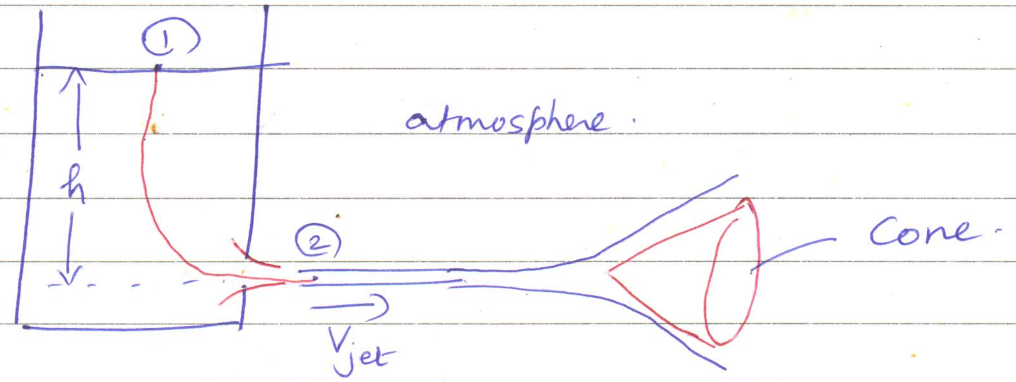
ESO 212 END-SEM EXAM SOLN. KEY

2010-I.

(1)



(a) To find V_{jet}, apply Bernoulli eqn between points (1) & (2) which lie on the same streamline.

$$\frac{p_1}{\rho} + g z_1 + \frac{V_1^2}{2} = \frac{p_2}{\rho} + g z_2 + \frac{V_2^2}{2}.$$

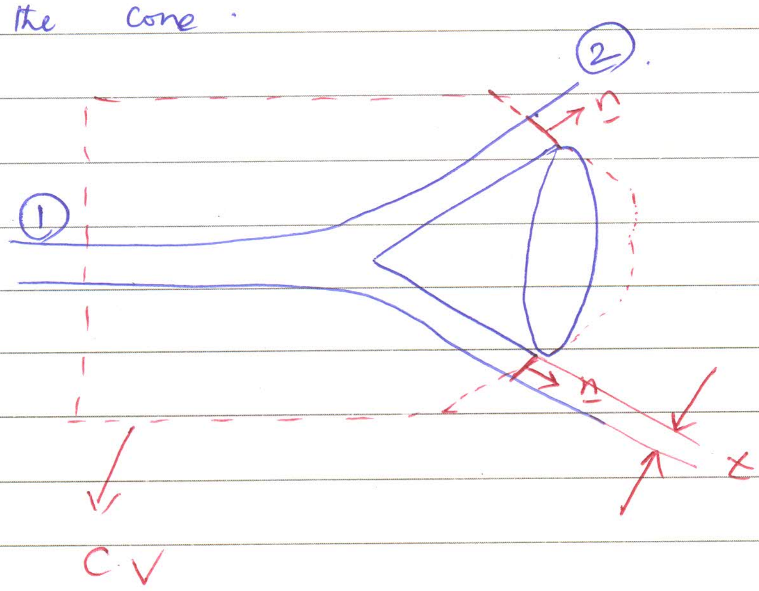
$$P_1 = P_2 = P_{atm}, \qquad V_1 \approx 0, \qquad (z_1 - z_2) = h.$$

$$\Rightarrow \quad V_2^2 = 2gh \quad \Rightarrow \quad V_2 = \sqrt{2gh}$$

$$V_2 = \sqrt{2 \times 9.8 \times 45.92}$$

$$\boxed{V_{jet} = V_2 = 30 \text{ m/s.}}$$ — 4 points

(b) It is convenient to be in the ref. frame of the cone.



mass consv. $\Rightarrow$ $-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$

$\rho_1 = \rho_2 = \rho.$

$$\Rightarrow \quad (V_j + V_c)\frac{\cancel{\pi} D_j^2}{4} = (V_2 + V_c)\,\underbrace{2\cancel{\pi} R t}$$

$$(30+14)\frac{(100\times 10^{-3})^2}{4} = (V_2+14)\, 2\times 230\times 10^{-3}\times 5.434\times 10^{-3}$$

area over which fluid is leaving normally to the C.S

$\Rightarrow V_2 = 30$ m/s (w.r.t stationary frame of ref)

4 Points

$V_2 = 44$ m/s in the ref. frame of the cone.

(c) Momentum balance (for the same c.v. shown in page 2)

steady:
x-component:

$$F_x = \int_{c.s.} u \, \rho \, \underline{v} \cdot \underline{n}$$

$$R_x = u_1 \left(-\rho V_1 A_1 \right) + u_2 \left(\rho V_2 A_2 \right).$$

$\downarrow$

external force on the c.v

$\Downarrow$

force to be applied on the cone.

In this problem:
$V_1 = V_2 = V_j$
$A_2 = A_1 = A_j$

$$R_x = -(V_j + V_c)\, \rho\, (V_j + V_c)\, A_j$$

$$+ (V_j + V_c) \cos 60^\circ \, \rho \, (V_j + V_c)\, A_j$$

$$\Rightarrow \quad R_x = \rho (V_j + V_c)^2 A_j \left[\cos 60 - 1 \right].$$

$$= 10^3 \, (44)^2 \, \frac{\pi}{4} (0.1)^2 \left(\frac{1}{2} - 1 \right)$$

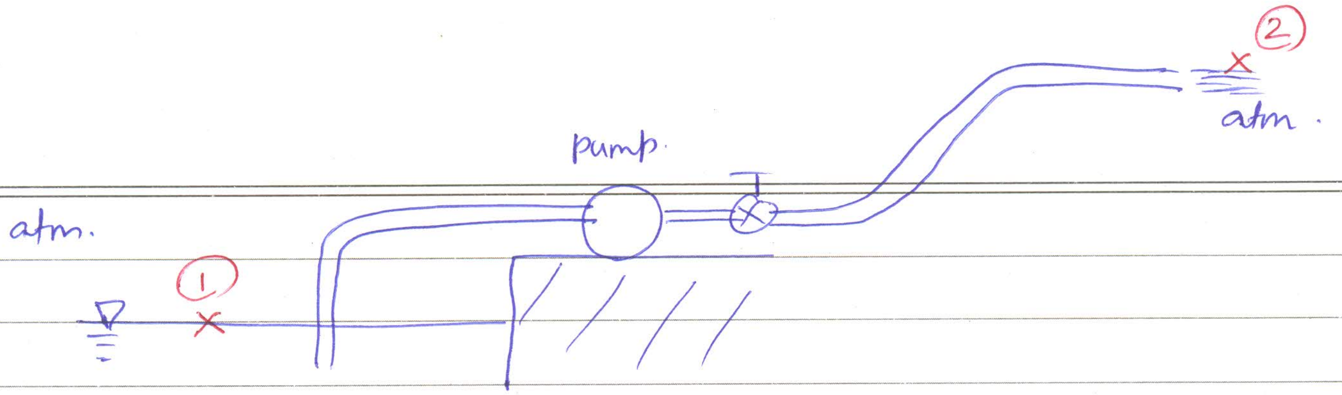
$$= -7602.65 \, N$$

$$= -7.6 \, kN.$$

$\boxed{6 \text{ points}}$

Force in the negative x direction that must be exerted on the cone $\Big\}$ = 7.6 kN.

(2) atm.



minor losses : (1) entrance

(2). 90° elbow − 1

(3). Gate valve.

(4) 45° elbow − 2.

Apply energy balance between (1) & (2) :

2 points

$$\left(\frac{p_1}{\rho} + \alpha\frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + g z_2\right) + \Delta h_{pump} = h_{lT} \begin{cases} h_{major} \\ h_{minor} \end{cases}$$

2 points

$$h_{maj} = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$h_{min} = \frac{\bar{V}^2}{2}\sum_i K_i$$

$p_1 = p_2 = p_{atm}$. $\bar{V}_1 = 0$; $\bar{V}_2 = 37$ m/s.

$\alpha_2 = 1$ (free jet).

NOTE : $\bar{V}_2$ at the nozzle exit is not the avg $\bar{V}$ in the pipe.

If you have used $\bar{V}_2$ for velocity in the pipe, you will be given half the total grade ~~for~~ if the procedure is correct.

$$\Rightarrow \Delta h_{pump} = g z_2 + \frac{\bar{V}_2^2}{2} + f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$+ \frac{\bar{V}^2}{2} \left[K_{ent} + K_{90^\circ} + 2 K_{45^\circ} + K_{gate} \right].$$

in the pipe:

$$\bar{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = 4.84 \text{ m/s.}$$ 2 points

$$Re = \frac{4.84 \times 100 \times 10^{-3} \times 10^{3}}{10^{-3}}.$$

$$Re = 4.84 \times 10^{5}.$$

$$\epsilon = 0.0015 \text{ mm.}$$

$$\Rightarrow \frac{\epsilon}{D} = 1.5 \times 10^{-5}.$$

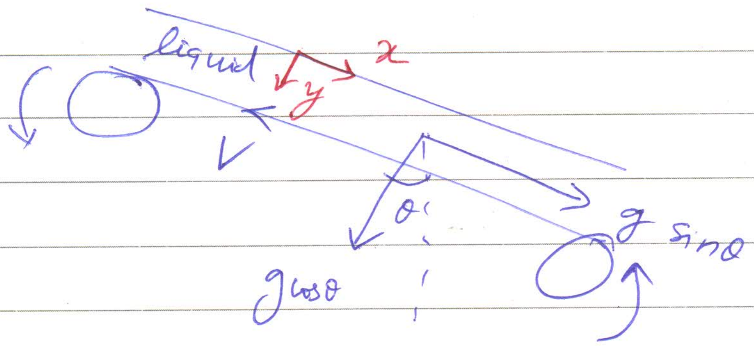
from $f - Re$ chart $\Rightarrow$. $f = 0.0135$. 2 points

$$\Delta h_{pump} = 2.249 \times 10^{3} \text{ m}^2/\text{s}^2$$ 2 points

$$\text{Power i/p} = \dot{m}\, \Delta h_{pump} = 10^3 \times \frac{38}{10^3} \times 2.249 \times 10^{3}$$

$$\boxed{\text{Power i/p to the pump} = 85.5 \text{ kW.}}$$ 2 points

(3)



Boundary conds.

(a) $V_x(y=0) = 0$

$V_x(y=b) = -V$

$\boxed{2 \text{ points}}$

(b) x-mom: $\mu \dfrac{d^2 V_x}{dy^2} + \rho g \sin\theta = 0$

$$\frac{d^2 V_x}{dy^2} = -\frac{\rho g \sin\theta}{\mu} \quad \boxed{2 \text{ points}}$$

$$\frac{dV_x}{dy} = -\frac{\rho g \sin\theta}{\mu} y + C_1$$

$\boxed{1 \text{ point}}$

$$V_x = -\frac{\rho g \sin\theta}{\mu}\frac{y^2}{2} + C_1 y + C_2$$

$\boxed{1 \text{ point}}$ $\boxed{1 \text{ point}}$

use BC's :⇒ $C_1 = \dfrac{\rho g \sin\theta}{2\mu} b - \dfrac{V}{b}$; $C_2 = 0$

$$V_x(y) = \frac{\rho g \sin\theta\, b^2}{2\mu}\left[\frac{y}{b} - \frac{y^2}{b^2}\right] - \frac{V}{b} y$$

$\boxed{1 \text{ point}}$

$$Q_{\text{(per unit width)}} = \int_0^b V_x \, dy$$

$$Q = \frac{\rho g \sin\theta \, b^3}{12\mu} - \frac{Vb}{2}. \quad \boxed{2 \text{ points}}$$

If $Q = 0$ $\quad V_c = \dfrac{\rho g \sin\theta \, b^2}{6\mu}$.

$$= \frac{10^3 \times 9.8 \times \frac{1}{\sqrt{2}} \times (0.01)^2}{6 \times 0.1}$$

$$\Rightarrow \boxed{V_c = 1.155 \text{ m/s}}. \quad \boxed{2 \text{ points}}$$

4 (a). Euler eqn : (steady)

1 $$\rho(\underline{v}\cdot\nabla)\underline{v} = -\nabla p + \rho\underline{g}.$$

Vector identity : $(\underline{v}\cdot\nabla)\underline{v} = \nabla\left(\frac{1}{2}\underline{v}\cdot\underline{v}\right) + \underbrace{(\nabla\times\underline{v})}_{\underline{\omega}}\times\underline{v}.$

$$\Rightarrow \left[\nabla\left(\frac{1}{2}\underline{v}\cdot\underline{v}\right) + \underline{\omega}\times\underline{v} + \frac{1}{\rho}\nabla p - \underline{g}\right]\cdot d\underline{r} = 0$$

along a streamline, $(\underline{\omega}\times\underline{v})\cdot d\underline{r} = 0$

$\underline{g} = -g\,\hat{\underline{k}}$

$$\Rightarrow \nabla\left(\frac{1}{2}\underline{v}\cdot\underline{v}\right)\cdot d\underline{r} + \frac{1}{\rho}\nabla p\cdot d\underline{r} - \underline{g}\cdot d\underline{r} = 0$$

$$\Rightarrow d\left(\frac{1}{2}\underline{v}\cdot\underline{v}\right) + d\left(\frac{p}{\rho}\right) + g\,dz = 0$$

$\Rightarrow$ along a streamline.

$$d\left[\frac{1}{2}v^2 + \frac{p}{\rho} + gz\right] = 0.$$

6 $$\Rightarrow \left(\frac{v^2}{2} + \frac{p}{\rho} + gz\right) = \text{Const. along a streamline.}$$

4 (b) Streamlines $\Rightarrow$ const ψ $\Rightarrow$ $d\psi = 0$.

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0.$$

slope of a streamline $\Rightarrow$ $\left.\frac{dy}{dx}\right|_{\psi} = \frac{-\partial \psi / \partial x}{\partial \psi / \partial y}$

$$= \frac{v}{u}.$$

equipotential $\Rightarrow$ const ϕ. $\Rightarrow$ $d\phi = 0$.

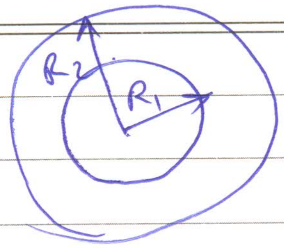
$$d\phi = u\,dx + v\,dy = 0.$$

$$\left.\frac{dy}{dx}\right|_{\phi} = -\frac{u}{v}.$$

multiplying the two slopes $\Rightarrow$

7 $\frac{v}{u} \times \left(-\frac{u}{v}\right) = -1$ $\Rightarrow$ Streamlines & equipotentials are orthogonal.

5 (a).



$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

$$\frac{dT}{dr} = \frac{C_1}{r^2} \Rightarrow T_1 = -\frac{C_1}{r} + C_2$$

$T = T_1$ at $r = R_1$

$T = T_2$ at $r = R_2$

unknown yet..

$$C_1 = \frac{T_1 - T_2}{\left(\frac{1}{R_2} - \frac{1}{R_1}\right)} \; ; \quad C_2 = \frac{-T_2 + T_1 \frac{R_1}{R_2}}{R_1\left(\frac{1}{R_2} - \frac{1}{R_1}\right)}$$

$$T = T_1 + \frac{T_2 - T_1}{\frac{1}{R_1} - \frac{1}{R_2}}\left(\frac{1}{R_1} - \frac{1}{r}\right).$$

$$Q_{th} = -(4\pi r^2)\, k \frac{dT}{dr} = \frac{4\pi k\,(T_1 - T_2)}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$$

$$R_{th,cond} = \frac{1}{4\pi k}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{— (3) points}$$

$$R_{th,conv} = \frac{1}{4\pi R_2^2 h} \quad \text{— (1) point}$$

$$\frac{d R_{tot}}{d R_2} = 0 \Rightarrow \boxed{R_{2c} = \frac{2k}{h}} \quad \text{— (3) points}$$

5 (b) $Bi = \frac{hR}{k} = \frac{1 \times 0.5 \times 10^{-3}}{1} = 5 \times 10^{-4} \ll 1$ (2)

$\Rightarrow$ Tempr. variation in the body negligible.

$$\theta^* = \frac{T - T_f}{T_0 - T_f} = \exp\left[\underbrace{-\frac{hA}{\rho V c} t}_{(2)\text{ points}}\right]$$

$T_0 = 30°$ $\quad$ $T = 80°$

$T_f = 100°$

$$\Rightarrow \frac{2}{7} = \exp\left[-\frac{2\cancel{\pi} R \cancel{L} \cdot 1}{\rho \cancel{\pi} R^2 \cancel{L} \cdot c} t\right]$$

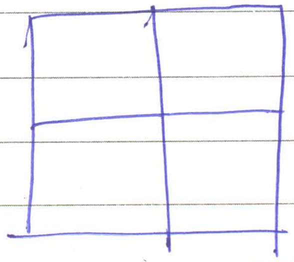
(1) point

$$= \exp\left[-\frac{\cancel{2}}{\cancel{2} \cdot 10^3 \frac{1}{\cancel{2}} 10^{-3} \frac{\cancel{4}}{2} 10^3} t\right]$$

$$\Rightarrow \frac{2}{7} = \exp\left[-\frac{1}{2000} t\right]$$

$\boxed{t \simeq 42 \text{ mins.}}$ (2) points

6 (a) $\quad C_f = \dfrac{C_1}{\sqrt{Re_L}}$ $\quad$ 1



$$F_A = \frac{C_1}{\sqrt{2L_1}\sqrt{\frac{U\rho}{\mu}}} \, 4A_1$$

$$F_B = \frac{C_1}{\sqrt{4L_1}} \frac{1}{\sqrt{\frac{U\rho}{\mu}}} \, 4A_1$$

$$\Rightarrow F_A = \sqrt{8}\, F_1$$

$$F_B = 2F_1$$

T

$$\Rightarrow F_A > F_B.$$

6 (b) density of napthalene at the surface : $= \rho_{A0}$.

$$pV = \frac{m}{M} RT$$

$$\rho_{A0} = \frac{m}{V} = \frac{p_A M}{RT} = \frac{\left(\frac{1}{760} \times 1.013 \times 10^5 \text{ Pa}\right) 128}{8314 \cdot 300}$$

$$\rho_{A0} = 6.8 \times 10^{-3} \text{ kg/m}^3$$ $\quad$ 2

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \rho_A}{\partial r}\right) = 0$$

6

$$\rho_A = \frac{C_1}{r} + C_2$$

$$\rho_A (r = R_1) = \rho_{A0}$$

$$\rho_A (r \to \infty) = 0$$

$$\Rightarrow \quad \rho_A = \frac{\rho_{A0}}{r} R_1.$$

$$\text{flux} = -D_{AB} \left.\frac{\partial \rho_A}{\partial r}\right|_{r=R_1} = D_{AB}\frac{\rho_{A0}}{R_1^2}$$

$$\text{Rate of evap.} = \text{flux} \cdot 4\pi R_1^2$$

$$= 4\pi R_1 D_{AB} \rho_{A0}.$$

$$= 4\pi \left(\tfrac{1}{2}\times 10^{-2}\right) \times \left(5\times 10^{-6}\right) \times \left(6.8\times 10^{-3}\right)$$

$$= 2.136 \times 10^{-9} \text{ kg/s.}$$

5

7.