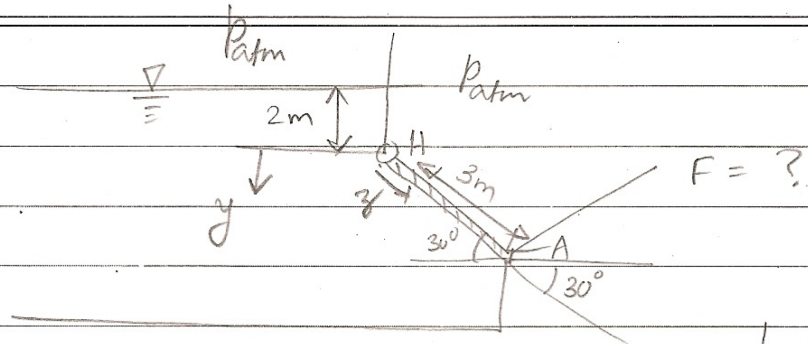
ESO 212 I Midsem Solution key

(1)



$y = z \sin 30^\circ$

$y = \frac{z}{2}$

$$F_R = W \int_{z=0}^{z=3} (2 + y) \rho g \, dz$$

$$F_R = \rho g W \int_{z=0}^{z=3} \left(2 + \frac{z}{2}\right) dz$$

$$F_R = 10^3 \times 9.8 \times 10 \left[2z + \frac{z^2}{4} \right]_{z=0}^{z=3}$$

$$F_R = 10^4 \times 9.8 \times \left[6 + \frac{9}{4} \right]$$

$$\boxed{F_R = 808500 \text{ N}}$$

line of action of resultant force:

$$z' F_R = \int_{z=0}^{z=3} z \, p \, dz$$

$$z' F_R = \rho g w \int_{z=0}^{z=3} \left(2 + \frac{z}{2}\right) z \, dz$$

$$z' F_R = \rho g w \left[z^2 + \frac{z^3}{6} \right]_0^3$$

$$z' F_R = 10^3 \times 10 \times 9.8 \left[9 + \frac{27}{6} \right]$$

$$z' \times 808500 = 10^4 \times 9.8 \left[13.5 \right]$$

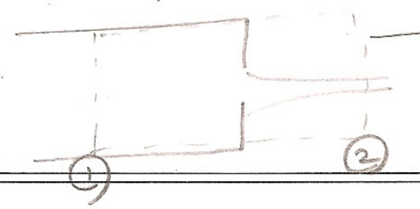
$$\Rightarrow \boxed{z' = 1.636 \, m}$$ —— (2) points

Moment about the hinge H.

$$z' F_R = F \times 3$$

$$\Rightarrow F = \frac{1.636 \times 808500}{3}$$

$$\boxed{F = 441 \, kN}$$



(2)

$$\dot{Q} = 20\ \frac{kg}{s} = \frac{20}{1000}\ kg \times \frac{m^3}{kg} \times \frac{1}{s}$$

$$\dot{Q} = 0.02\ m^3/s$$

$$\text{Area } A_1 = \frac{\pi}{4} \times 0.12^2$$

$$V_1 = \frac{0.02}{\frac{\pi}{4} \times 0.12^2} = 1.77\ m/s$$

mass balance:

$$V_2 \times \frac{\pi}{4} \times 0.025^2 = 0.02\ m^3/s$$

$$\Rightarrow V_2 = 40.74\ m/s$$

mom. balance:

$$F_x + 800 \times 10^3 \times \left(\frac{\pi}{4} \times 0.12^2\right)$$

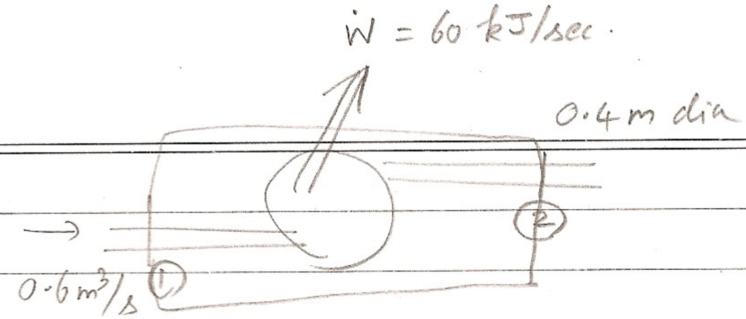
$$= -20 \times 1.77$$

$$+ 20 \times 40.74$$

$$\Rightarrow \boxed{F_x = -8268.4N}$$

$\Rightarrow$ force must be applied in the −ve x direction.

(3)



$z_1 \approx z_2$

energy balance: $\frac{p_1}{\rho} + \frac{V_1^2}{2} + z_1 g = \frac{p_2}{\rho} + \frac{V_2^2}{2} + z_2 g$

$+ W_{shaft} + W_{loss}$

neglect

$V_1 A_1 = V_2 A_2$ $\qquad W_{shaft} = \frac{\dot{W}_s}{\dot{m}}$

$V_2 = V_1 \frac{A_1}{A_2}$ $\qquad = \frac{\dot{W}_s}{\rho \dot{Q}}$ $\qquad V_1 = \frac{0.6}{\frac{\pi}{4} 0.3^2}$

$V_2 = V_1 \frac{D_1^2}{D_2^2}$

$V_1 = 8.48$ m/s

$$\frac{p_1 - p_2}{\rho} = \frac{\dot{W}_s}{\rho \dot{Q}} + \frac{1}{2}(V_2^2 - V_1^2)$$

$$(p_1 - p_2) = \frac{\dot{W}_s}{\dot{Q}} + \frac{1}{2} \rho V_1^2 \left[\frac{D_1^4}{D_2^4} - 1\right]$$

$$= \frac{60 \times 10^3}{0.6} + \frac{1}{2} 10^3 \left[\frac{0.3^4}{0.4^4} - 1\right] (8.48)^2$$

$(p_1 - p_2) = 75421$ Pa $=$ **75.42 kPa**