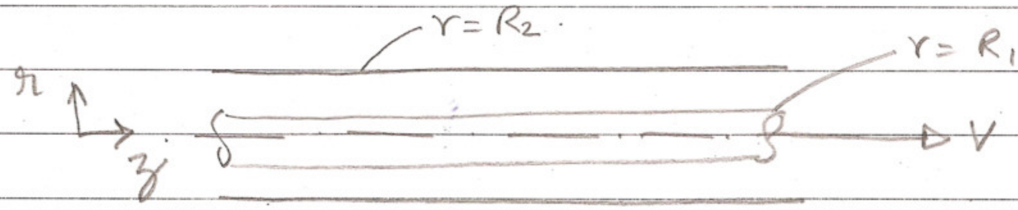
ESO212 II Midsem Solution Key.

(1)



given $\dfrac{dp}{dz} = 0$; Steady, fully-developed.

From the given conditions ⇒ flow is axisymmetric.

$V_\theta = 0$. $\dfrac{\partial V_z}{\partial z} = 0$ ⇒ continuity ⇒ $V_r = 0$

so $V_z = V_z(r)$.

z-mom eqn in cyl. coordinates

$$\rho\left[\underbrace{\cancel{\frac{\partial V_z}{\partial t}}}_{\text{Steady}\to 0} + \underbrace{\cancel{V_r \frac{\partial V_z}{\partial r}}}_{\text{from conty}\to 0} + \underbrace{\cancel{\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}}}_{\text{axisymm}\to 0} + \underbrace{\cancel{V_z \frac{\partial V_z}{\partial z}}}_{\text{fully develop}\to 0}\right]$$

$$= \underbrace{-\cancel{\frac{\partial p}{\partial z}}}_{0} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}}}_{0} + \underbrace{\cancel{\frac{\partial^2 V_z}{\partial z^2}}}_{0}\right]$$

given P_0 in both chambers

axisymmetry

fully developed

$$\Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial V_z}{\partial r}\right] = 0$$

$$\frac{\partial}{\partial r}\left[r\frac{\partial V_z}{\partial r}\right] = 0 \quad \Rightarrow \quad r\frac{\partial V_z}{\partial r} = C_1$$

$$\frac{\partial V_z}{\partial r} = \frac{C_1}{r} \quad \Rightarrow \quad V_z = C_1 \ln r + C_2$$

(2)

B.C :

$$V_z(r=R_1) = V$$
$$V_z(r=R_2) = 0.$$

$$V = C_1 \ln R_1 + C_2$$
$$0 = C_1 \ln R_2 + C_2$$

Subtract $V = C_1 \ln R_1 - C_1 \ln R_2$

$$V = C_1 \ln \frac{R_1}{R_2} \Rightarrow C_1 = \frac{V}{\ln R_1/R_2}$$

$$C_2 = -C_1 \ln R_2$$

$$\Rightarrow C_2 = -\frac{\ln R_2 \; V}{\ln R_1/R_2}$$

$$V_z = C_1 \ln r + C_2$$

$$= \frac{V}{\ln \frac{R_1}{R_2}} \ln r - \frac{V \ln R_2}{\ln R_1/R_2}$$

2

$$\boxed{V_z(r) = \frac{V}{\ln R_1/R_2} \ln\left(\frac{r}{R_2}\right)}$$

4 points

shear stress exerted by the fluid on the wire

$$\tau_{rz} = \mu \left.\frac{\partial v_z}{\partial r}\right|_{r=R_1} \quad ; \quad \frac{\partial v_z}{\partial r} = \frac{C_1}{r} = \frac{V}{\ln \frac{R_1}{R_2}} \, \frac{1}{r}$$

$$= \frac{\mu V}{R_1} \, \frac{1}{\ln \frac{R_1}{R_2}}$$

$$\left.\frac{\partial v_z}{\partial r}\right|_{r=R_1} = \frac{V}{\ln \frac{R_1}{R_2}} \, \frac{1}{R_1}$$

force per unit length on the wire $= \tau_{rz} \cdot (2\pi R_1)$

$$\boxed{F = \frac{2\pi \mu V}{\ln \frac{R_1}{R_2}}} \quad \text{———} \quad \boxed{\text{3 points}}$$

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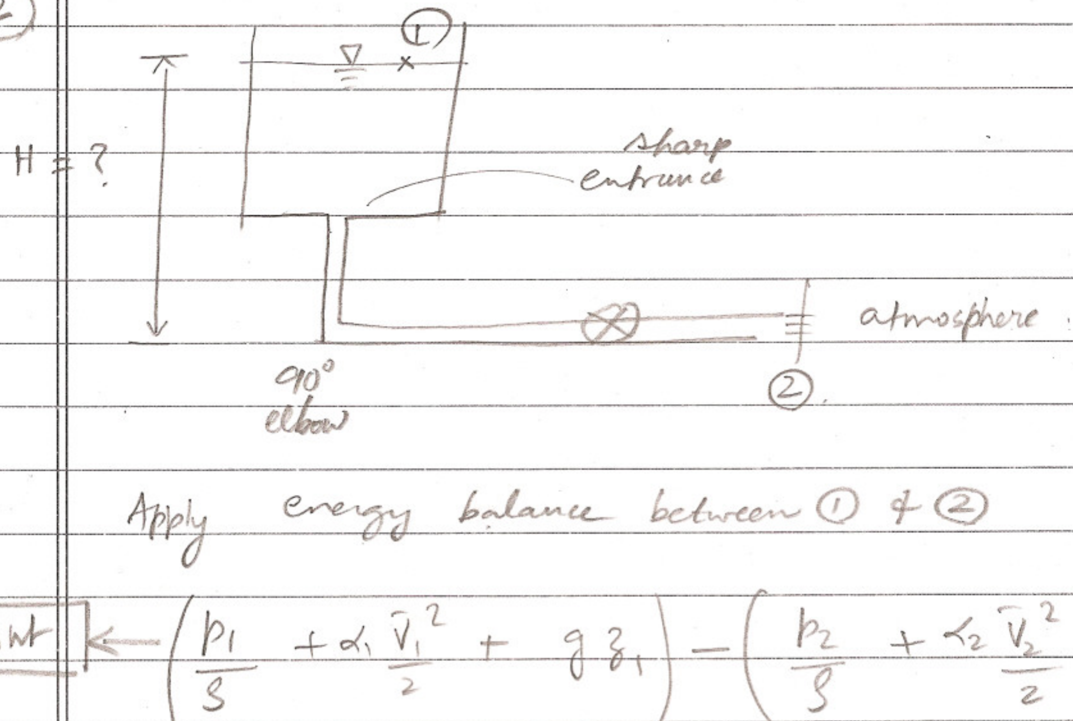
(2)



Apply energy balance between (1) & (2)

$$\boxed{\text{1 point}} \longleftarrow \left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right)$$

$$= f \frac{L}{D} \frac{\bar{V}_2^2}{2} + \frac{\bar{V}_2^2}{2} \sum_i K_i$$

$\alpha \approx 1$; $\bar{V}_1 \simeq 0$; $P_1 = P_2 = P_{atm}$. $z_1 - z_2 = H$. } $\boxed{1 \text{ point}}$

$$H = \frac{\bar{V}_2^2}{2g} + f \frac{L}{D} \frac{\bar{V}_2^2}{2g} + \frac{\bar{V}_2^2}{2g} \left(0.5 + 0.9 + 0.3 \right)$$

$$\left. \begin{array}{l} Re = 10^5 \\ D = 5\times10^{-2}\,m \\ \rho = 10^3\ kg/m^3 \\ \mu = 10^{-3}\ kg/m^3 \end{array} \right\} \Rightarrow 10^5 = \frac{5\times10^{-2} \times 10^3 \times \bar{V}_2}{10^{-3}}$$

$\Rightarrow$

$\bar{V}_2 = 2\ m/s$. — $\boxed{1 \text{ point}}$

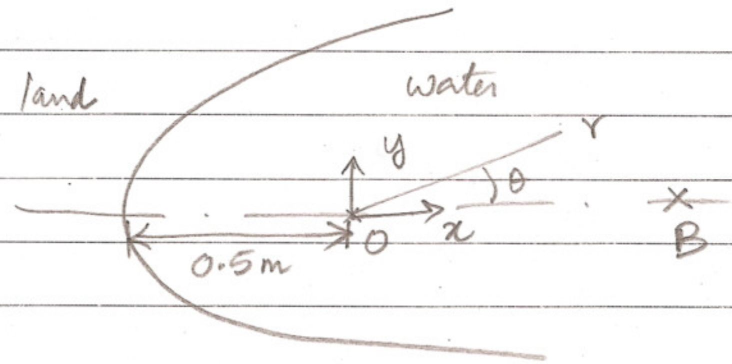
for $Re = 10^5$, $f = 0.018$ — $\boxed{1 \text{ point}}$
(smooth pipe)

$$H = \frac{\bar{V}_2^2}{2g} \left[1 + 0.018 \times \frac{1}{5\times10^{-2}} + 0.5 + 0.9 + 0.3 \right]$$

$$H = \frac{4}{2\times9.8} \times 3.06$$

$\boxed{H = 0.624\ m}$ — $\boxed{2 \text{ points}}$

③



To get a Rankine half-body $\Rightarrow$ superposition of a source & uniform flow

$\boxed{\text{1 point}}$

$$\psi = Uy + m\theta = Uy + m\tan^{-1}\left(\frac{y}{x}\right).$$

$$\psi = Ur\sin\theta + m\theta. \qquad ; \; r = \sqrt{x^2+y^2}$$

$\boxed{\text{1 point}}$

$$u = \frac{\partial \psi}{\partial y} = U + m\frac{x}{x^2+y^2} = U + \frac{m}{r}\cdot\frac{x}{r}$$

$$u = U + \frac{m}{r}\cos\theta.$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{m}{r}\sin\theta$$

<u>Stagnation pt</u>: ("nose" of the half body): $u = 0, v = 0.$

$\boxed{\text{1 point}}$

$\Rightarrow \theta = \pi, \; r = \dfrac{m}{U}$; let $a = m/U$.

(or)

$x = -m/U, \; y = 0.$

given $a = 0.5\ m \Rightarrow \frac{m}{U} = 0.5$

$\boxed{1 \text{ point}}$ $m = \frac{Q}{2\pi \cdot b} = \frac{0.35}{2\pi\ (1)} = 0.0557\ m^2/s$

$\boxed{1 \text{ point}}$ $U = \frac{m}{a} = \frac{0.0557}{0.5} = 0.1114\ m/s$

given $V_B = 0.25\ \frac{m}{s} = U + \frac{m}{r_{OB}} \underbrace{\cos 0}_{1}$

$\Rightarrow 0.25 = 0.1114 + \frac{0.0557}{r_{OB}}$

$\boxed{2 \text{ points}}$ $\Rightarrow \boxed{r_{OB} = 0.4\ m}$

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