ESO212 Fluid Mechanics & Rate Processes **July-Nov 2010**

II Mid-semester Exam **1 hour; 20 points**

Useful Information

- $g = 9.8$ m/s^2, density of water = 10^3 kg/m^3, viscosity of water = 10^{-3} Pa s.
- Kinetic energy correction factor $\alpha = 2$ for laminar flow, $\alpha = 1$ for turbulent flow in pipes.
- $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
- Loss coefficient K for: entrance = 0.5, elbow (90°) = 0.9, gate valve (fully open) = 0.3, gate valve (half open) = 5.

Clearly state all the assumptions you make

1. An infinitely long cylindrical wire of radius R_1 is pulled (with constant velocity V) through a circular cylindrical tube of radius R_2 filled with a liquid of viscosity μ and density ρ, as shown in figure 1. The wire is placed coaxially in the tube, i.e. their axes of revolution coincide. The cylindrical tube connects two large reservoirs of the liquid maintained at a constant pressure P_0. Determine:

   (a) The steady velocity profile in the fully-developed region of the cylindrical tube, neglecting entrance and exit effects. Give brief reasons for why certain terms are neglected in the Navier-Stokes equations (see over-leaf for Navier-Stokes equations in cylindrical coordinates). [4 points]

   (b) The force per unit length required to pull the wire through the cylindrical tube. (Neglect contributions to the force from the liquid in the reservoirs.) [3 points]

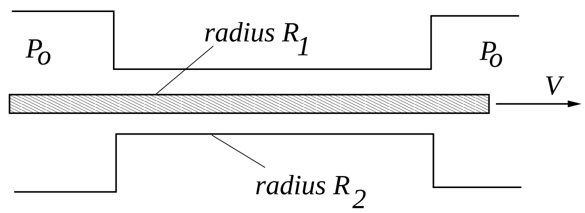

Figure 1: **Problem 1**

2. Consider a water tank which is connected to a pipe system as shown in figure 2. The pipe walls are *smooth*, and the pipe system has a sharp-edged entrance, one 90° elbow, and a fully-open gate valve. Water exits from the pipe to atmospheric pressure. The pipe diameter is 5 cm, and the total length of the pipe is 1 m. Calculate the minimum height H of the water in the tank above the pipe system discharge (H shown in the figure), such that the Reynolds number of flow in the pipe is 10^5. [6 points]

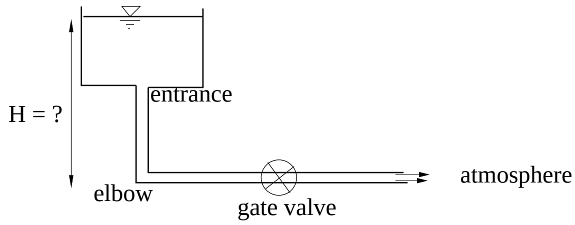

Figure 2: **Problem 2**

3. A swimming pool is approximated by a Rankine half-body shape as shown in figure 3. At point O, which is 0.5 m from the left edge of the swimming pool, there is a line source (into the paper) of water delivering 0.35 m^3/s per meter of depth into the paper. Assuming potential flow, find the coordinates of point B along the axis where the water velocity is 25 cm/s. [7 points]

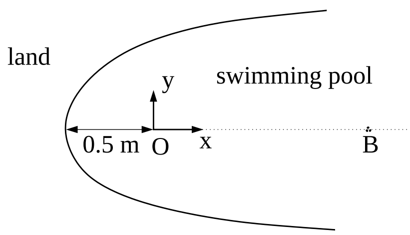

Figure 3: **Problem 3**

Navier-Stokes equations in cylindrical coordinates:

Continuity:

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$

r-momentum:

$$\rho\left[\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right]=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]+\rho g_r$$

z-momentum:

$$\rho\left[\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right]=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

θ-momentum:

$$\rho\left[\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_\theta v_r}{r}\right]=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]+\rho g$$