IIT KANPUR **ESO212 FLUID MECHANICS & RATE PROCESSES** **2010-I**

End-semester Exam **3 hours; 80 points**

USEFUL INFORMATION

(All required information is provided with the question paper)

• Density of water = 10^3 kg/m^3, viscosity of water = 10^{-3} Pa s, $g = 9.8$ m/s^2, $P_{atm} = 1.013 \times 10^5$ Pa.

• Kinetic energy correction factor $\alpha = 2$ for laminar flow, $\alpha = 1$ for turbulent flow in pipes; Loss coefficient K for: entrance = 0.78, elbow (90°) = 0.9, elbow (45°) = 0.4; gate valve (fully open) = 0.3.

• Vector Identity: $(\mathbf{v} \cdot \nabla)\mathbf{v} = \nabla(\frac{1}{2}\mathbf{v} \cdot \mathbf{v}) + (\nabla \times \mathbf{v}) \times \mathbf{v}$

Navier-Stokes *x*-momentum equation in Cartesian coordinates:

$$\rho \left[\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right] = -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right] + \rho g_x$$

Steady conduction equation in spherical coordinates:

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right] = 0$$

Steady diffusion equation in spherical coordinates:

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \rho_A}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial \rho_A}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 \rho_A}{\partial \phi^2} \right] = 0$$

Clearly state all the assumptions you make

1. A large tank containing water (shown in figure 1) has a small smoothly contoured orifice, from which a water jet exits with a velocity of V_{jet}. The height of water in the tank h as shown in the figure is 45.92 m. The diameter of the jet is 100 mm, and this jet moves horizontally to the right, where it is deflected by a cone that moves to the left at $V_{cone} = 14$ m/s. The thickness t of the liquid film that leaves the cone at the radius R (= 230 mm) is $t = 5.434$ mm. Neglect effects of gravity for flow around the cone. Determine:

   (a) The velocity V_{jet} of the liquid jet that comes out of the orifice, neglecting losses. [4 points]

   (b) The velocity of the liquid that leaves the cone at $R = 230$ mm. [4 points]

   (c) The horizontal component of the external force on the cone required to maintain its motion. [6 points]

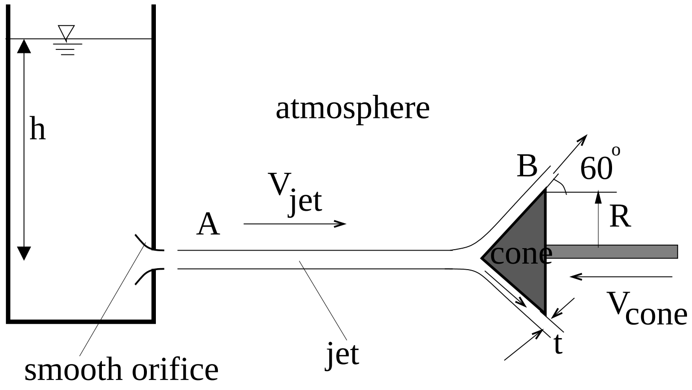

Figure 1: **Problem 1**

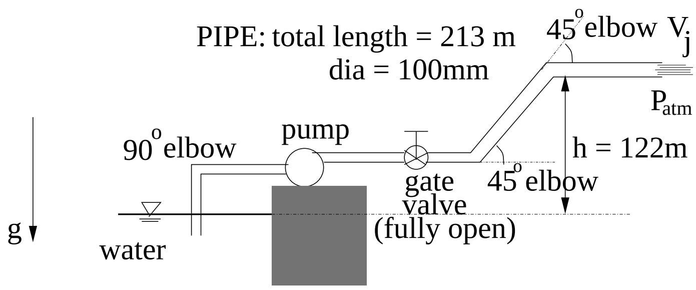

Figure 2: **Problem 2**

2. Water is to be pumped from a reservoir using a pipe system shown in figure 2. The flow rate in the pipe must be 0.038 m^3/s and water must leave the exit of the pipe via a nozzle at a velocity V_j = 37 m/s. The dimensions and and fittings involved in the pipe system are indicated in the figure. The wall roughness factor $\varepsilon = 0.0015$ mm. Neglect losses at the exit nozzle. Determine the power input (in kW) to the pump required to achieve this flow. [12 points]

3. A belt of width W moves at a velocity V as shown in figure 3. A very viscous liquid fills the gap b between the belt and a stationary plate. Assume steady, fully-developed laminar flow between the belt and the plate, and do not neglect the effect of gravity. For this system:

   (a) State the boundary conditions to solve the problem. [2 points]

   (b) Derive the expression for velocity profile in the liquid [6 points]

   (c) Derive the expression for volumetric flow rate of the liquid [2 points].

   (d) If $\mu = 0.1$ Pa s, $\rho = 10^3$ kg/m^3, $b = 1$ cm, $\theta = 45^\circ$, determine the velocity (in m/s) of the belt above which there is a net volumetric flow rate in the direction of the belt motion. [2 points]

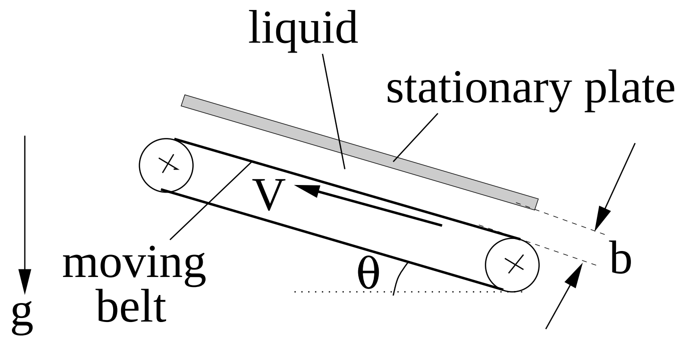

Figure 3: **Problem 3**

4. (a) Derive the Bernoulli equation for steady incompressible flow from the Euler equation for an inviscid fluid. Between which two points is the derived Bernoulli equation valid ? [7 points]

   (b) Prove that for a 2-D incompressible, irrotational flow the streamlines and equipotential lines are always orthogonal. [7 points]

5. (a) Consider a solid sphere of radius R_1 maintained at a temperature T_1 in a fluid at an ambient bulk temperature of T_0 (see figure 4 A). The convective heat transfer coefficient characterizing the heat loss at the boundary is h. This sphere is now wrapped with an insulating material of thermal conductivity k, such that the total radius becomes R_2 (figure 4 B).

   i. Derive an expression for the thermal resistance of the insulating layer. [3 points]

   ii. Is there a critical insulation thickness for this case ? If so, derive an expression for the critical insulation thickness. [4 points]

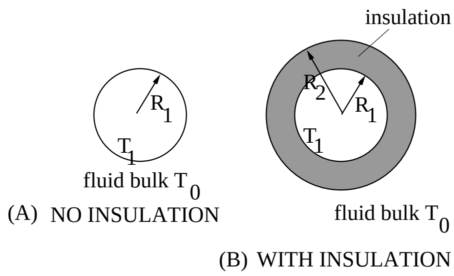

Figure 4: **Problem 5(a)**

   (b) A rice noodle in the shape of a long wire (1 mm diameter, $k = 1$ W/(m K), $C = 4000$ J/(kg K), $\rho = 2000$ kg/m^3) of initial temperature 30°C is dipped in a vessel containing boiling water (100°C). The heat transfer coefficient for the noodle-water interface is $h = 1$ W/(m^2 K). Calculate the Biot number for this case.

Based on the Biot number, what conclusion can you reach for the temperature distribution in the noodle ? Determine the time it takes for the center of the noodle to reach 80°C, when it is considered to be cooked. [7 points]

6. (a) Consider the steady laminar boundary layer flow past the top surface of square plate arrangements shown in figure 5. Compared to the friction drag on a single plate (area A, drag force F_1), how much larger is the drag on four plates together as in configurations (A) and (B) shown in figure 5. Will the drag on the two arrangements be the same ? Explain. [7 points]

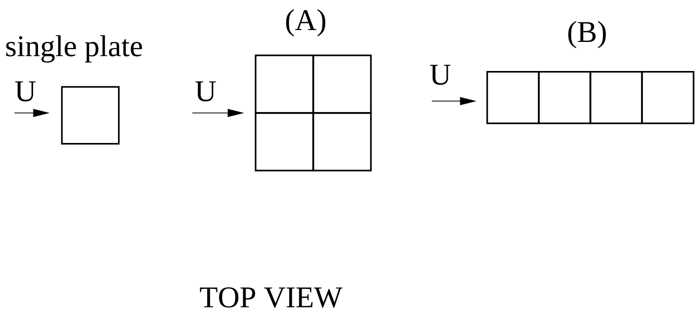

Figure 5: **Problem 6(a)**

(b) A naphthalene ball of diameter 1 cm is suspended in a large room with still pure air at 27°C and 1 atm. The surface temperature of naphthalene can be assumed to be 27°C and its vapor pressure at this temperature is 1 mm Hg. Assume naphthalene vapor to be an ideal gas. Determine the steady rate of evaporation of naphthalene (in kg/s). Assume that the radius of the naphthalene ball diameter remains constant in your calculation. Molecular weight of naphthalene is 128 kg/kmol; $R = 8314$ m^3 Pa/(kmol K); D for naphthalene in air is 5×10^{-6} m^2/s. [7 points]