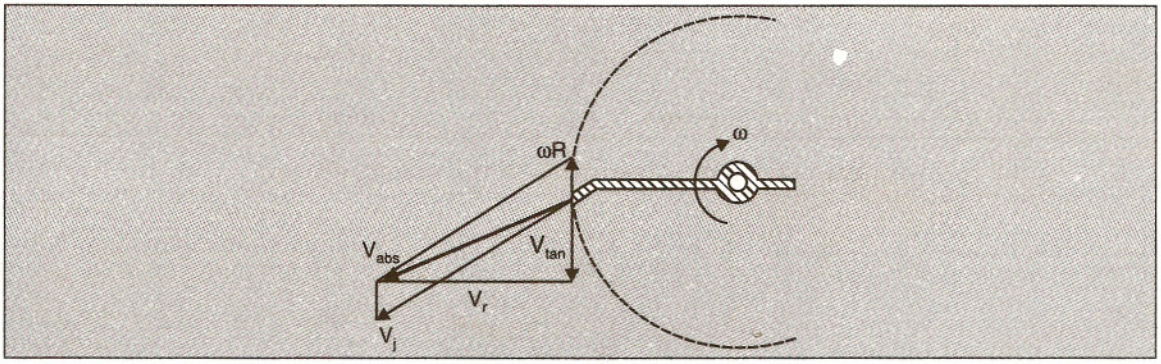
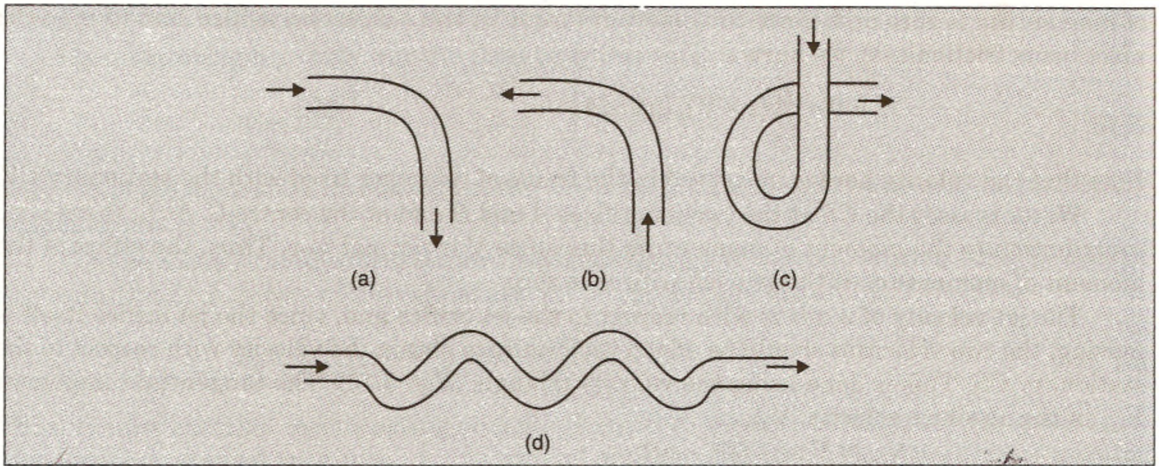


Fig. 5.15. Velocity triangle to obtain the velocity of water with respect to the stationary *CV*.

PROBLEMS

5.1 Water flows through constant area pipes of the forms shown. Mark the directions of the resultant forces, if any, on the pipes if friction is neglected.

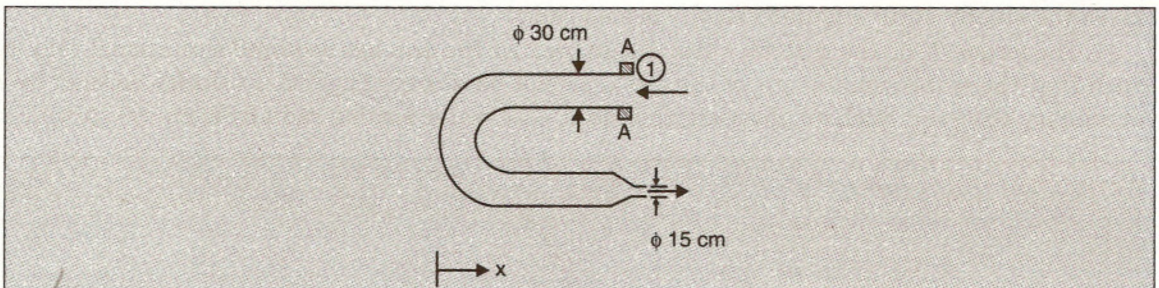


Tut 5.2 If the pressure across a straight free jet of water is atmospheric everywhere, why does a person feel a force when he places his hand against such a jet.

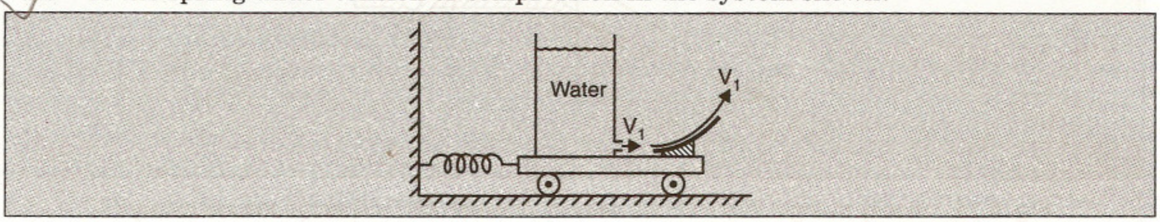
5.3 It is argued that a person on a fireboat feels a higher reaction when he directs the water jet against a solid surface on the coast than when he discharges it in the air. Is this correct?

5.4 Obtain the horizontal force acting at the flange *AA* of the nozzle assembly shown if the pressure at point 1 is 10^5 Pa *gauge* and the water issues as a free jet into the atmosphere.

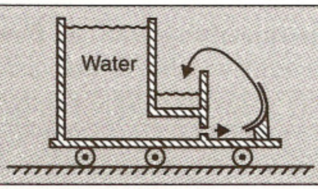
Tut Take $\mathring{Q} = 2\ \mathrm{m^3/s}$.



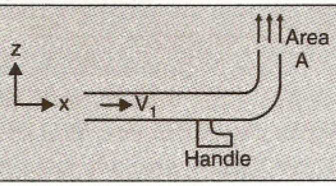
Tut 5.5 Is the spring under tension or compression in the system shown?



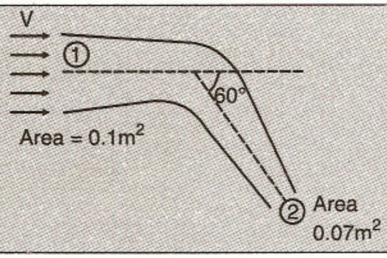
5.6 A newly graduated engineer dreams up of an ingeneous way of propelling fresh water tankers in the *Thar* desert. A jet of water issuing from a specially designed tanker as shown is deflected back into it by a vane. The thrust by the jet propels the railroad with no loss of precious water. Is his idea feasible?



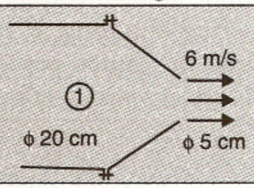
5.7 A person holds a hose through which a liquid of density ρ flows. If the fluid issues as a free jet into the atmosphere, obtain the vertical component of the force experienced by the person. Does this act upwards or downwards?



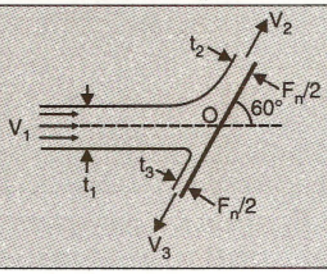
5.8 Water flows through the reducing elbow shown at the rate of 1 m^3/s. The gauge pressure at 1 is 0.1 MPa and that at 2 is 0.09 MPa. What is the resultant force on the elbow? Neglect the weight of water.



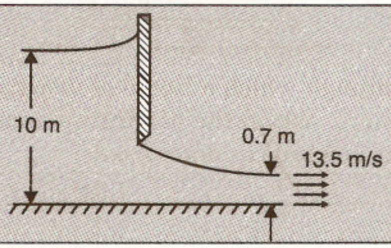
5.9 A jet of water issues out of a fireman's nozzle at 6 m/s. If the gauge pressure at section 1 is 1.8×10^4 Pa, estimate the force in each of the eight bolts connecting the nozzle to the pipe. Note that each bolt is in tension, i.e., the nozzle has a tendency to pull on the hose. Does this mean that the fireman holding the hose is tugged forward? Explain.



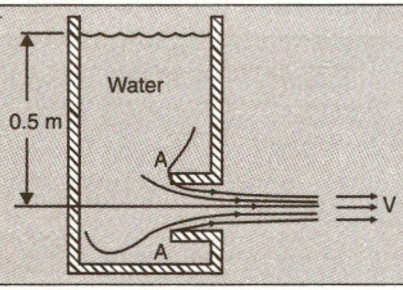
5.10 Assuming that $V_1 = V_2 = V_3$ and that the force exerted by the water on the stationary plate shown acts *normally*, obtain the volume flow rates $\mathring{Q}_2$ and $\mathring{Q}_3$ in terms of the incoming flow rate $\mathring{Q}_1$. Also obtain the normal force F_n.



5.11 The sluice gate on a dam is raised to allow the flow of water as shown. Estimate the force acting on the gate per unit width. Assume 1-D flow downstream of the gate and the pressure distributions to be hydrostatic far upstream and downstream.

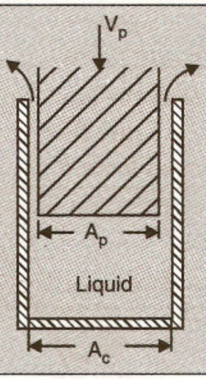


5.12 *Borda's mouthpiece:* A tank has a circular re-entrant outlet near its bottom as shown. Such an outlet is called Borda's mouthpiece. Water issues from this as a jet having a uniform velocity V of 3.13 m/s. The water jet does not fill the tube completely and is surrounded by air. Find the area of the jet as a fraction of the mouthpiece area. Use the fact that with such a mouthpiece, the velocity along all the walls is negligible.



5.13 Consider a tank with a simple sharp-edged orifice. In this case, the velocity at the wall near the orifice is no longer negligible. Show that the 'contraction' of the jet from this orifice is smaller than of that from the Borda's mouthpiece (Prob. 5.12).

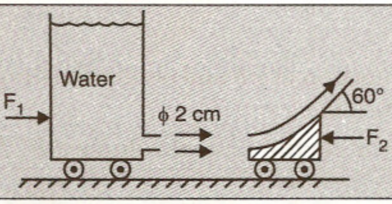
5.14 *Hydraulic damper:* A hydraulic damper consists of a piston (area A_p) moving in a slightly larger cylinder (area A_c). Find the force F resisted by the piston when it moves at a constant velocity V_p. Take the gauge pressure at the bottom of the cylinder to be $F/A_p - \rho V_p^2/2$ (Prob. 7.52). Neglect viscous effects. Note that the flow is unsteady.



5.15 (a) Obtain the forces F_1 and F_2 required to prevent motion of the tank and the vane shown. The water velocity in the jet is constant at 4.5 m/s. (b) Find F_1 and F_2 if the vane is held

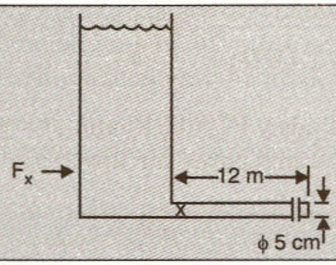
Tut

stationary but the tank moves to the left at a constant speed of 2 m/s. The velocity of water *with respect to the tank* is 4.5 m/s.

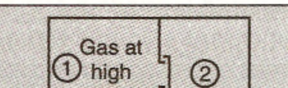


5.16 Consider the tank of Prob. 5.15. The jet of water issues at a constant relative velocity V_j. Show by applying the momentum theorem that the tank will move to the left at a constant velocity *only* if a finite horizontal force is applied to it (otherwise it accelerates). Contrast this behaviour with that of a rotating sprinkler (Example 5.7).

5.17 An incompressible fluid is supplied to a large tank from where it flows out through a long pipe of length 12 m and diameter 5 cm. At time $t = 0$, a valve is closed slowly so that for a short interval of time thereafter, the velocity is given by $(10 - 5\sqrt{t})$ m/s, where t is in seconds. Find the horizontal force F_x (as a function of time) required to hold the tank in place.



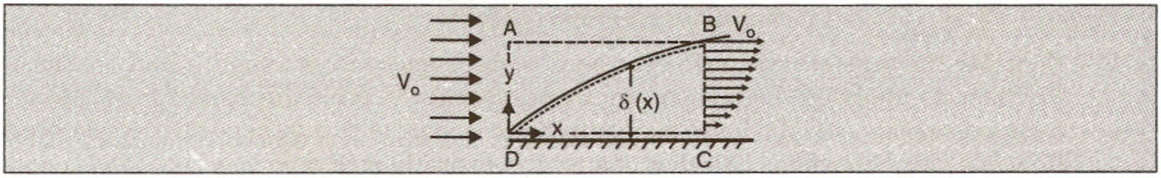
5.18 A partitioned tank on frictionless wheels as shown contains a gas at high pressure in one chamber and a gas at much lower pressure in the other. The plug separating the two is removed at $t = 0$ when the tank is at rest. The tank accelerates to the left for a short time and thereafter moves at a constant velocity for some time. If we assume that the opening between the two chambers is so small that the pressures in them are essentially constant for a short duration, show that the above behaviour is consistent with the momentum equation.



5.19 *Boundary layer:* Fluid flowing at a uniform velocity V_0 at constant pressure encounters a flat plate as shown and a boundary layer results (see Sec. 1.7). The x-component of the fluid velocity at any section BC is given by $V_x = V_0 f$, where f is a function of y/δ and is less than *one* between points C and B. At point B and above, $f \simeq 1$. Since δ is small, the pressure inside the boundary layer may be assumed to be constant everywhere. (This will be shown in Sec. 13.2). Using the control volume DBC show that the drag force on the plate per unit width is $-\int_0^\delta \rho(V_x - V_0)V_x dy$. Note that mass crosses the CS across BD (see Prob. 4.9).

Assume f to be a linear function of y/δ and obtain the drag force in terms of an appropriate dimensionless drag coefficient.

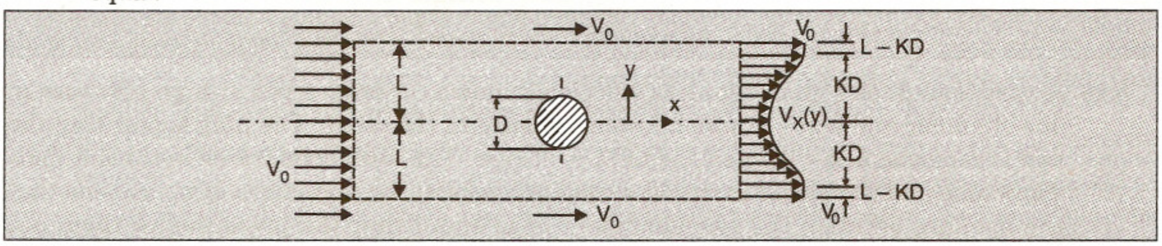
Repeat the above steps taking the control volume $ABCD$.



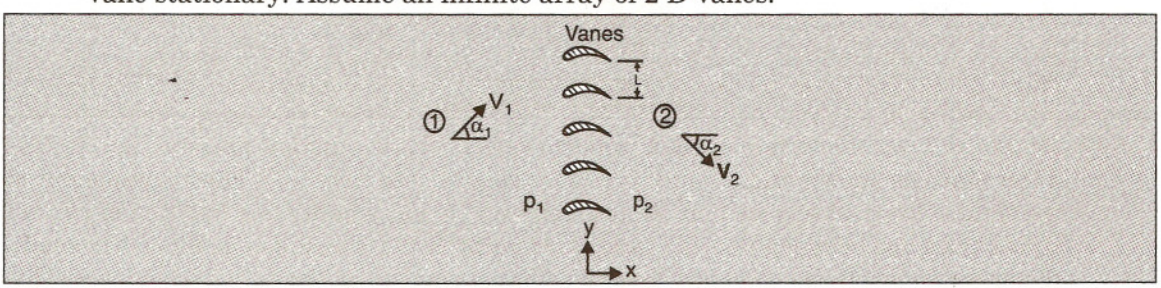
5.20 *Wake-survey method:* An experimental method of measuring the force exerted on a solid body consists of placing it in a uniform stream of fluid and studying the velocity pattern downstream. Using the CV shown, obtain the following dimensionless equation for the drag force on a cylinder of diameter D:

$$C_D = \frac{x\text{-force on cylinder}}{\frac{1}{2}\rho V_0^2 \ (WD)} = 4\int_0^K V_x^* \left(1 - V_x^*\right) dy^*$$

where $W =$ width of the cylinder, $V_x^* = V_x/V_0$ and $y^* = y/D$. (Hint: The fluid bleeds through the sides of the CV and the pressures far upstream and downstream can be taken as equal.



5.21 A series of identical turning-vanes are used in the wind-tunnel bends to keep the flow smooth and 1-D. If the velocities at points just upstream and downstream are as shown, and if the corresponding pressures are p_1 and p_2, obtain the force required to keep a vane stationary. Assume an infinite array of 2-D vanes.

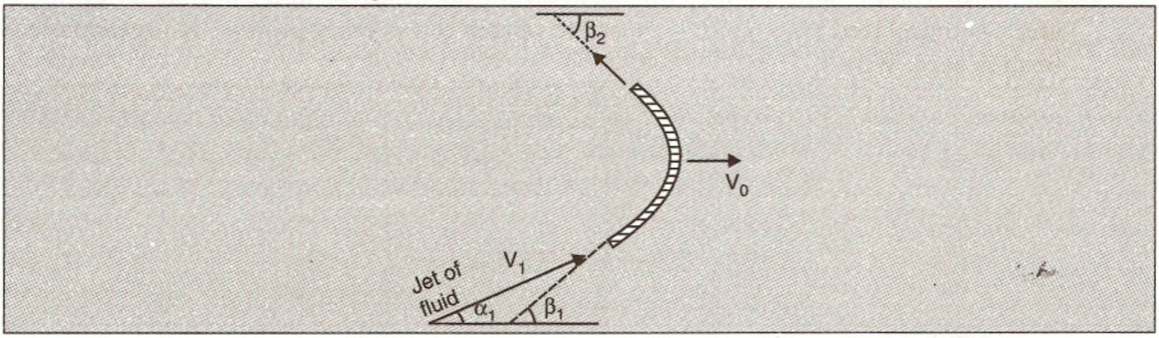


5.22 A jet of fluid at velocity V_1 is directed towards the vane shown (which is moving at velocity V_0) such that the vane sees the fluid entering tangentially to it. The fluid leaves with the same *relative* velocity at the exit of the vane, again tangentially (as observed by the moving vane). Obtain

(a) V_1 in terms of V_0, β_1 and α_1,

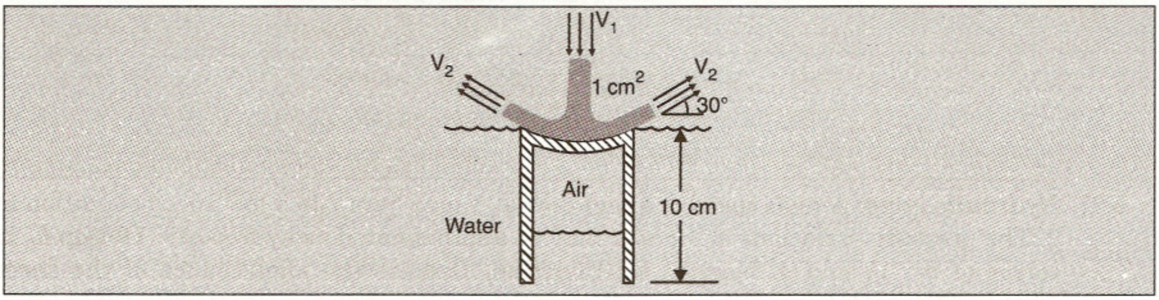
(b) the magnitude and direction of the exit velocity (you may not be able to write this explicitly), and

(c) the x-force exerted by the fluid on the vane.

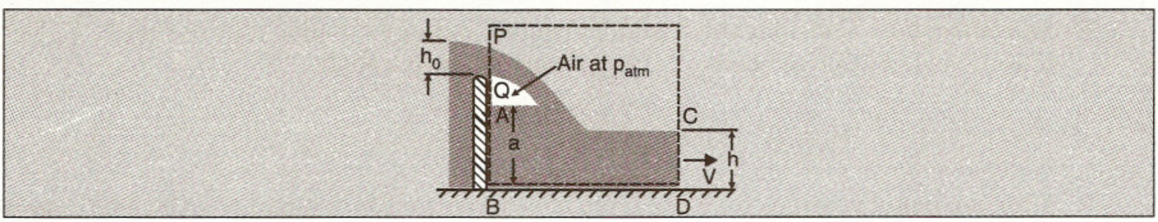


5.23 A child playing with a water pistol directs the jet of water on the circular base of an inverted glass vase floating with its open end down in a bucket as shown. The vase weighs 200 gm and has a cross-sectional area of 50 cm^2. Assume that the velocity V_2 equals V_1. Obtain the vertical force on the vase due to the jet of water. Neglect the weight of water in the *CV*.

Obtain V_1 required to just submerge the vase as shown. Assume the vase walls to be thin. What is the height to which water rises inside the vase?

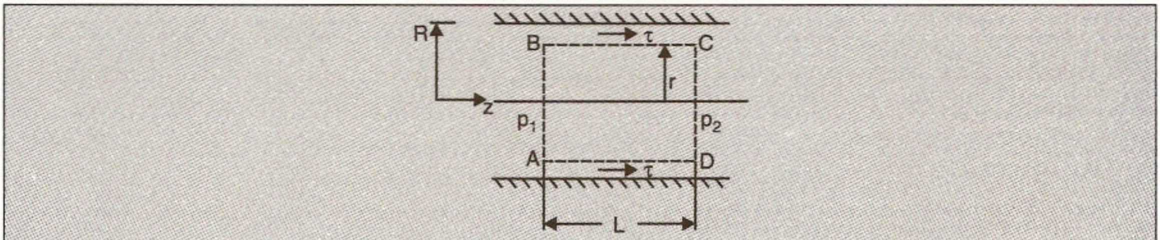


5.24 Water discharges over a weir into a channel having the same width. It is observed that a region of still water backs up to height a at the back of the weir as shown. Assuming that the water discharges horizontally over the weir and that the pressure variation at *AB* and *CD* are hydrostatic, obtain a in terms of V, h and h_0. Neglect frictional stresses. Also take the pressure across *PQ* as atmospheric (see Sec. 7.6).

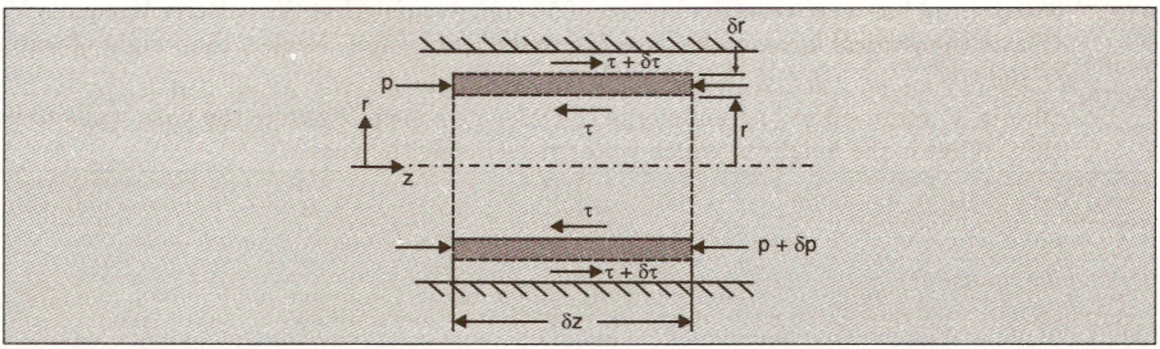


5.25 *Poiseuille flow:* Consider a cylindrical *CV* in a circular pipe carrying a fully-developed laminar flow as shown. A uniform pressure p_1 acts on face *AB* and similarly a pressure p_2 acts on face *CD*. A shear stress τ acts on the curved surface. Apply the momentum equation to obtain $-\dfrac{p_1 - p_2}{2}\dfrac{r}{L} = \tau$

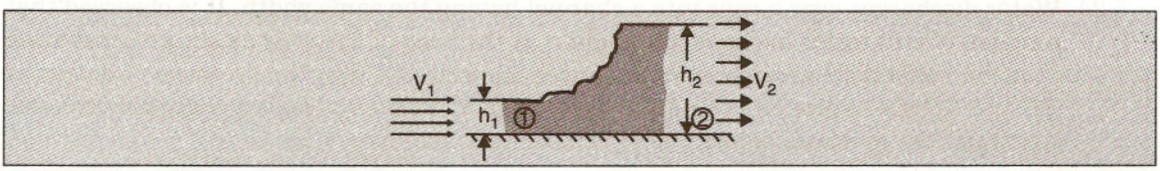
Using the relationship $\tau = \mu dV_z/dr$ between the shear stress and the velocity gradient, and assuming that $(p_1 - p_2)/L$ = constant, obtain the velocity profile. (Hint: Convective term is zero. Why?)



5.26 If the cylindrical *CV* in Prob. 5.25 is replaced by a cylindrical shell as shown, obtain the relationship between p and τ.



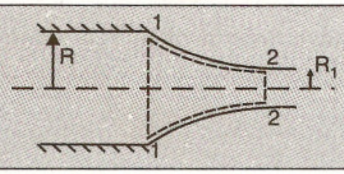
5.27 *Hydraulic jump:* A high speed channel flow at 1 may 'jump' to a low speed condition at 2. The pressure variations at 1 and 2 may be approximated as hydrostatic. Obtain h_2 in terms of h_1, V_1 and g. Neglect wall friction. Discuss the significance of the three mathematical solutions.



5.28 *Jet contraction:* Consider the liquid jet coming out of a circular pipe of radius R. The velocity distribution at section 1-1 is assumed to be parabolic

$$V_z = 2\bar{V}\left(1 - \frac{r^2}{R^2}\right)$$

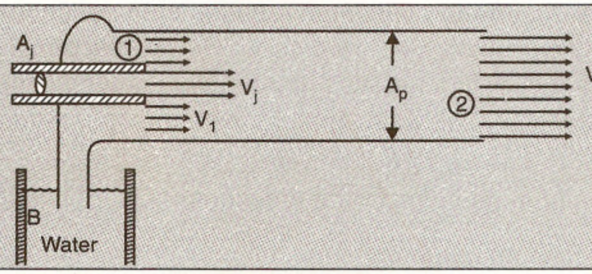
After the liquid emerges from the pipe, its velocity profile changes and the jet diameter decreases as shown, till at section 2-2 the velocity is uniform across the jet. Assuming that the pressure is approximately atmospheric over the *CV* shown, use the continuity and momentum equations to show that $R_j = \sqrt{3}\ R/2$.



5.29 *Ejector pump:* A high-speed water jet issuing from a pipe of area A_j drags along the surrounding water such that the device shown can be used as a pump. If the velocity profiles are assumed as 1-*D* at sections 1 and 2, relate V_2 to V_j and V_1. If shear stresses at the pipe wall are neglected, and the pressure is assumed uniform across the entire section 1, use the momentum equation to show that

$$p_2 - p_1 = \rho\left(\frac{A_j}{A_p}\right)\left(1 - \frac{A_j}{A_p}\right)(V_1 - V_j)^2$$

Note that p_1 is lower than p_2 confirming that the device is indeed a pump.

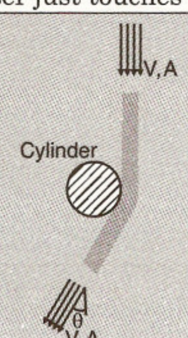


5.30 In Example 5.3, we have drawn the pressure profile at time t along the vertical and horizontal legs as linear. Show that this must be so.

5.31 Consider the flow of water below the sluice gate as discussed in Prob. 4.26. Again assuming 1-*D* flow, obtain another relationship between $\delta\ (x, t)$ and $V_x(t)$. Assume hydrostatic pressure variation downstream of the gate and neglect frictional losses.

For steady flow, show that δ cannot vary continuously with x. The depth of water δ admits only two values, one δ_0 and the other related to δ_0 by the hydraulic jump relation of Prob. 5.27.

5.32 *Coanda effect:* When a jet of water just touches a curved surface it attaches itself to the



CHAPTER 5

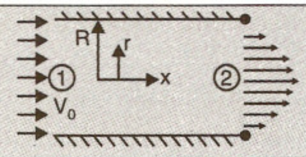
surface and is bent through an angle θ as shown. This is called the Coanda effect. Obtain the magnitude and direction of the force acting on the cylinder assuming that the jet velocity is unchanged. Neglect gravity effects.

A similar bending of a liquid stream is observed when it is poured out of a vessel.

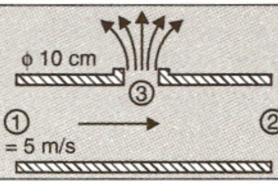
5.33 The velocity profile at the entrance of a pipe is flat, as shown. At section 2, it is parabolic and is given by

$$V = V_m\left(1 - \frac{r^2}{R^2}\right)$$

Obtain the drag force F acting on the fluid in terms of the pressures p_1 and p_2 and ρ, V_0 and R using the momentum correction factor. Verify the results by direct integration.



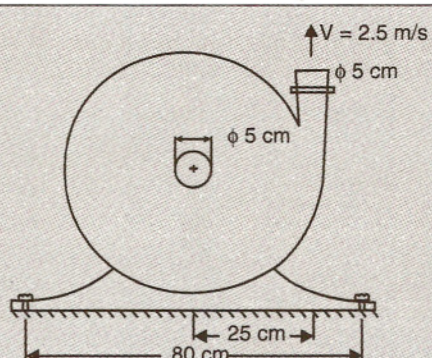
5.34 *Manifolds:* Water flows through a pipe with a hole at the side. One third of the water coming in, issues *vertically* as a spray at section 3. Find the difference in the pressures at sections 1 and 2 for steady flow conditions. Neglect frictional forces and any axial momentum lost at section 3. Does the pressure increase or decrease downstream?



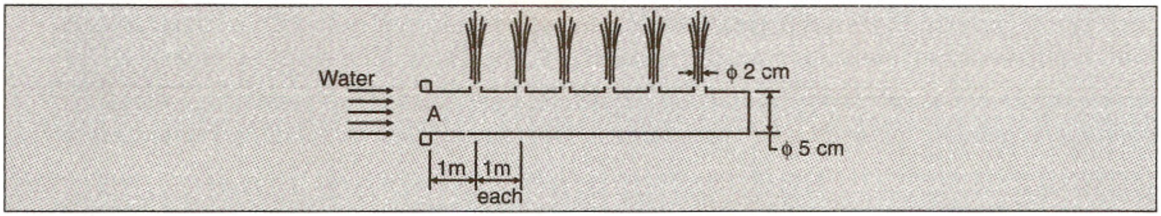
5.35 Compute the torque required to prevent the sprinkler of Example 5.7 from rotating.

5.36 Show that the contribution to $\oiint_{CS} \rho \mathbf{V}_{\text{abs}} \cdot d\mathbf{A}$ of any jet in Example 5.7 is $\rho \mathbf{V}_j A_j$.

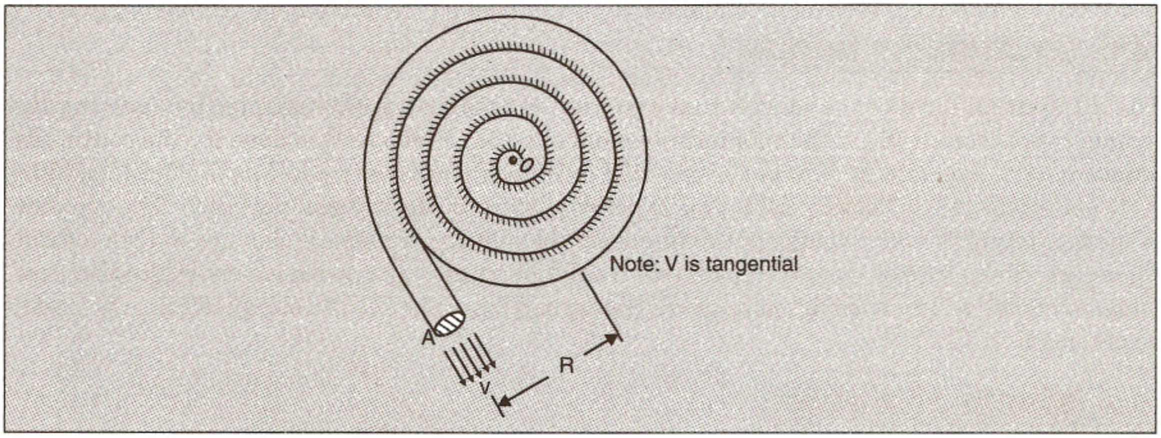
5.37 A pump takes in water axially near the centre and delivers it at a higher pressure from an exit port at 2.5 m/s. Two bolts on each side, as shown, fasten it securely to the base. Compute the tensile and compressive loadings in the bolts due to the unbalanced torque alone.



5.38 A cooling system for a central air-conditioning plant uses a 5 cm dia. pipe. Water enters at A and issues vertically at the six 2 cm dia. nozzles as shown. Assuming that the water velocity at each of the nozzles is *approximately* 6 m/s, compute the bending moment at the flange at A due to the flow of water alone.



5.39 A toy cracker, "chakri", consists of a thin tube containing a combustible powder. The tube is wound spirally about O as shown. If the combustible material burns at the rate of $\mathring{m}$ and if the density of the combustion products is ρ find the torque required to prevent rotation when the cracker is ignited at A. Also estimate the initial rate of rotation of the wheel. Assume $\mathring{m}$ small so that unsteady, non-inertial effects are negligible.



5.40 Find the point of action of the normal force F_n due to the water jet in Prob. 5.10. This is the point at which an external force (equal to F_n) must be applied to prevent rotation of the plate.

6

EQUATION OF MOTION

6.1 EQUATION OF MOTION

The differential form of the momentum equation is obtained in this chapter by applying the momentum theorem (Eq. 5.2) to an infinitesimal control volume as was done for the continuity equation. The *equation of motion* so obtained is applicable to every point in the fluid enabling us to obtain the entire velocity field. This is not possible with the integral approach. The equation of motion requires a knowledge of the forces on the surface of a small element within a fluid. Therefore, the nature of these forces and their relation to the velocity field must be studied first. Only 2-*D* flows will be considered in the following sections. The results obtained thus can easily be extended to 3-*D* flows.

6.2 STRESS AT A POINT

In Sec. 5.1 it was seen that the surface force at a point in a fluid is expressed in terms of a stress defined as the force per unit area. In a stationary fluid the pressure (which is a compressive

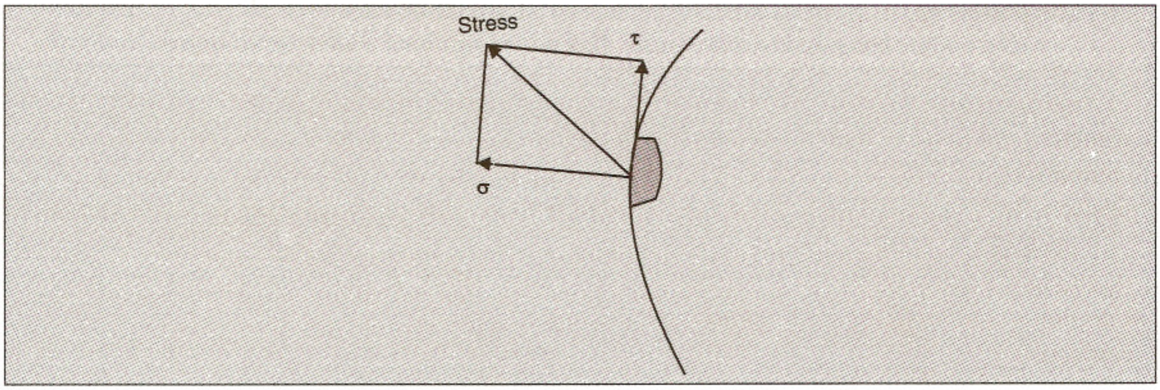


Fig. 6.1. Stress acting on a surface of an element of fluid and its two components, in a 2-*D* flow field.